The Bearable Compositeness of Leptons

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Outline

- Lepton observables in the Effective Field Theory language
  - precision, constraints, anomalies
- Strong dynamics at the multi-TeV scale:
  - Fermion partial compositeness
  - The resulting flavor structure
  - [Higgs compositeness]
- Confronting partial compositeness with lepton data:
  - neutrino masses
  - charged lepton flavor and CP violation

A model of leptons

Weinberg '67

$$\mathcal{L}_{lep} = \sum_{\alpha=e,\mu,\tau} \left[\overline{l_{L\alpha}} i\gamma^\mu D_\mu l_{L\alpha} + \overline{e_{R\alpha}} i\gamma^\mu D_\mu e_{R\alpha} - (y_\alpha \overline{l_{L\alpha}} H e_{R\alpha} + h.c.) \right]$$

- Standard Model (SM) symmetries:
  - ✳ $U(1)_e$ x $U(1)_{¥}$ x $U(1)_{□}$= $U(1)_L$ x orthogonal combinations
  - ✳ CP invariance
- Precise SM parameters: flavor-dependent masses m_e , $m_{¥}$, $m_{□}$, as well as flavor-universal gauge interactions, α , θ_w , G_F
- Yet, neutrino flavor eigenstates oscillate into one another
  - ✳ a striking effect, but induced by tiny masses: due to new physics very weakly mixed with the SM

    Super-Kamiokande '98
  - ✳ $U(1)_L$ and CP symmetries still resist to experimentalists (presently 2σ effect for CP violation)

New physics in effective operators

The SM is an effective theory valid up to scale ₴

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_{D=5} + \frac{1}{\Lambda^2}\mathcal{L}_{D=6} + \ldots$$

Effective description appropriate as long as the new degrees of freedom are heavier than the electroweak scale, v = 246 GeV

A unique D=5 operator, inducing neutrino masses

$$\frac{1}{\Lambda}\mathcal{L}_{D=5} = \frac{(m_\nu)_{\alpha\beta}}{v^2} l_{L\alpha} l_{L\beta} H H + h.c.$$

Weinberg '79

The D=5 operator can break all global symmetries: lepton number $U(1)_L$, lepton flavor numbers, and CP .

However, it is a very weak breaking :

$$\frac{v^2}{(m_\nu)_{\alpha\beta}} = 10^{15}\ \text{GeV}\ \frac{0.03\ \text{eV}}{(m_\nu)_{\alpha\beta}}$$

The hope is that some SM symmetries are broken at lower scales:

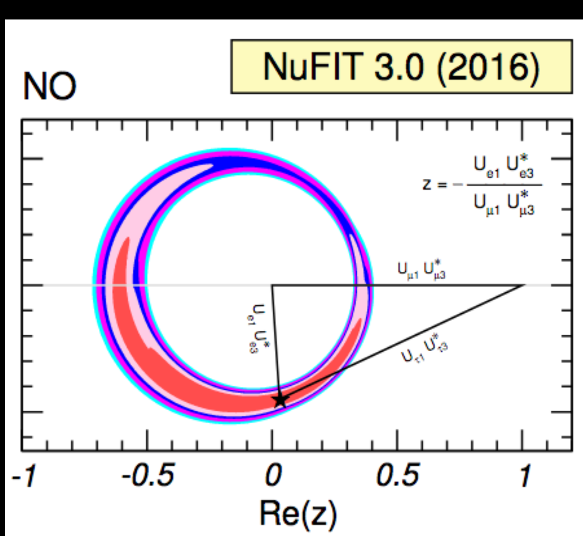
$\Lambda_{D=5} >> \Lambda_{D=6} \sim 10$ TeV

Neutrino oscillation data

Mass squared differences known precisely, up to one sign: $m_3 > m_{1,2}$ (Normal Ordering) or $m_3 < m_{1,2}$ (Inverted Ordering)

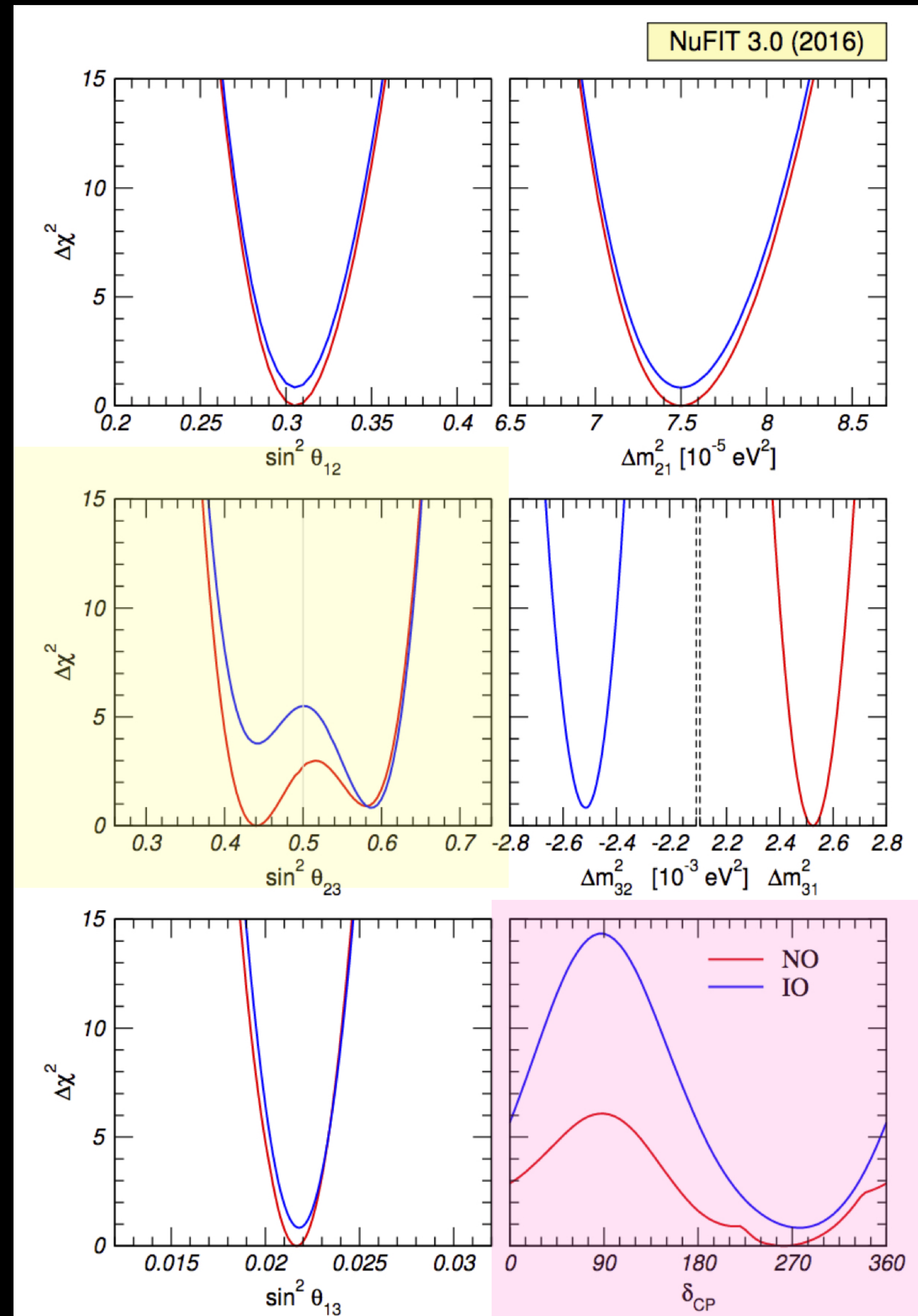
□$_{13}$ measured (in 2012, from reactor □'s) as precisely as □$_{12}$ (solar □'s), □$_{23}$ (atmospheric □'s) is not precisely determined yet (slight preference for non-maximal value, from accelerator □'s)

Leptonic CP-violation around the corner ? Some values of □ already disfavoured at 2□



See also analog fits by de Salas,Forero, Ternes,Tortola,Valle '17 Capozzi,Lisi,Marrone,Palazzo '18

Esteban,Gonzalez-Garcia,Maltoni,Martinez-Soler,Schwetz '16



Charged lepton flavor/CP violation

Electromagnetic Dipole operator: $\mathcal{L}_{D=6} \supset \frac{C^{e\gamma}_{ij}}{\Lambda^2} \overline{e_{Li}} \sigma^{\mu\nu} e_{Rj} F_{\mu\nu} \frac{v}{\sqrt{2}}$

Flavor violation frontier:
μ to e transitions →
indexes i,j =1,2 or 2,1

$$BR(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13} \ (90\%\text{C.L.})$$

MEG data 2009-2013, EPJC 2016
ultimate sensitivity › 10^{-14}
(future is μ-to-e conversion on nuclei)

CP violation frontier:
electric dipole moment
(EDM) of the electron →
imaginary part for i,j =1,1

$$|d_e| < 1.1 \cdot 10^{-29} e\,cm \ (90\%\text{C.L.})$$

ACME data, Nature 2018

Puzzle: Electroweak scale hierarchy problem → new physics close to TeV → too large flavor/CP violation → precision low-energy experiments to test larger scales → does one reintroduce the hierarchy problem or not ?

Anomalous magnetic dipole moment

- Muon MDM: real part of the dipole with indexes i,j = 2,2

reviewed e.g. by Knecht '14
Jegerlehner '18

- 3 to 4□discrepancy: a¥^{exp} - a¥^{SM} □(31±8) 10^{-10}

- Intense activity to reduce the SM theoretical uncertainty. Expected change in the uncertainty is much smaller than the discrepancy.

Passera,Marciano,Sirlin '08-'10

- One experiment only dominates the present measurement; two new projects aim to reduce by a factor 4 the experimental uncertainty

E821 (Brookhaven) '06

E989 (Fermilab proposal) '10
now data-taking !
g-2 (J-PARC proposal) '10

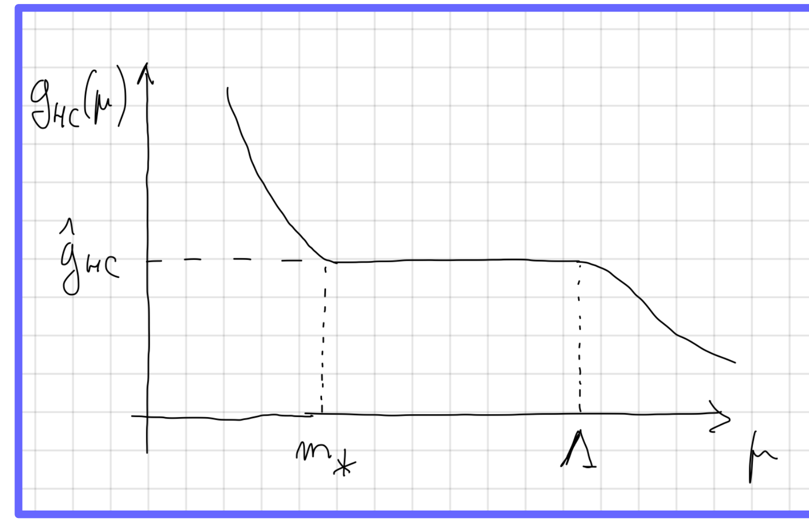
- The discrepancy can be explained by (flavour & CP conserving) new physics with the size of SM one-loop electroweak contribution (typically within the LHC reach)

Partial compositeness: motivations

- *How SM fermions acquire a mass?*
  - ✓ *Why masses and mixings are hierarchical? Why not true for neutrinos?*
  - ✓ *What is the nature of electroweak symmetry breaking?*
- *Coupling SM fermions with a new strongly-coupled sector*
  - ✓ *dynamics may induce hierarchy from anarchy*
  - ✓ *flavour violating effects can be suppressed by this hierarchy*
  - ✓ *a large top quark Yukawa is possible, by operators relevant in the infrared*
  - ✓ *if the Higgs is composite, the electroweak scale is protected from the UV physics*
- *Compositeness in lepton sector can be tested well beyond the LHC reach*
  - ✓ *great precision of low-energy experiments*

Partial Compositeness (PC) abridged

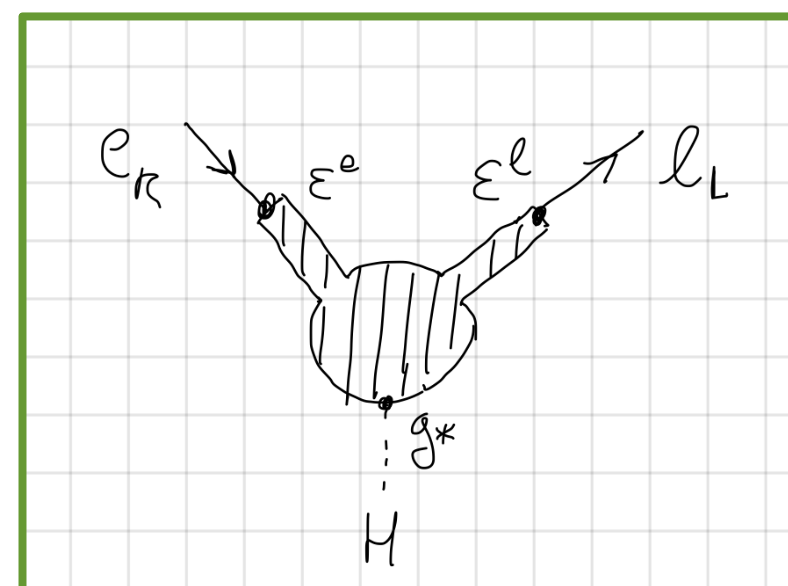
- Scale m* ~ few TeVs generated by a strongly-coupled sector that confines (dimensional transmutation)
- Approximate scale-invariance protects m* << Λ
- SM fermions ψ weakly mix linearly with composite operators O's at scale Λ
- Anarchical values of λ's in the UV become hierarchical in the IR, according to the anomalous dimensions of O's
- Each SM fermion acquires a degree of compositeness 0< ε^{Ψ} < 1 (PC)
- Composite among composite states: 1< g* < 4π
- Each Yukawa coupling is controlled by the product of the left- and right-handed ε's



$$\mathcal{L}_{PC} = \lambda^{\psi}_{ia} \overline{O}^{\psi}_{a} \psi_i + h.c.$$

$$\mu \frac{d}{d\mu} \lambda^{\psi} \simeq (\Delta_{O^{\psi}} - 5/2) \lambda^{\psi}$$

$$\lambda^{\psi}(m_*) \simeq \lambda^{\psi}(\Lambda) \left(\frac{m_*}{\Lambda} \right)^{\Delta_{O^{\psi}} - 5/2} \equiv g^* \epsilon^{\psi}$$



$$\mathcal{L}^{eff}_{PC} = y^e_{ij} \overline{\ell_L}_i e_{Rj} H + h.c.$$

$$y^e_{ij} \simeq g^* \epsilon^{\ell}_i \epsilon^e_j$$

Values of the ε's from fermion masses

$$y^e_{ij} \simeq g^* \epsilon^\ell_i \epsilon^e_j$$

$$m^e_i \simeq \epsilon^\ell_i \epsilon^e_i g_* \frac{v}{\sqrt{2}} \,, \qquad U^\ell_{ij} = \min\left(\frac{\epsilon^\ell_i}{\epsilon^\ell_j}, \frac{\epsilon^\ell_j}{\epsilon^\ell_i}\right)$$

$$\epsilon^\psi_i \in \left[\frac{\sqrt{2} m^\psi_i}{g_* v}, 1\right]$$

Input values at μ=1 TeV
from Xing, Zhang, Zhou 2011

fermion masses (GeV)	$\epsilon^\psi_i/\epsilon^\psi_j$
$m_e = 0.490 \cdot 10^{-3}$	$2.8 \cdot 10^{-6}/g_* \leq \epsilon^{\ell,e}_1/\epsilon^{\ell,e}_2 \leq 1$
$m_\mu = 0.103$	$2.8 \cdot 10^{-6}/g_* \leq \epsilon^{\ell,e}_1/\epsilon^{\ell,e}_3 \leq 1$
$m_\tau = 1.76$	$5.9 \cdot 10^{-4}/g_* \leq \epsilon^{\ell,e}_2/\epsilon^{\ell,e}_3 \leq 1$
$m_u = 1.2 \cdot 10^{-3}$	$\epsilon^q_1/\epsilon^q_2 = \lambda_C = 0.225$
$m_c = 0.54$	$\epsilon^q_2/\epsilon^q_3 = \lambda^2_C = 0.051$
$m_t = 148$	$\epsilon^u_1/\epsilon^u_2 = 0.010$
$m_d = 2.4 \cdot 10^{-3}$	$\epsilon^u_2/\epsilon^u_3 = 0.072$
$m_s = 0.05$	$\epsilon^d_1/\epsilon^d_2 = 0.21$
$m_b = 2.4$	$\epsilon^d_2/\epsilon^d_3 = 0.41$

In quark sector g* unique free parameter. Note that δ_{CKM} can be large.

PC of leptons not fixed by $m_{e,\mu,\tau}$ only → need to specify neutrino sector

Neutrino masses

Neutrino mass from compositeness

Below compositeness scale:

$$\mathcal{L}_{m_*} \supset \frac{m_\nu}{v^2} \ell\ell HH + h.c.$$

If strong dynamics breaks lepton number, then Naive Dimensional Analysis (NDA) gives:

$$m_\nu \simeq \frac{(g_* \epsilon^\ell v)^2}{m_*} \gtrsim \frac{m_\tau^2}{m_*}$$

→ Strong dynamics must preserve lepton number:

$U(1)_L$ is broken only by weak, external couplings with $\Delta L \neq 0$:

$\mathcal{L}_{PC}$	spurion	$U(1)_L$
$\lambda^\ell \ell (O_{L=1})^\dagger$	λ^ℓ	0
$\tilde\lambda (O_{L=\delta})^\dagger$	$\tilde\lambda$	δ
$\tilde\lambda \ell (O_{L=\delta+1})^\dagger$	$\tilde\lambda$	δ
$\tilde\lambda \ell\ell (O_{L=\delta+2})^\dagger$	$\tilde\lambda$	δ
$\dots$	$\dots$	$\dots$

$$\tilde\lambda(m_*) \simeq \tilde\lambda(\Lambda_L) \left(\frac{m_*}{\Lambda_L}\right)^{\gamma_{O_L}}$$

Neutrino mass m_ν requires $\Delta L = -2$, that can be obtained by one or several insertions of $\Delta L = \delta$

Various $\Delta L \neq 0$ couplings are PC realizations of usual neutrino mass mechanisms!
If one assumes that λ's have anarchic flavor structure in the UV,
then all cases reduce to 3 possible flavor structures for m_ν

Neutrino flavor structure (I)

$$m^\nu \text{ quadratic in } \epsilon^\ell_k \; : \quad m^\nu_{ij} = \epsilon^\ell_i \epsilon^\ell_j \tilde{\epsilon} \, \frac{(g_* v)^2}{m_*} \propto \begin{pmatrix} (\epsilon^\ell_1)^2 & \epsilon^\ell_1 \epsilon^\ell_2 & \epsilon^\ell_1 \epsilon^\ell_3 \\ \dots & (\epsilon^\ell_2)^2 & \epsilon^\ell_2 \epsilon^\ell_3 \\ \dots & \dots & (\epsilon^\ell_3)^3 \end{pmatrix}$$

It can be realized e.g. for $\mathcal{L}_{PC} = \lambda^\ell_i \ell_i O^\dagger_{L=1} + \tilde{\lambda} O^\dagger_{L=\delta} \,,\qquad \tilde{\epsilon} \sim \tilde{\lambda}^{-2/\delta}$

The neutrino flavor structure is the same as for $(m_e m_e^\dagger)_{ij} \sim \epsilon^\ell_i \epsilon^\ell_j$

Large neutrino mixing implies that the 3 lepton doublets have similar degree of PC:

$$\frac{\epsilon^\ell_2}{\epsilon^\ell_3} \simeq 1 \,, \qquad 0.2 \lesssim \frac{\epsilon^\ell_1}{\epsilon^\ell_{2,3}} \lesssim 1$$

Thus, neutrino oscillation data reduce the allowed range of ε's by orders of magnitude!

Neutrino flavor structure (II)

$$m_\nu \text{ linear in } \epsilon^\ell_k \;: \quad m^\nu_{ij} = \underline{(\epsilon^\ell_i \tilde\epsilon_j + \epsilon^\ell_j \tilde\epsilon_i)} \, \frac{(g_* v)^2}{m_*} \propto \begin{pmatrix} \epsilon^\ell_1 & \epsilon^\ell_2 & \epsilon^\ell_3 \\ \epsilon^\ell_2 & \epsilon^\ell_2 & \epsilon^\ell_3 \\ \epsilon^\ell_3 & \epsilon^\ell_3 & \epsilon^\ell_{3,2} \end{pmatrix}$$

It can be realized e.g. for $\mathcal{L}_{PC} = \lambda^\ell_i \ell_i O^\dagger_{L=1} + \tilde\lambda_j \ell_j O^\dagger_{L=-1} \,, \qquad \tilde\epsilon_j \sim \tilde\lambda_j$

or for $\mathcal{L}_{PC} = \lambda^\ell_i \ell_i O^\dagger_{L=1} + \tilde\lambda_{jk} (\ell_j \ell_k)_S O^\dagger_{L=0} \,, \qquad \tilde\epsilon_j \sim \sum_k \tilde\lambda_{jk} \epsilon^\ell_k$

Neutrino oscillation data imply $\quad |\epsilon^\ell_1| \lesssim |\epsilon^\ell_2| \sim |\epsilon^\ell_3|$

Special case, motivated by $m_e << m_{\mu,\tau}$: $\quad |\epsilon^\ell_1| \ll |\epsilon^\ell_2| \sim |\epsilon^\ell_3|$

→ normal ordering of neutrino masses & suppressed neutrino-less 2β decay

Neutrino flavor structure (III)

m_ν independent from ϵ^ℓ_k : $$m^\nu_{ij} = \tilde{\epsilon}_{ij} \frac{(g_* v)^2}{m_*} \propto \mathcal{O}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

It can be realized e.g. for $$\mathcal{L}_{PC} = \tilde{\lambda}_i \ell_i O^\dagger_{L=0} \,, \qquad \tilde{\epsilon}_{ij} \sim \tilde{\lambda}_i \tilde{\lambda}_j$$

or for $$\mathcal{L}_{PC} = \tilde{\lambda}_{ij} (\ell_i \ell_j)_T O^\dagger_{L=0} \,, \qquad \tilde{\epsilon}_{ij} \sim \tilde{\lambda}_{ij}$$

The neutrino flavor structure is anarchical: all matrix entries scale from UV to IR with the same anomalous dimension $\rightarrow$ large mixing is automatic

The charged lepton flavor structure is independent from the neutrino one: $U(1)_L$ violation is decoupled from violations of charged lepton flavor/CP/universality

For all 3 neutrino flavor structures, one can show that anarchic Partial Compositeness implies large CP-violating phases

Flavor/CP violation

Dipole operator in PC framework

$$\mathcal{L}_{D=6} \supset \frac{C_{ij}^{e\gamma}}{\Lambda^2} \overline{e_{Li}} \sigma^{\mu\nu} e_{Rj} F_{\mu\nu} \frac{v}{\sqrt{2}}$$

As in the case of Yukawa couplings and neutrino masses, the Wilson coefficient can be estimated by Naive Dimensional Analysis (NDA):

$$\frac{C_{ij}^{e\gamma}}{\Lambda^2} \frac{v}{\sqrt{2}} = \frac{c_{ij}^{e\gamma}}{m_*^2} \left(\frac{g_*}{4\pi}\right)^2 \epsilon_i^\ell \, \epsilon_j^e \, e \, \frac{g_* v}{\sqrt{2}} = \frac{c_{ij}^{e\gamma}}{m_*^2} \left(\frac{g_*}{4\pi}\right)^2 \frac{\epsilon_i^\ell}{\epsilon_j^\ell} m_j \, e$$

order one coefficient

strong loop-factor

Higgs coupling

scale dimension

photon coupling

lepton composite fraction

relevant ratio of PC parameters

Other lepton operators in PC framework

Effective operator	Wilson coefficient
$Q_{eW}^{ij} = \left(\bar{\ell}^i_L \sigma^{\mu\nu} e^j_R\right) \sigma^I H W^I_{\mu\nu}$	$\frac{C^{eW}_{ij}}{\Lambda^2} = \frac{1}{16\pi^2}\frac{g_*^3}{m_*^2}\epsilon^\ell_i\epsilon^e_j g\, c^{eW}_{ij} = \frac{1}{16\pi^2}\frac{g_*^2}{m_*^2}\frac{\epsilon^\ell_i}{\epsilon^\ell_j}\frac{\sqrt{2}m^e_j}{v} g\, c^{eW}_{ij}$
$Q_{eB}^{ij} = \left(\bar{\ell}^i_L \sigma^{\mu\nu} e^j_R\right) H B_{\mu\nu}$	$\frac{C^{eB}_{ij}}{\Lambda^2} = \frac{1}{16\pi^2}\frac{g_*^3}{m_*^2}\epsilon^\ell_i\epsilon^e_j g'\, c^{eB}_{ij} = \frac{1}{16\pi^2}\frac{g_*^2}{m_*^2}\frac{\epsilon^\ell_i}{\epsilon^\ell_j}\frac{\sqrt{2}m^e_j}{v} g'\, c^{eB}_{ij}$
$Q_{eH}^{ij} = (H^\dagger H)\left(\bar{\ell}^i_L e^j_R H\right)$	$\frac{C^{eH}_{ij}}{\Lambda^2} = \frac{g_*^3}{m_*^2}\epsilon^\ell_i\epsilon^e_j\, c^{eH}_{ij} = \frac{g_*^2}{m_*^2}\frac{\epsilon^\ell_i}{\epsilon^\ell_j}\frac{\sqrt{2}m^e_j}{v}\, c^{eH}_{ij}$
$Q_{H\ell}^{(1)ij} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{\ell}^i_L\gamma^\mu \ell^j_L\right)$	$\frac{C^{H\ell(1)}_{ij}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^\ell_i\epsilon^\ell_j\, c^{H\ell(1)}_{ij}$
$Q_{H\ell}^{(3)ij} = \left(H^\dagger \sigma^I i\overleftrightarrow{D}_\mu H\right)\left(\bar{\ell}^i_L\sigma^I\gamma^\mu \ell^j_L\right)$	$\frac{C^{H\ell(3)}_{ij}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^\ell_i\epsilon^\ell_j\, c^{H\ell(3)}_{ij}$
$Q_{He}^{ij} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{e}^i_R\gamma^\mu e^j_R\right)$	$\frac{C^{He}_{ij}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^e_i\epsilon^e_j\, c^{He}_{ij} = \frac{1}{m_*^2}\frac{2m^e_i m^e_j}{v^2}\frac{1}{\epsilon^\ell_i\epsilon^\ell_j}\, c^{He}_{ij}$
$Q_{\ell\ell}^{ijmn} = \left(\bar{\ell}^i_L\gamma_\mu \ell^j_L\right)\left(\bar{\ell}^m_L\gamma^\mu \ell^n_L\right)$	$\frac{C^{\ell\ell}_{ijmn}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^\ell_i\epsilon^\ell_j\epsilon^\ell_m\epsilon^\ell_n\, c^{\ell\ell}_{ijmn}$
$Q_{\ell e}^{ijmn} = \left(\bar{\ell}^i_L\gamma_\mu \ell^j_L\right)\left(\bar{e}^m_R\gamma^\mu e^n_R\right)$	$\frac{C^{\ell e}_{ijmn}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^\ell_i\epsilon^\ell_j\epsilon^e_m\epsilon^e_n\, c^{\ell e}_{ijmn} = \frac{1}{m_*^2}\frac{2m^e_m m^e_n}{v^2}\frac{\epsilon^\ell_i\epsilon^\ell_j}{\epsilon^\ell_m\epsilon^\ell_n}\, c^{\ell e}_{ijmn}$
$Q_{ee}^{ijmn} = \left(\bar{e}^i_R\gamma_\mu e^j_R\right)\left(\bar{e}^m_R\gamma^\mu e^n_R\right)$	$\frac{C^{ee}_{ijmn}}{\Lambda^2} = \frac{g_*^2}{m_*^2}\epsilon^e_i\epsilon^e_j\epsilon^e_m\epsilon^e_n\, c^{ee}_{ijmn} = \frac{1}{g_*^2 m_*^2}\frac{4m^e_i m^e_j m^e_m m^e_n}{v^4\epsilon^\ell_i\epsilon^\ell_j\epsilon^\ell_m\epsilon^\ell_n}\, c^{ee}_{ijmn}$

Experimental bounds for 2-lepton operators

	Upper bound on $\lvert C \rvert$ for $\Lambda = 1\,\text{TeV}$	Observable
$C^{e\gamma}_{12,21}$	2.1×10^{-10}	$\mu \to e\gamma$
$C^{e\gamma}_{13,31}$	2.4×10^{-6}	$\tau \to e\gamma$
$C^{e\gamma}_{23,32}$	2.7×10^{-6}	$\tau \to \mu\gamma$
$\text{Im}\, C^{e\gamma}_{11}, \ \text{Re}\, C^{e\gamma}_{11}$	~~3.8~~ 0.48 $\times 10^{-12}, \ 2.4 \times 10^{-6}$	$d_e, \Delta a_e$
$\text{Im}\, C^{e\gamma}_{22}, \ \text{Re}\, C^{e\gamma}_{22}$	$8.4 \times 10^{-3}, \ 1.8 \times 10^{-5}$	$d_\mu, \Delta a_\mu$
$\text{Im}\, C^{e\gamma}_{33}, \ \text{Re}\, C^{e\gamma}_{33}$	$4.4 \times 10^{-1}, \ 3.2$	$d_\tau, \Delta a_\tau$
$C^{eH}_{12,21}$	3.5×10^{-5}	$\mu \to e\gamma$ (2-loop)
$C^{eH}_{13,31}$	3.0×10^{-1}	$\tau \to e\gamma$ (1- and 2-loop)
$C^{eH}_{23,32}$	3.4×10^{-1}	$\tau \to \mu\gamma$ (1- and 2-loop)
$\text{Im}\, C^{eH}_{11}, \ \text{Re}\, C^{eH}_{11}$	~~6.5~~ 0.82 $\times 10^{-7}, \ 8.4 \times 10^{-2}$	$d_e, \Delta a_e$ (2-loop)
C^{He}_{12}	$4.9(39) \times 10^{-6}$	$\mu\, Au \to e\, Au$ ($\mu \to eee$)
C^{He}_{13}	$1.5(1.8) \times 10^{-2}$	$\tau \to eee$ ($\tau \to e\mu^+\mu^-$)
C^{He}_{23}	$1.3(1.5) \times 10^{-2}$	$\tau \to \mu\mu\mu$ ($\tau \to \mu e^+ e^-$)
$C^{H\ell(1,3)}_{12}$	$4.9(37) \times 10^{-6}$	$\mu\, Au \to e\, Au$ ($\mu \to eee$)
$C^{H\ell(1,3)}_{13}$	$1.4(1.8) \times 10^{-2}$	$\tau \to eee$($\tau \to e\mu^+\mu^-$)
$C^{H\ell(1,3)}_{23}$	$1.3(1.5) \times 10^{-2}$	$\tau \to \mu\mu\mu$ ($\tau \to \mu e^+ e^-$)

Another analogous bounds for 4-lepton operators

These bounds translate into constraints on the PC parameters: m*, g*, and the ε's

Charged lepton flavor violation

Most stringent bound from radiative μ-to-e transitions

$$\mathrm{Br}\,(\mu \to e\gamma) = 48\pi^2 \frac{v^6}{\Lambda^4 m_\mu^2} \left(|C_{12}^{e\gamma}|^2 + |C_{21}^{e\gamma}|^2\right) \qquad |C^{e\gamma}_{12,21}| < \underline{2 \cdot 10^{-8}} \left(\frac{\Lambda}{10\ \mathrm{TeV}}\right)^2$$

In the PC frame, the optimal choice of parameters to weaken this constraint is

$$\frac{\epsilon_1^\ell}{\epsilon_2^\ell} = \sqrt{\frac{m_e}{m_\mu}} \quad \Rightarrow \quad |c^{e\gamma}_{12,21}| \left(\frac{g_*}{4\pi}\right)^2 \left(\frac{10\ \mathrm{TeV}}{m_*}\right)^2 \underline{< 2 \cdot 10^{-3}}$$

Huge progress, still this number is much smaller than one: flavor anarchy is problematic ...
… this suggests that strong dynamics preserves flavor numbers (familiar from QCD).

In this case, flavor violation resides only in the external couplings λ's:

$$U(1)_L^3 \quad : \quad \epsilon_i^\ell \epsilon_j^e \to \min(\epsilon_i^\ell \epsilon_i^e, \epsilon_j^\ell \epsilon_j^e) \quad \Rightarrow \quad |c^{e\gamma}_{12,21}| < 2 \cdot 10^{-3} \sqrt{\frac{m_\mu}{m_e}} \simeq \underline{0.03}$$

The residual tuning can be avoided raising m_* or lowering g_* slightly

Charged lepton CP violation

Most stringent bound from electron EDM

$$\frac{d_e}{2} = \frac{\mathrm{Im}C_{11}^{e\gamma}}{\Lambda^2}\frac{v}{\sqrt{2}} \qquad |\mathrm{Im}C_{11}^{e\gamma}| < 0.5 \cdot 10^{-10}\left(\frac{\Lambda}{10\ \mathrm{TeV}}\right)^2$$

In the PC frame, the bound becomes milder :

$$|\mathrm{Im}c_{11}^{e\gamma}|\left(\frac{g_*}{4\pi}\right)^2\left(\frac{10\ \mathrm{TeV}}{m_*}\right)^2 < 0.5 \cdot 10^{-4}$$

Still, this number << 1 suggests that strong dynamics preserves CP (familiar from QCD). In this case, CP violation resides only in the external couplings λ's.

The EDM is significantly suppressed if strong dynamics also preserves flavor numbers:

$$U(1)_L^3 \times CP \ : \qquad |\mathrm{Im}c_{11}^{e\gamma}|\left(\frac{g_*}{4\pi}\right)^2\left(\frac{10\ \mathrm{TeV}}{m_*}\right)^2 < 0.5 \cdot 10^{-4}\frac{m_\mu}{m_e} \simeq 0.01$$

A more effective alternative is to allow for multiple composite scales: $m_*^e >> m_*$

Lepton Yukawas arise well above m_* , where Higgs and top Yukawa arise.

In this case the electron EDM can be strongly suppressed, without tuning.

Anomalous magnetic dipole moments

The long-standing discrepancy w.r.t. the SM in the muon MDM

$$\frac{e}{4m_\mu}\Delta a_\mu = \frac{\mathrm{Re} C_{22}^{e\gamma}}{\Lambda^2}\frac{v}{\sqrt{2}} \qquad \mathrm{Re}\, C_{22}^{e\gamma} \simeq \underline{10^{-3}} \left(\frac{\Lambda}{10\ \mathrm{TeV}}\right)^2$$

In the PC frame, the discrepancy can be accommodated for

$$\mathrm{Re}\, c_{22}^{e\gamma} \left(\frac{g_*}{4\pi}\right)^2 \left(\frac{10\ \mathrm{TeV}}{m_*}\right)^2 \simeq \underline{8}$$

This large number suggests a resonance below the 10 TeV scale.

A few remarks:

Radiative decays, EDMs, MDMs constrain also **D=6 operators other than the dipole,** via operator mixing at one or two loops.

The ratio $m_*/g_* \approx f$ measures the **tuning needed for the EW scale:**

$$\frac{v^2}{f^2} \sim \frac{(g_* v)^2}{m_*^2} \simeq 0.1 \left(\frac{g_*}{4\pi}\right)^2 \left(\frac{10\ \mathrm{TeV}}{m_*}\right)^2$$

In general, each SM fermion ψ can mix with **resonances of different mass m_*^ψ ...**

Conclusions

- Precision lepton observables are exploring the multi-TeV scale
- Partial compositeness explain fermion mass hierarchies, and thus mitigates the flavor problem
- Three specific neutrino flavor patterns emerge from the composite dynamics
- Flavor and CP constraints push the compositeness scale above the range preferred by naturalness
- A symmetry $U(1)^3$ x CP greatly reduces the tension. Alternative solution is to allow for multiple flavor scales above m_*
- Current anomalies (muon g-2, B semi-leptonic decays) are flavor and CP conserving ! But, they require composite states below m_*, except for the b-to-s violation of lepton universality...