Fourdimensional Universe where Time is Taken as a Length, and Dark Energy

Nathalie Olivi-Tran

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Nathalie Olivi-Tran
Laboratoire Charles Coulomb,
Universite de Montpellier,
UMR CNRS 5221, place E.Bataillon,
34095 Montpellier Cedex 05, France
email: nolivi@univ-montp2.fr,
nathalie.olivi-tran@umontpellier.fr

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Abstract

If we use the Friedmann-Lemaitre-Robertson-Walker model for a vacuum dominated universe, we see that time may be a function of a length, the so called co-moving distance. In the FLRW model, this function is injective, so for each time t there is one co moving distance $a(t)$. Straightforwardly, we can assume that time has the dimension of a length, even if, in a fourdimensional universe, the four dimensions are not isotropic. Taking account of this hypothesis, we can solve the EPR paradox for entangled states. Moreover, a simple model of antimatter can also be made: antimatter deforms the universe inwards while matter deforms the universe outwards. Dark matter is also deduced to be a track of massive moving objects within the local curvature of the universe. And finally, taking account that our universe is fourdimensional and that time may be the fourth dimension and has the dimension of a length, the mystery of dark energy is solved. Indeed, in a fourdimensional universe, what we call dark energy has a positive pressure which comes naturally from the expansion of the universe.

1 Introduction

Up to now, time has been considered as a special dimension. But, noone thought of linking time with a spatial dimension. In this book chapter, we will see that it is easy to show that time is a function of the cosmological comoving distance. This can be made by using the Friedmann-Lemaitre-Robertson-Walker model. So, one can consider that we are in a curved three dimensional (with three spatial dimensions) universe. Straightforwardly, we can embbed this curved three dimensional universe in a fourdimensional euclidian space.

Once we have shown that time has the dimension of a length, several paradoxes of physics find their solutions. The Heisenberg uncertainty principle is shown to be only an approximation, which was valid at the beginning of the universe. The EPR paradox is also solved: entangled state are only transient. Antimatter has a temporal dimension which is negative. The accelerated expansion of the universe is explained by the fact that time is a logarithmic function of the comoving distance. And finally, dark matter is the track of massive object in the local curvature of the universe and dark energy is positive if we consider that our threedimensional universe is embedded in a fourdimensional euclidian space.

2 Time and comoving distance

2.1 The FLRW model

One of the most studied cosmological model is the FLRW (Friedmann- Lemaitre-Robertson-Walker) cosmological model. In this model, the metric may be written [1]:

$$ds^2 = c^2dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi)] \tag{1}$$

where c is the speed of light, t the time, k the local curvature and r, θ and ϕ define an appropriate system of coordinates. Therefore, the quantity $a(t)$ represents the cosmic expansion factor and gives the rate at which two points of fixed comoving coordinates (r_1, θ_1, ϕ_1) and (r_2, θ_2, ϕ_2) increase their mutual distance as $a(t)$ increases [1]. By solving Einstein's equations for the FLRW metric given by equation (1), one may find out the time dependence of $a(t)$. Indeed, if the matter content of the Universe can be described by a perfect fluid, such equations reduce to the system of two equations [1]:

$$(\frac{\frac{da}{dt}}{a})^2 = H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2} \tag{2}$$

$$-(\frac{\frac{d^2a}{dt^2}}{a}) = \frac{4\pi G}{3}(\rho + 3p) \tag{3}$$

where G is the gravitational constant, ρ is the density of the fluid, Λ is the cosmological constant, H the Hubble parameter and p is the pressure of the fluid, in the case of a vacuum dominated universe. The aim of this study is to relate the local curvature to the local time. This may be obtained using equation (2):

$$dt = \frac{da}{Ha} \tag{4}$$

with

$$a^2(t) = -k(H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3})^{-1} \tag{5}$$

therefore the relation of the local time with the local curvature may be written:

$$t = \frac{lna}{H} = \frac{1}{2H}ln(-k(H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3})^{-1}) \tag{6}$$

Therefore, the behavior of the local time t as a function of the local curvature k depends on the respective values of H, G and Λ. If we analyze equation (6), we see that the local time t cannot be equal to zero except for the special case where $H^2 - \frac{8\pi G}{3}\rho - \frac{\Lambda}{3}$ and $k = -1$. In this last case, the local time is always equal to zero. This is unphysical although it may happen in the FLRW model for the critical density $\rho_c = 3H^2/8\pi G$ and in the absence of a cosmological constant term. More explicitly, if we use equation (4) by integrating it on given domain it leads to :

$$\int_{t=0}^{t=t'} dt = \int_{a(t=0)}^{a(t=t')} \frac{da}{Ha} da \tag{7}$$

which gives:

$$t' = \frac{1}{2H}\ln(-k(H^2 - \frac{8\pi G}{3}\rho(t') - \frac{\Lambda}{3})^{-1}) \tag{8}$$
$$-\frac{1}{2H}\ln(-k(H^2 - \frac{8\pi G}{3}\rho(t=0) - \frac{\Lambda}{3})^{-1})$$

Here again the same analysis as for equation (6) applies. Moreover, the difference between the two parts of the right hand side of equation (8) cannot be equal to zero except for $\rho(t) = \rho(t = 0)$ and $k(t) = k(t = 0)$ which is once again a particular case. Therefore, in the general case, t cannot be equal to zero. We can interpret the fact that the coordinates t, r, θ and ϕ which define a local referential may be replaced by the coordinates k, r, θ and ϕ which are also define a local referential. Physically, it means that the three dimensional curved space represented by the three coordinates r, θ, ϕ is embedded in a four dimensional space which mean curvature is not known but which fourth dimension is represented by the coordinate k. As k is the local curvature it may be related to the local radius of curvature. This means that because of the relation between time t and k given by equation (6), the irreversibility of time is no more intrinsic to it but only due to the expansion of the universe. Now let us make the hypothesis that the definition of local time in the FLRW cosmological model is the same as the definition of time in quantum mechanics. The previous paragraph states that time t cannot be equal to zero except for a critical density.

2.2 The Heisenberg uncertainty principle

If we place ourselves in the general case where time cannot be equal to zero and depends on the local curvature, let us begin the demonstration which usually leads to the uncertainty principle of Heisenberg [2]:

$$\psi(\mathbf{r}, t) = \frac{1}{(2\pi)^3/2} \int g(\mathbf{k}) e^{i[\mathbf{k_w}.\mathbf{r} - \omega(k_w)t]} d^3k_w \tag{9}$$

which may be simplified in one dimension into:

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(k_w) e^{i[k_w x - (\hbar k_w^2/2m)t]} dk_w \tag{10}$$

where kw is the wavenumber, $\psi(r, t)$ is the wavefunction of the particle of mass m, is Planck's constant, The demonstration made in order to obtain the Heisenberg uncertainty principle makes the hypothesis that $t = 0$. This allows one to simplify this last equation into a simple Fourier transform. But, as obtained in equation (6), t has a non zero value and cannot be equal to zero except in very special cases. Therefore, the simplification which leads to a simple Fourier transform in the demonstration of the uncertainty principle of Heisenberg [2] reduces to an approximation. Indeed, the demonstration remains valid with the approximation that:

$$(\hbar k^2/2m)t << kx \tag{11}$$

The numerical value of is equal to $1.054589.10^{-34} J.s.$ So the approximation is almost always valid. This is in good agreement with all the experiments which have been made in quantum mechanics. As a conclusion, we can say that the local time can be related to the local curvature in the FLRW cosmological model, and therefore that the local time cannot be equal to zero except in very special cases of the values of the constants which enter the FLRW model. This means also that our curved universe may be seen as embedded in a four dimensional space where the fourth local coordinate is the local radius of curvature. Using the fact that local time cannot be equal to zero, we showed also that the demonstration of the uncertainty principle of Heisenberg is only a very good approximation. Indeed, the demonstration of the uncertainty principle of Heisenberg is not as simple as a Fourier transform.

3 The EPR paradox and four dimensions

Consider as above systems A and B each with a Hilbert space HA , HB . Let the state of the composite system be:

$$|\Psi >_A \in H_A \otimes H_B \tag{12}$$

In general there is no way to associate a pure state to the component system A. However, it still is possible to associate a density matrix. Let

$$\rho_T = |\Psi >< \Psi| \tag{13}$$

which is the projection operator onto this state. The state of A is the partial trace of ρ_T over the basis of system B:

$$\rho_A \overset{\text{def}}{=} \sum_j < j|_B (|\Psi >< \Psi|)|j >_B = \text{Tr}_B \rho_T \tag{14}$$

ρ_A is sometimes called the reduced density matrix of ρ on subsystem A .Colloquially, we trace out system B to obtain the reduced density matrix on A. For example, the density matrix of A for the entangled state discussed above is

$$\rho_A = (1/2)(|0>_A < 0|_A + |1>_A < 1|_A) \tag{15}$$

This demonstrates that, as expected, the reduced density matrix for an entangled pure ensemble is a mixed ensemble. Also not surprisingly, the density matrix of A for the pure product state $|\psi>_A \otimes |\phi>_B$ discussed above is

$$\rho_A = |\psi>_A < \psi|_A. \tag{16}$$

In general, a bipartite pure state ρ is entangled if and only if one, meaning both, of its reduced states are mixed states.

Now, we make the hypothesis (as shown in references [3, 4]) that our three dimensional curved universe is embedded in four dimensional space with no curvature: time has the dimension of a length and is related to the curvature of the universe at the point of measurement. Therefore it is possible to apply to the four dimensional space an orthonormal basis (x, y, z, t) which is a 'universal' referential in the four dimensional space. Each of these coordinates are related to time and to the three coordinates of space in our three dimensional universe by making a change of referential. We showed also that the Heisenberg uncertainty principle is only an approximation because time cannot be equal to zero except at the beginning of constitution of the universe: the Big Bang. Therefore it is possible to make a simultaneous measure in two locations of our universe (taking into account that we defined a universal referential).

Consequently, equation (14) becomes:

$$\rho_A = \sum_{j_t} < j_t|_B(|\Psi >< \Psi|)|j_t >_B \tag{17}$$

$$\rho_B = \sum_{j_{t+\Delta t}} < j_{t+\Delta t}|_A(|\Psi >< \Psi|)|j_{t+\Delta t} >_A \tag{18}$$

$|j_t>$ may be written $\exp(-it)$ and $|j_{t+\Delta t}>$ may be written $\exp(-i(t+\Delta t))$ because at different locations the time is different thus the base vectors are different. So, if time evolves differently for two entangled states separated by space in our three dimensional universe, the density matrix of each state will have different evolutions. As a conclusion, we can say that entanglement for entangled states in different spatial locations is only transient.

We showed in previous articles [3, 4] that time is a function of curvature. With this assumption and by making a dimensional analysis of Einstein's Fields equations we found that time had the dimension of a length.

Therefore, our three dimensional curved universe may be embedded in a four dimensional universe with no curvature and moreover it possible to define an 'universal' referential. With this referential, as it is universal, two events may be simultaneous. Moreover, we showed that Heisenberg's uncertainty principle is only an approximation, so it is also possible to measure exactly location and impulsion of a given object at the same time. With all this hypotheses, all deriving from the fact that our three dimensional universe is embedded in a four dimensional space with an 'universal' referential we showed here that for two entangled states separated by space (at two different locations of the four dimensional space (x, y, z, t) and (x', y', z', t') their state 'entangled' is only transient as time evolves differently for the two states. Indeed one cannot neglect time in the case of entanglement: time is related to the local curvature of our three dimensional universe (and in the local referential of our universe) and this time differs for two different locations due to the fluctuations of the curvature of space due to the presence of objects with mass (and to the non homogeneous character of the universe). The final conclusion is that entanglement for particles separated by space in our three dimensional universe, is only transient. Therefore, if one measures at time t an entangled particle the corresponding other entangled particle, at time $t + \Delta t$ will not follow the two states law for entanglement because of the difference of time evolution at the two different locations.

4 Expansion of universe and four dimensions

I make here the hypothesis that our three dimensional universe with physical laws is embedded in a fourdimensional space where no physical law exist. This is consistent with my former publication [3, 4]. Presently, the measure of the rate of expansion of our universe shows that its inflation is accelerating. But our theory [3, 4, 5] showed that the time measured in our universe is a function of the comoving distance at the location where measurement is made; moreover this function is a logarithmic one. We will show in this paper that the only fact that time is a logarithmic function of the comoving distance implies that the measured rate of expansion is accelerating. Black holes leads to a leak of matter and light leading to a instantaneous negative pressure within the universe and therefore solving the paradox of an inflating universe and a negative pressure given by the Friedmann equation. But now, another question is arising: how can the universe expand? what is the 'force' which makes it expand? We will also show here that Heisenberg's uncertainty principle is a possible explanation of the transition from a four dimensional space with no physical laws to our universe with all presently physical known laws. This explanation is almost the same as for the Big Bang.

I will work here within the Friedmann cosmological model. The scale factor $a(t)$ or cosmic scale factor of the Friedmann equations is a function of time which represents the relative expansion of the universe. It is the time dependent factor that relates the proper distance for a pair of objects moving with the Hubble flow in an expanding or contracting FLRW (Friedmann-Lemaitre- Robertson-Walker) universe.

The evolution of the scale factor is a dynamical question, determined by the equations of general relativity, which are represented in the case of a locally isotropic, locally homogeneous universe by the Friedmann equations. The Hubble parameter is defined by:

$$H = \frac{\dot{a}(t)}{a(t)} \tag{19}$$

where the dot represents a time derivative. Current evidence suggests that the expansion rate of the universe is accelerating which means that the first derivative of $a(t)$ is increasing over time. This implies also that any given galaxy recedes from us with increasing speed over time. The Friedmann scale factor relates the proper distance at an arbitrary time t to their distance at some reference time. If I make the hypothesis that our three dimensional universe is embedded in a four dimensional euclidean space, distances are no more relative because we may define an absolute referential (x, y, z, w) where w is the fourth coordinate. In a previous article by Olivi-Tran and Gauthier [3] we showed that time t is related to the local curvature k. In this case, time t may be written (see reference [3] for details of calculations):

$$t = \frac{1}{2H} \ln\left(\frac{k}{(-H^2 + \frac{8\pi G}{3}\rho + \frac{\Lambda}{3})}\right) \tag{20}$$

where G is the gravitational constant, ρ is the density (energy) of the fluid which increases [6], Λ is the cosmological constant, H the Hubble parameter. Because H is at square, time t increases but with a logarithmic behavior. Therefore it is obvious that, as time t has a logarithmic behavior, that it is time which will decelerate. Straightforwardly, in the four dimensional space defined above, distances may be absolute. As a consequence an absolute distance divided by a decelerating time t leads to an appearing acceleration of the expansion of universe. The fact that the Friedmann equation:

$$-(\frac{\frac{d^2a}{dt^2}}{a}) = \frac{4\pi G}{3}(\rho + 3p) \tag{21}$$

implies that there is an apparent negative pressure p in our universe may be explained by the presence of black holes. Let us make the hypothesis that black holes are real 'holes' within the hypersurface of our universe with boundaries oriented outside the universe (see for that reference [8]). Therefore, black holes will have a very large mass [6] and will attract light and matter (because of the curvature of their boundaries). Moreover, because the hypersurface of our universe is 'broken', black holes will make disappear matter and light that they attract because they are in contact with the fourdimensional space with no physical laws. Therefore, there is a leak of matter and energy which implies an apparent negative instantaneous pressure. I have to add that paradoxically, the energy within our universe increases: see for that the paper by Darias and Olivi-Tran [6]

But how can the universe expand itself ? Black holes, following my theory, make physical laws disappear (see above). But, up to now, there have been no publications on how can the outside of our universe (where no physical laws exist, following my theory) transform into a part of our universe. To help to understand that, a very simple way is Heisenberg's uncertainty principle (which is only strictly valid if time $t = 0$ [3], i.e. outside our universe):

$$\Delta E \Delta t >= \hbar/2 \tag{22}$$

where E is the energy and t the time. At the boundaries of our universe, if we take the limit from outside to the boundaries, time is equal to zero. So equation (22) is valid. Therefore, E will grow to ∞; but at the very moment where the limit of the boundaries is reached, equation (22) is no more valid,t will be a function of the comoving distance and E will take a finite value. The same phenomenon could have happened at the time $t = 0$ of the universe, i.e. the Big Bang, except that we do not know why the uncertainty principle becomes valid, maybe a perturbation in the still fourdimensional space in which our three dimensional universe has grown?

I showed here that the acceleration of the expansion of the universe is due to the logarithmic increase of time as a function of the radius of curvature of our universe. Apparent pressure is negative , as predicted by Friedmann's equation because there is a leak of matter and light due to black holes. The transition from a fourdimensional space with no physical laws to a three dimensional curved universe which is embedded in the first, is due to Heisenberg's uncertainty principle.

5 Antimatter and four dimensions

So the simple fact that we consider that our three dimensional universe is embedded in a fourdimensional space leads to all the previous results. Another of our previous published article [6], showed that spherical cups placed on a sphere see their kinetic energy increase because of the straight character of the instantaneous velocity vector (tangential to the sphere). In fact, if you look at Fig. 4 in this article [6], you see that the resulting velocity vector at the very moment of the interaction between two cups is oriented from the center of the sphere towards the outside of the sphere. This means, if we generalize this twodimensional calculation to a three dimensional hypersphere, that two particles interacting within the hypersphere will see their velocity vector oriented outwards the sphere like if they tried to escape from the threedimensional hypersphere. A direct conclusion of this extrapolation from a two dimensional sphere to a three dimensional sphere may that our universe (which is inflating) is concave (positive curvature) and that its inflation is due to the interaction of 'particles' at its surface. Now let us imagine that we have a particle made of classical matter. The previous reasoning make us think that the deformation of the hypersurface of the universe is concave (the radius of curvature is directed outwards the universe: positive curvature).

Feynman several decades ago [7] , proposed that antimatter particles were particles travelling back in time. This is fully consistent with my theory of time being related to the radius of curvature (local or of the whole universe): antimatter particles have the same mass as their associated matter particles (see for that ref. where the mass given by the Higgs scalar field is a function of T^2 where T is time and is related to the local curvature [8]). But the deformation due to antimatter is locally convex. These facts explains:

- that a convex location (antiparticle) and a concave location (particle) which interact will disappear both

- that a convex location (antiparticle) will have a very short life time because of the inflation of universe

I think that many representations of massive objects within our universe lead to misunderstanding of this concave character of the deformation of the surface of our universe.

As a conclusion, I shall say that antimatter deforms the universe inwards (negative curvature) and matter outwards (positive curvature). A previous calculation in two dimensions let us think that our universe is concave. But the question whether the universe is open or closed remains; with a small own preference for a closed universe (what would exist without an hypersurface? the fourdimensional space; but can we mix both?).

6 Dark matter and four dimensions

In astronomy and cosmology, dark matter is a type of matter hypothesized to account for a large part of the total mass in the universe. Dark matter cannot be seen directly with telescopes; evidently it neither emits nor absorbs light or other electromagnetic radiation at any significant level [10]. Instead, its existence and properties are inferred from its gravitational effects on visible matter, radiation, and the large scale structure of the universe. Dark matter is estimated to constitute 84% of the matter in the universe and 23% of the mass-energy. Dark matter came to the attention of astrophysicists due to discrepancies between the mass of large astronomical objects determined from their gravitational effects, and mass calculated from the 'luminous matter' they contain; such as stars, gas and dust. Subsequently, other observations have indicated the presence of dark matter in the universe, including the rotational speeds of galaxies, gravitational lensing of background objects by galaxy clusters, and the temperature distribution of hot gas in galaxies and clusters of galaxies. The model of dark matter that I present here is based on the fluctuations of the hyper-surface of our universe. Our universe is three dimensional and curved, therefore it may be embedded in a four dimensional space with no curvature. The hyper-surface of our universe is superimposed to our present universe. But this hyper-surface is not 'smooth', this means that it may contain fluctuations. These fluctuations are due to real matter (due to the changes of curvature due to gravitation) but also to dark matter (or at least 'are' dark matter). Indeed, the local changes in the curvature of our universe may lead to what we call dark matter. This dark matter, although it is only a local deformation of the hyper-surface of our universe, may deviate radiations and can account for gravitational effects. In mathematical terms, Heisenberg's uncertainty principle writes (which is The possible nature of dark energy and dark matter 1061 only strictly valid if time $t = 0$ [3], i.e. outside our universe):

$$\Delta E \Delta t >= /2 \tag{23}$$

where E is the energy and t the time. At the boundaries of our universe, if we take the limit from outside to the boundaries, time is equal to zero. So equation (23) is valid. Therefore, E will grow to ∞; but at the very moment where the limit of the boundaries is reached, equation (23) is no more valid, t will be a function of the radius of curvature of the universe [3] and E will take a finite value. The fact that t takes locally a finite value but that this value is random leads to local deformations of the hyper-surface of our universe (i.e. of our universe itself, see above). These deformations evolve with the expansion of the universe, thus there are not real matter, they only have gravitational effects (due to curvature) and deviate radiations (which follow the curvature of our universe).

7 Dark energy and four dimensions

The question which is arising since the observation of the acceleration of the expansion of the universe is: do we really have to revise the Cosmological Standard Model? Indeed, type Ia Supernovae (SNeIa), anisotropies in the cosmic microwave background radiation (CMBR), and matter power spectrum inferred from large galaxy surveys suggest a revision of this model. Up to now, the most commonly accepted theory implies that cold dark matter (CDM) represents 25.8% of the universe matter content and dark energy 69.4% of the energy content (last Planck's results). This theory is explained within the ΛCDM model. But, a model would be to analyze the behavior of time during the expansion of the universe. Previous publications suggest that time could be a function of lengths: indeed, by making a simple dimensional analysis of Einstein's field equations, one finds that the dimension of time equals the dimension of a length. A possible explanation of the acceleration of the expansion of the universe would be to consider cosmic time as a function of the comoving distance.

Let us make the corresponding calculation within the frame of the Roberston Walker model in the case of the homogeneous vacuum dominated universe. The two equations (Friedmann equations) in this model are:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho \tag{24}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{25}$$

The cosmological constant term can be omitted if we make the following replacement

$$\rho \to \rho + \frac{\Lambda c^2}{8\pi G} \tag{26}$$

$$p \to p - \frac{\Lambda c^4}{8\pi G} \tag{27}$$

Therefore the cosmological constant can be interpreted as arising from a form of energy which has negative pressure, equal in magnitude to its (positive) energy density:

$$p = -\rho c^2 \tag{28}$$

Such form of energy generalization of the notion of a cosmological constant is known as dark energy. In fact, in order to get a term which causes an acceleration of the universe expansion, it is enough to have a scalar field which satisfies

$$p < -\frac{\rho c^2}{3} \tag{29}$$

Another theory to explain equation (29) is to analyze the pressure p.

I make here the hypothesis that our three dimensional universe with physical laws is embedded in a fourdimensional space where no physical law exist. This is consistent with my former publication [3, 5]. In the Friedmann model, the pressure p is negative. But the dimensions of p are

$$f/r^2 \tag{30}$$

where f is a force (the force due to the energy of vacuum) and r^2 is a twodimensional surface. If our universe is threedimensional, the pressure would write

$$f/r^3 \tag{31}$$

where r^3 is the hypersurface of our universe (which corresponds to the universe at time t). To obtain (30), one has to integrate (31) over r, which leads to a negative pressure. Indeed, the relation between the pressure p and what we call the force density (the force f divided by a threedimensional surface r^3) is:

$$-2\frac{f}{r^2} = \int \frac{f}{r^3}dr \tag{32}$$

So, if the pressure $p = f/r^2$ calculated within the Friedmann equations is negative (in the case of a vacuum dominated universe), the corresponding force density f/r^3 is positive. This seems logical as the energy of vacuum is positive and the force necessary to accelerate the expansion of the universe has to be applied on a threedimensional hypersurface. Indeed, our universe is threedimensional and curved. The hypersurface of the universe corresponds to the universe at a given comoving time t. The 'force' of the vacuum can only account for the acceleration of the expansion of the universe if applied to its hypersurface (which is threedimensional). In conclusion the pressure $p = f/r^2$ is negative but the force density f/r^3 is positive.

The fact that the pressure p in the second Friedmann equation is negative can be explained by the dimensions of p itself. Indeed, p is a force divided by a twodimensional surface. While only a force divided by a threedimensional surface (a force density) can account for the acceleration of the expansion of the universe. And that because the hypersurface of our universe is threedimensional and curved. The relation between the pressure $p = f/r^2$ and the force density f/r^3 is a simple derivation.

8 Conclusion

As you can see by reading this book chapter, the only fact that time has the dimension of a length and that the threedimensional curved universe may be embedded in a fourdimensional euclidian space solves many paradoxes of physics. Indeed, the EPR paradox, the nature of antimatter, the accelerated expansion of the universe, dark matter and dark energy are explained within this frame. Now, it may be possible to apply this theory to particle physics. It is possible that the spatial nature of time could explain other mysteries of physics than those of cosmological physics [11, 12, 13].

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