

Exotic energy injection
in the CMB from
Ultra Compact Mini Halos

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T³ webinar

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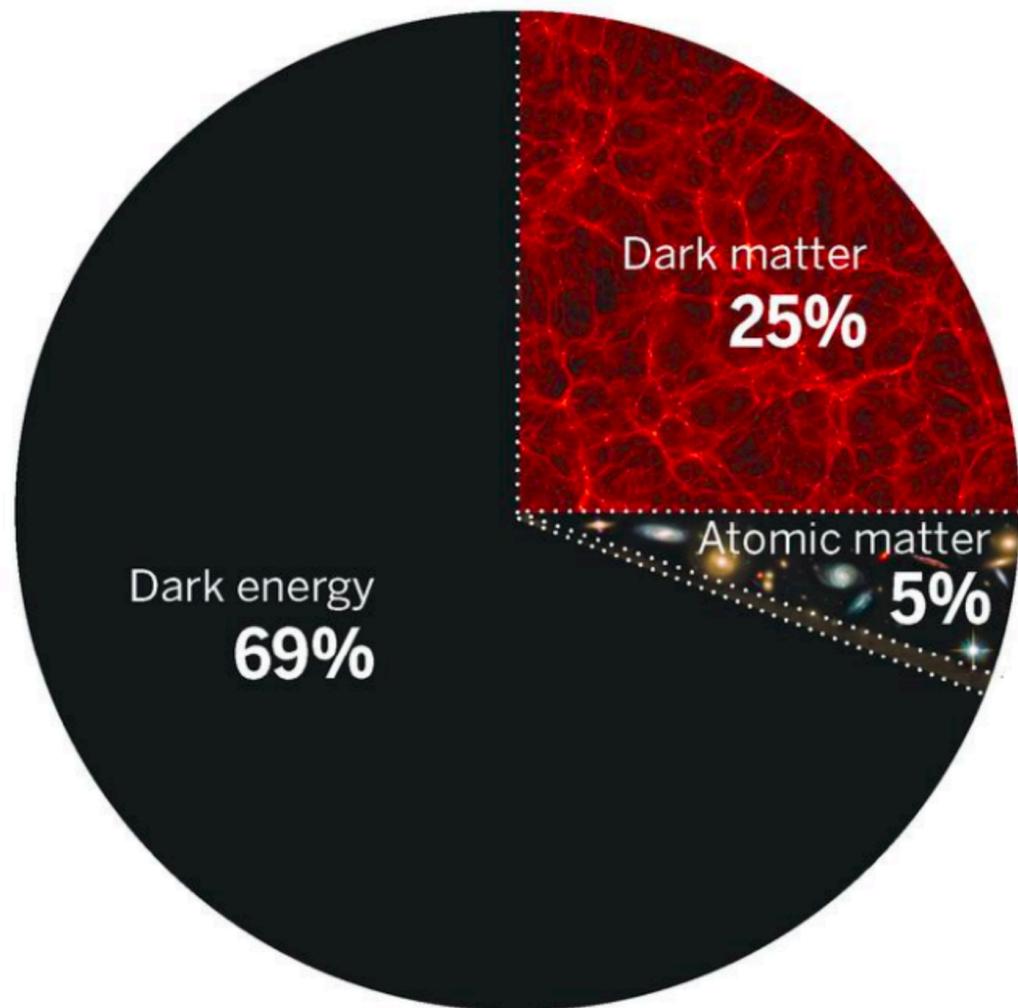
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Electromagnetic signatures of DM in the CMB

The problem of Dark matter



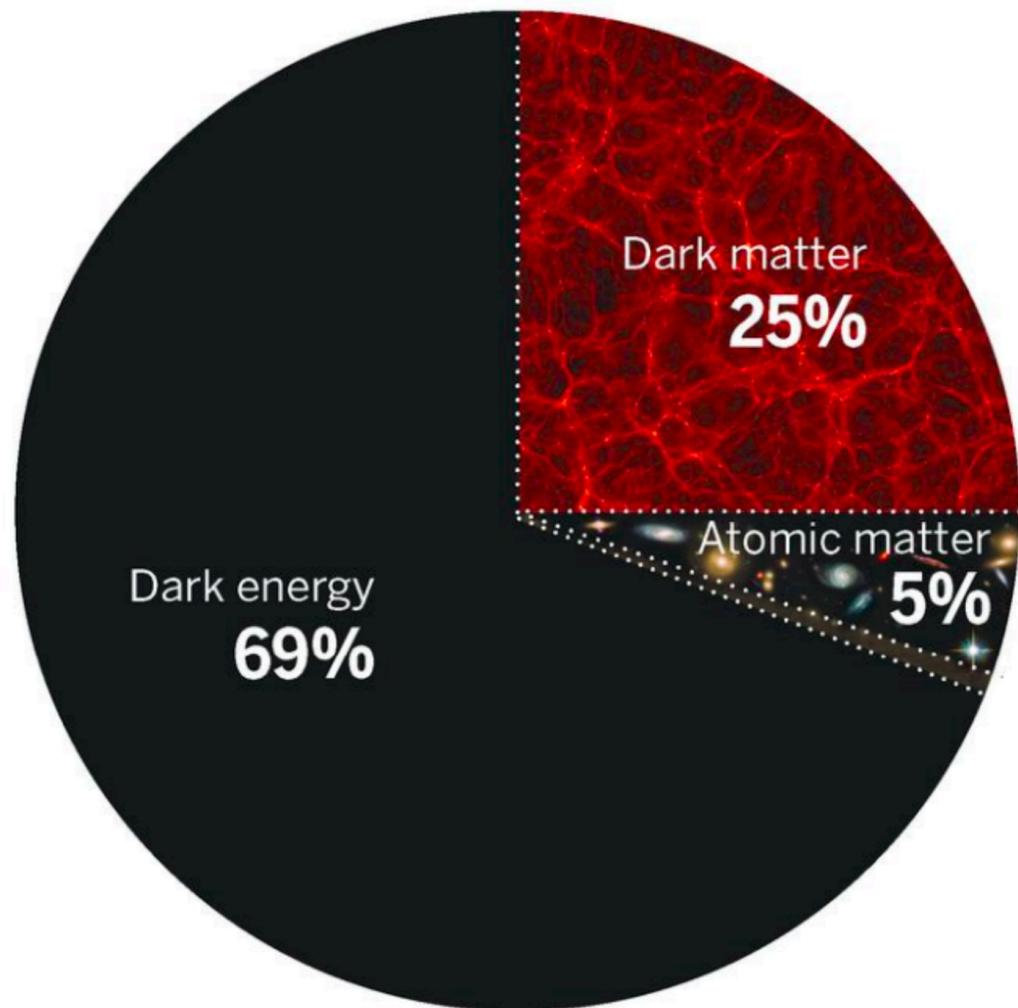
Many compelling **evidences** of dark matter from its gravitational impact

- Galactic rotation curves
- Gravitational lensing
- Structure formation
- The CMB anisotropies

Several **candidates** have been proposed

- Axions, axion-like particles
- Sterile neutrinos
- Primordial Black Holes (PBHs)
- Weakly Interacting Massive Particles (WIMPs)

The problem of Dark matter



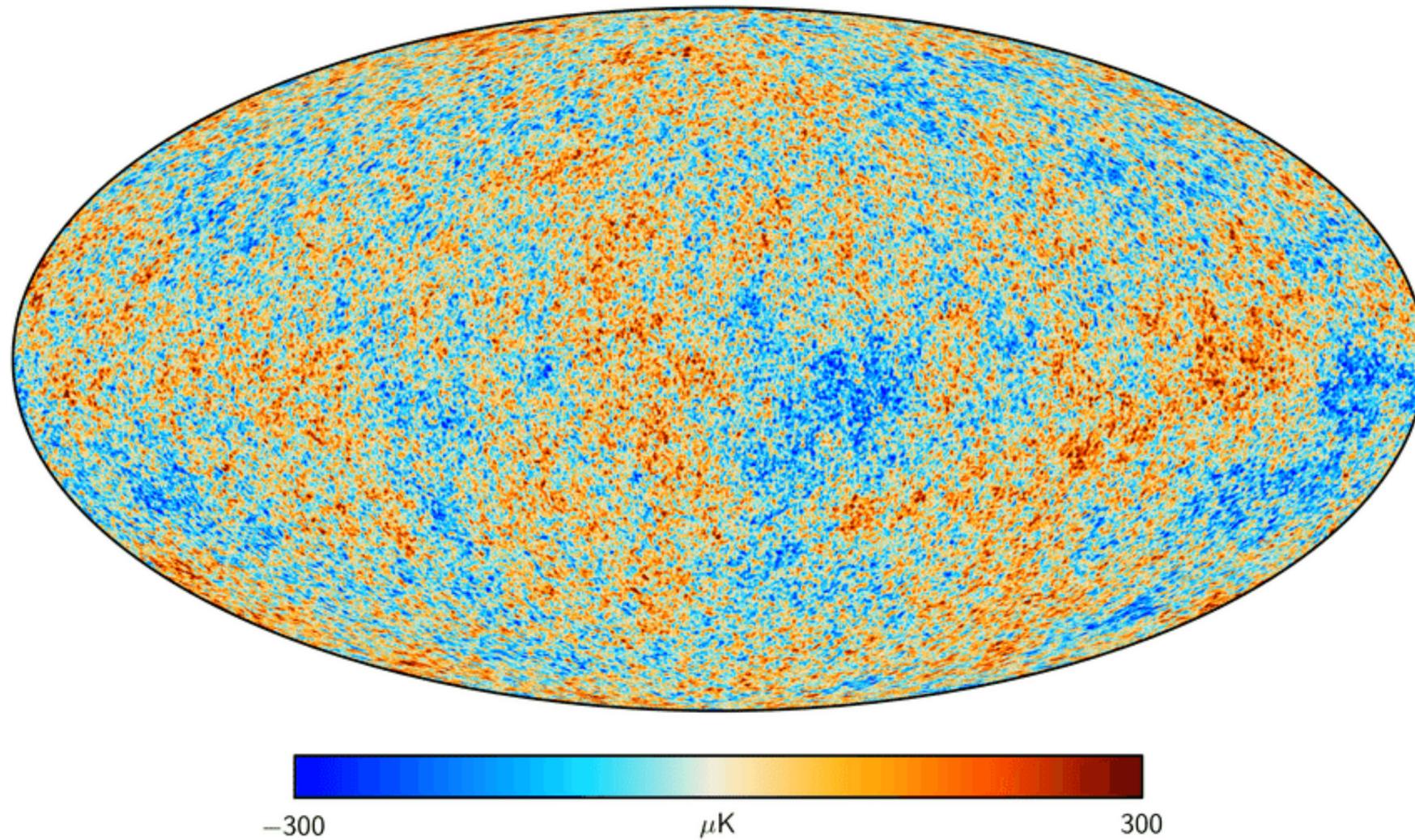
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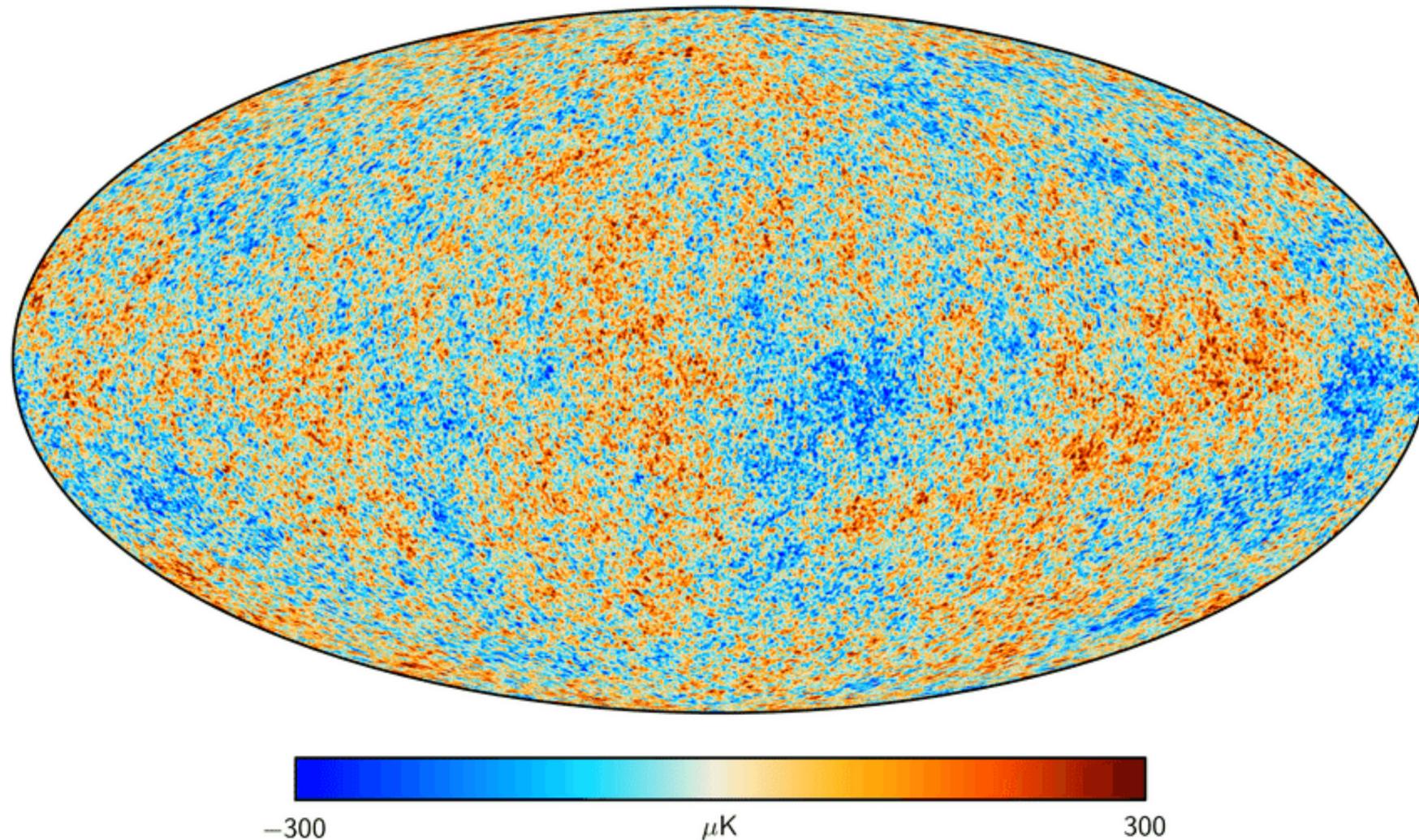
The CMB in a nutshell



Snapshot of **photon inhomogeneities at last scattering** around $z \sim 1100$, when free electrons and protons recombine to form neutral hydrogen

Most precise probe of the **DM relic density** $\Omega_{\text{cdm}} h^2 = 0.1198 \pm 0.012$

The CMB in a nutshell

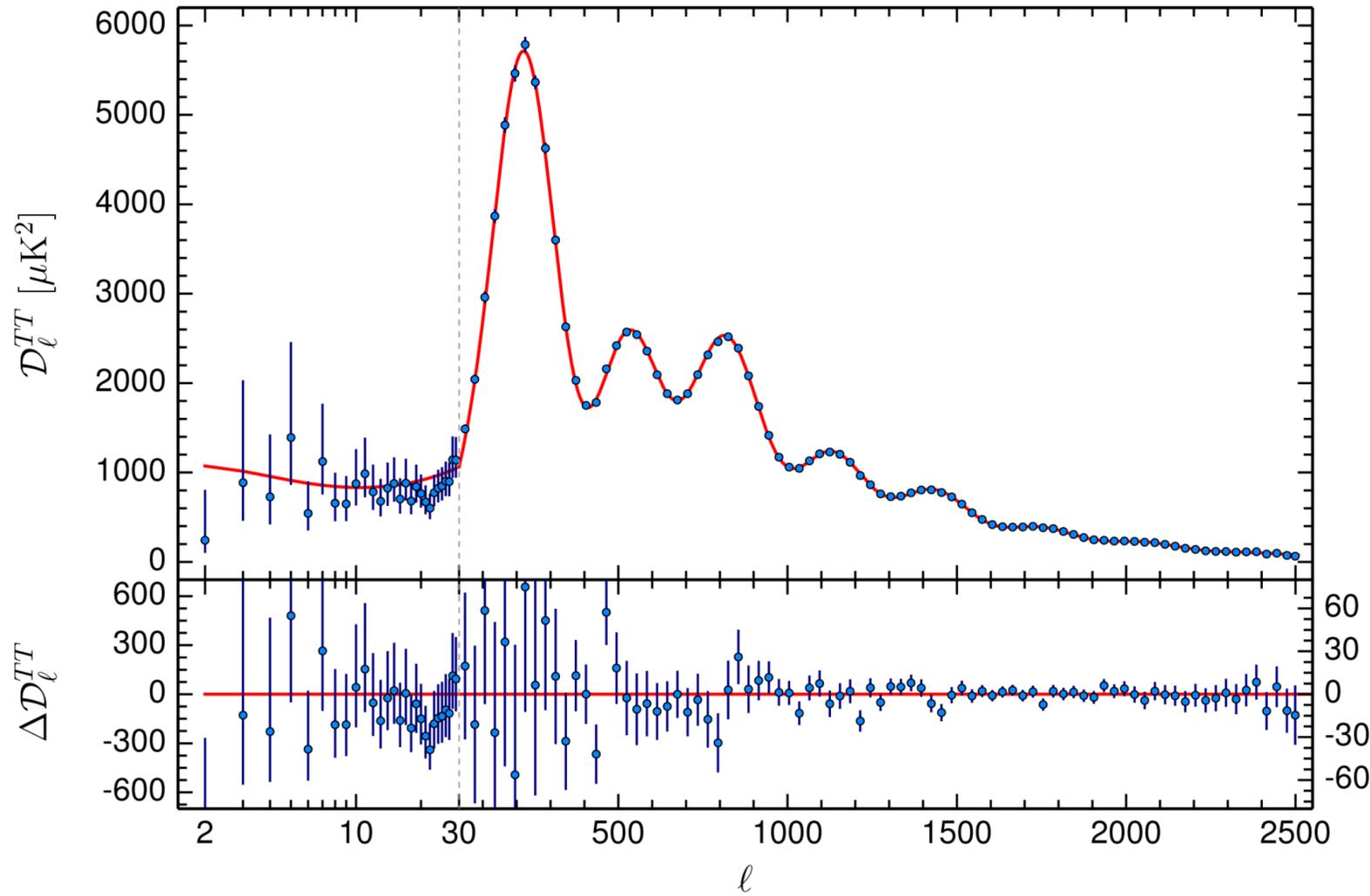


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Most precise probe of the **DM relic density** $\Omega_{\text{cdm}} h^2 = 0.1198 \pm 0.012$

Could the CMB tell us more about DM properties?

The CMB in a nutshell



2-point correlation function of temperature fluctuations

$$\langle \Theta(\hat{n})\Theta(\hat{n}') \rangle = \sum_{\ell} \frac{2\ell + 1}{4\pi} C_{\ell}^{TT} P_{\ell}(\hat{n} \cdot \hat{n}')$$

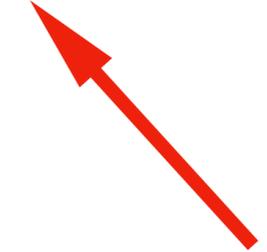
with $\Theta(\hat{n}) = \frac{\delta T(\hat{n})}{T}$

$$\mathcal{D}_{\ell}^{TT} \equiv \ell(\ell + 1)C_{\ell}^{TT} \sim \int d \log k \Theta_{\ell}^2(\tau_0, k) \mathcal{P}_{\mathcal{R}}(k)$$

Temp. transfer functions
(Boltzmann-Einstein eqs.)
(CLASS code)



Primordial spectrum
(Inflation)



The CMB in a nutshell

Line-of-sight solution

$$\Theta_\ell(\tau_0, k) = \int_\tau^{\tau_0} d\tau S_T(\tau, k) j_\ell(k(\tau_0 - \tau))$$

Source function

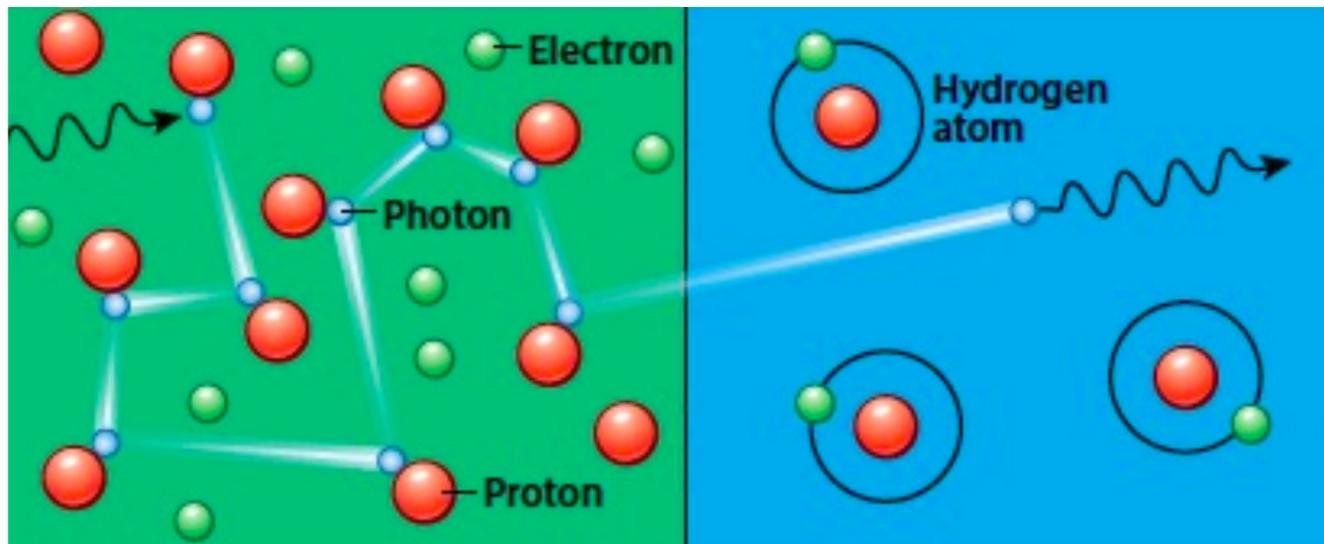
$$S_T(\tau, k) \equiv \underbrace{g(\Theta_0 + \Psi)}_{\text{SW}} + \underbrace{\partial_\tau(gv_b/k)}_{\text{Doppler}} + \underbrace{e^{-\kappa}(\dot{\Phi} + \dot{\Psi})}_{\text{ISW}}$$

Visibility function and optical depth

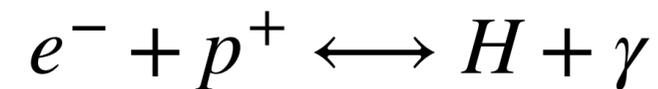
$$g(\tau) \equiv -\dot{\kappa}(\tau)e^{-\kappa(\tau)}, \quad \kappa(\tau) = \int_\tau^{\tau_0} d\tau a\sigma_T n_e$$

The CMB is highly sensitive to the free electron density through $g(\tau)$ and $\kappa(\tau)$
Energy injection from DM could affect n_e around recombination

The Peebles model of recombination



Hydrogen recombination



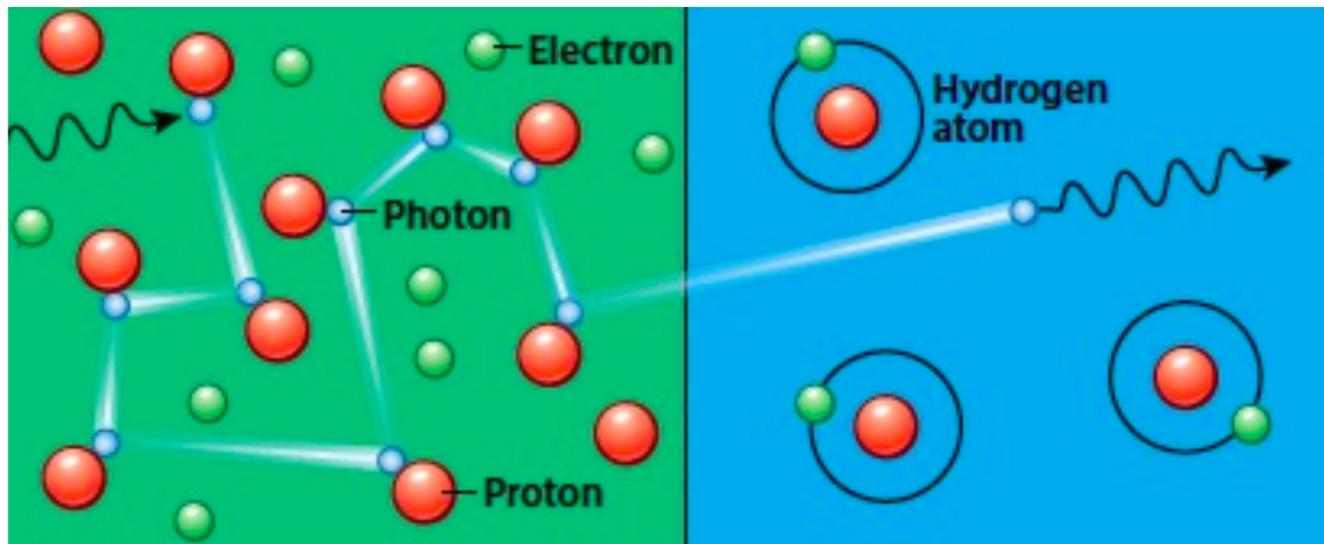
Track $x_e = n_e/n_H$ and T_M

Effective three-level system:

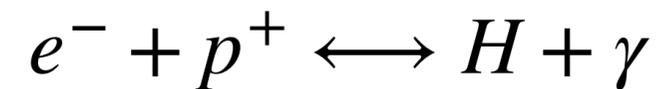
- Efficient recombination happens to $n \geq 2$
- From $n = 2$, it can reach ground state either via Ly- α $2p \rightarrow 1s$ or $2s \rightarrow 1s$ two-photon processes
- But they can be re-ionized by ambient CMB photons before reaching ground state

→ **C = prob. to $n = 2 \rightarrow 1$ before being ionized**

The Peebles model of recombination



Hydrogen recombination



Track $x_e = n_e/n_H$ and T_M

$$\frac{dx_e}{dz} = \frac{C(T_M)}{(1+z)H} \left[\alpha(T_M) n_H x_e^2 - (1-x_e) \beta(T_M) \right]$$

$$\frac{dT_M}{dz} = \frac{1}{(1+z)} \left[2T_M + \gamma(x_e)(T_M - T_{\text{CMB}}) \right]$$

RECFAST code: Solves Peebles eqs. together with Helium recombination
Also adds fudge factors to match complicated multilevel calculations

Exotic energy injection in the CMB

Restrict to the case of **s-wave DM annihilations**

Injected energy into the plasma per volume and time:

$$\left. \frac{dE}{dVdt} \right|_{\text{DM}}(z) \equiv n_{\text{pairs}} \Gamma_{\text{ann}} E_{\text{ann}} f(z) = \langle \rho_{\text{DM}} \rangle^2 (1+z)^6 p_{\text{ann}}$$

with $p_{\text{ann}} = f(z) \frac{\langle \sigma v \rangle}{m_{\text{DM}}}$

Plasma properties (Dark Ages code) → ← Particle physics

Three approximations:

- Fraction of absorbed energy can be taken to be **z-independent** $f_{\text{eff}} \equiv f(z = 600)$
- Timescale of interactions much smaller than expansion timescale (**on-the-spot approx**)
- **Neglect** boost of annihilation signal due to **non-linear structures** (2nd part)

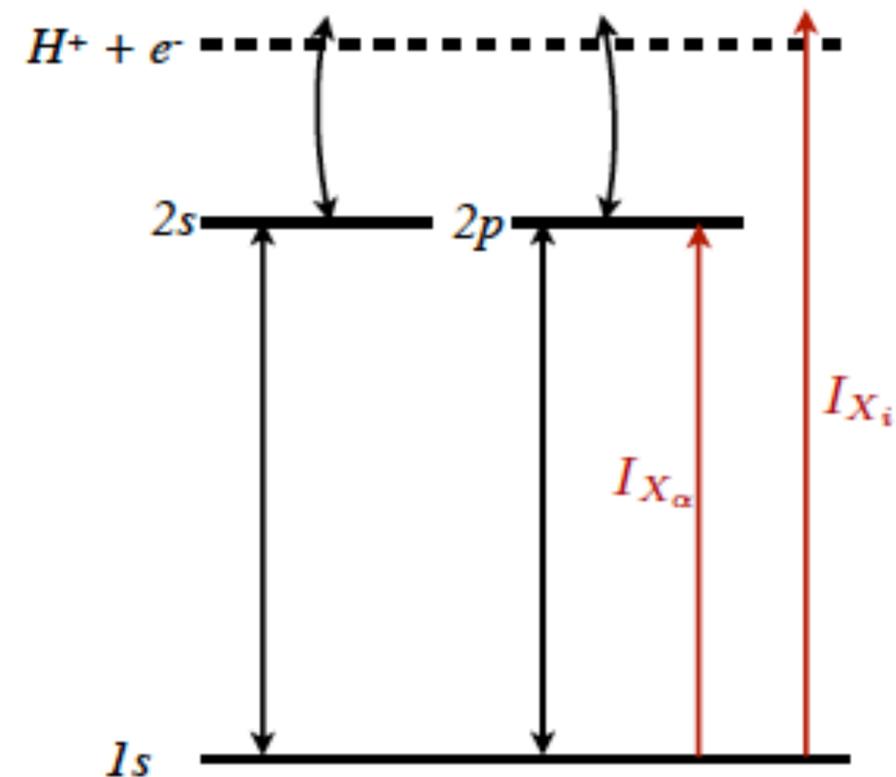
Exotic energy injection in the CMB

DM annihilations have three effects:
ionization, excitation and heating

$$\frac{dx_e}{dz} = \frac{dx_e}{dz} \Big|_{\text{st}} + I_{X_\alpha} + I_{X_i}$$

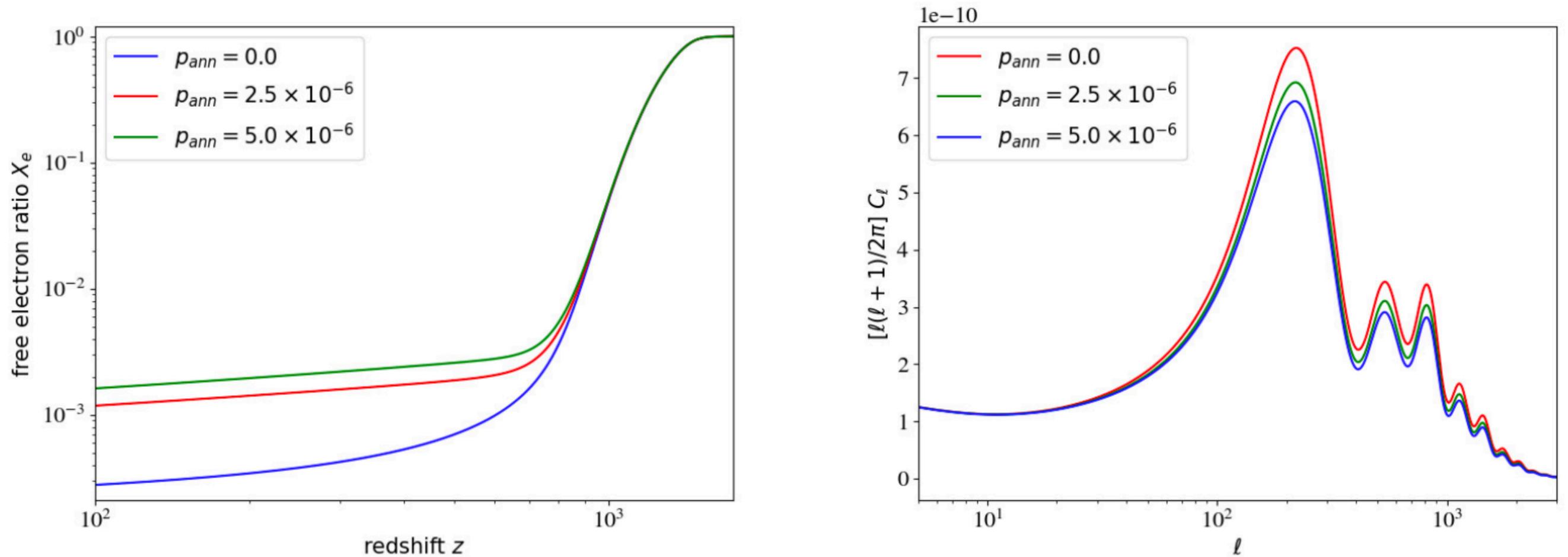
$$\frac{dT_M}{dz} = \frac{dT_M}{dz} \Big|_{\text{st}} + K_h$$

with $I_{X_\alpha}, I_{X_i}, K_h \propto \mathbf{p}_{\text{ann}}$



Giesen et al. (2012) 1209.0247v2

Exotic energy injection in the CMB

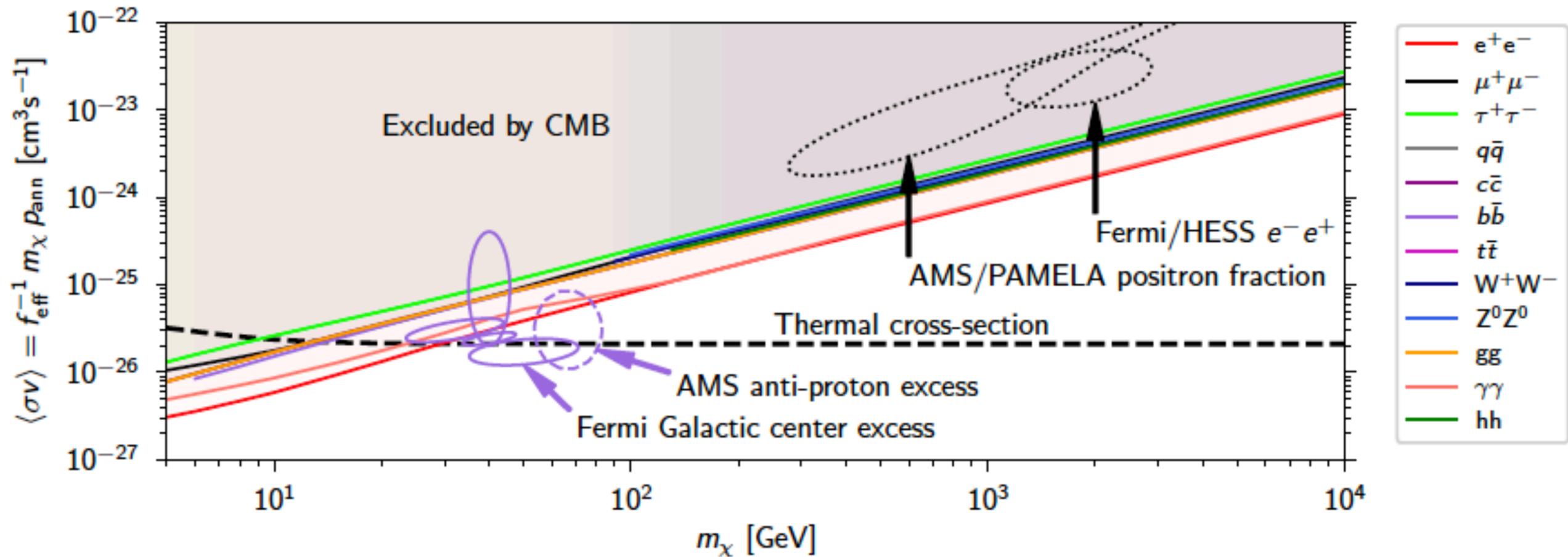


Most recent constraints from PlanckTTTEEE+lensing+BAO

$$p_{ann} < 3.2 \times 10^{-28} \text{ cm}^3 \text{ s}^{-1} \text{ GeV}^{-1} \quad (95 \% \text{ C.L.})$$

Planck (2018) 1807.06209v3

Exotic energy injection in the CMB



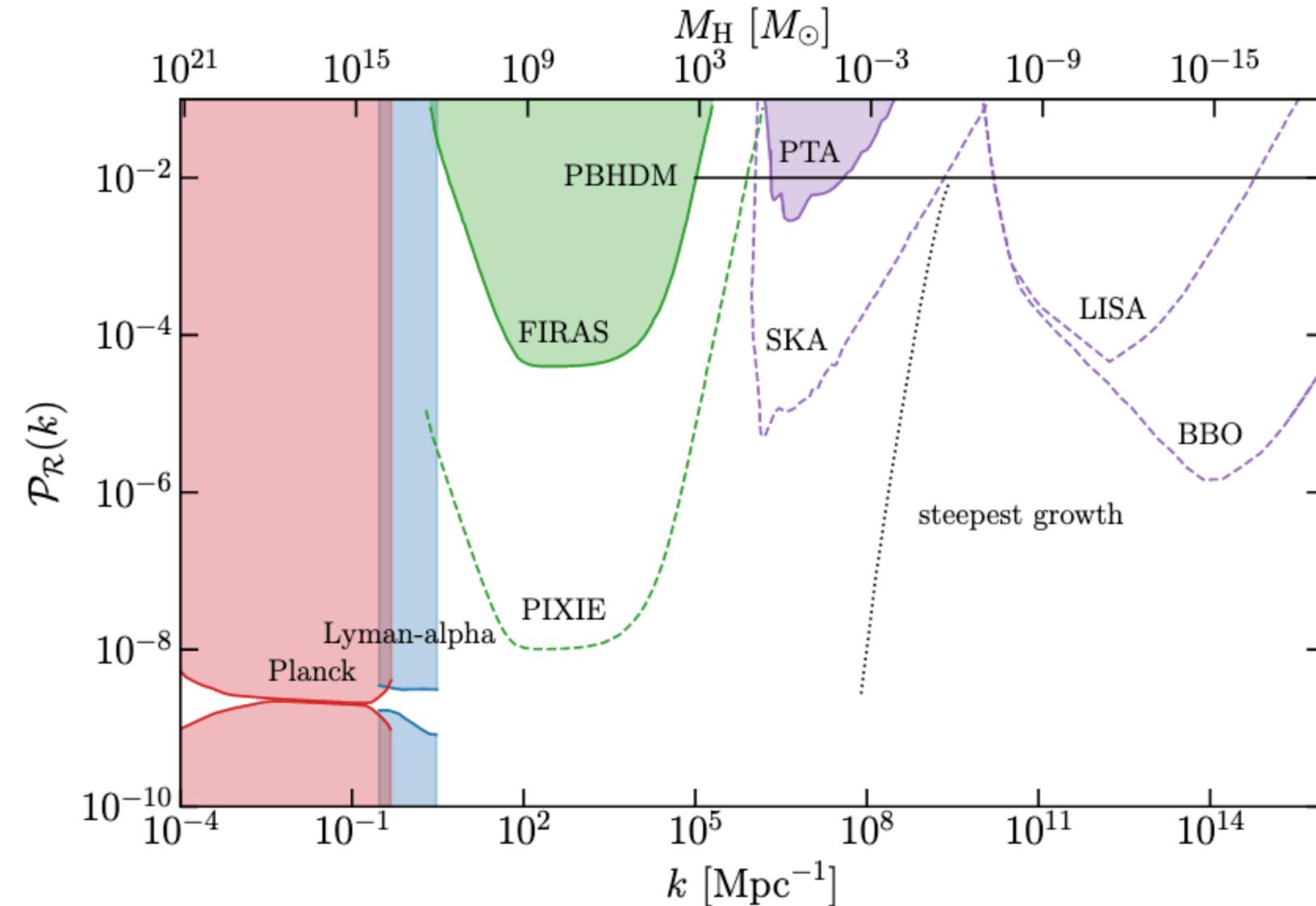
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Planck (2018) 1807.06209v3

Constraints on the small-scale primordial spectrum from UCMHs

Current constraints on the primordial spectrum



CMB, LSS and Ly- α observations constrain $\mathcal{P}_{\mathcal{R}}$ to a **nearly scale-invariant** form

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}$$

with $n_s \simeq 0.96$, $A_s \simeq 2.2 \times 10^{-9}$ and $k_* = 0.05 \text{ Mpc}^{-1}$

At scales $k \gtrsim 1 \text{ Mpc}^{-1}$, only **upper bounds** from spectral distortions (FIRAS) and Pulsar Timing Array (PTA)

[Green et al. \(2020\) 2007.10722v3](#)

Many models predict an **enhancement of power at small scales** (Examples: early matter dominated era or fast rolling scalar fields)

Generic properties of Ultra Compact Mini Halos

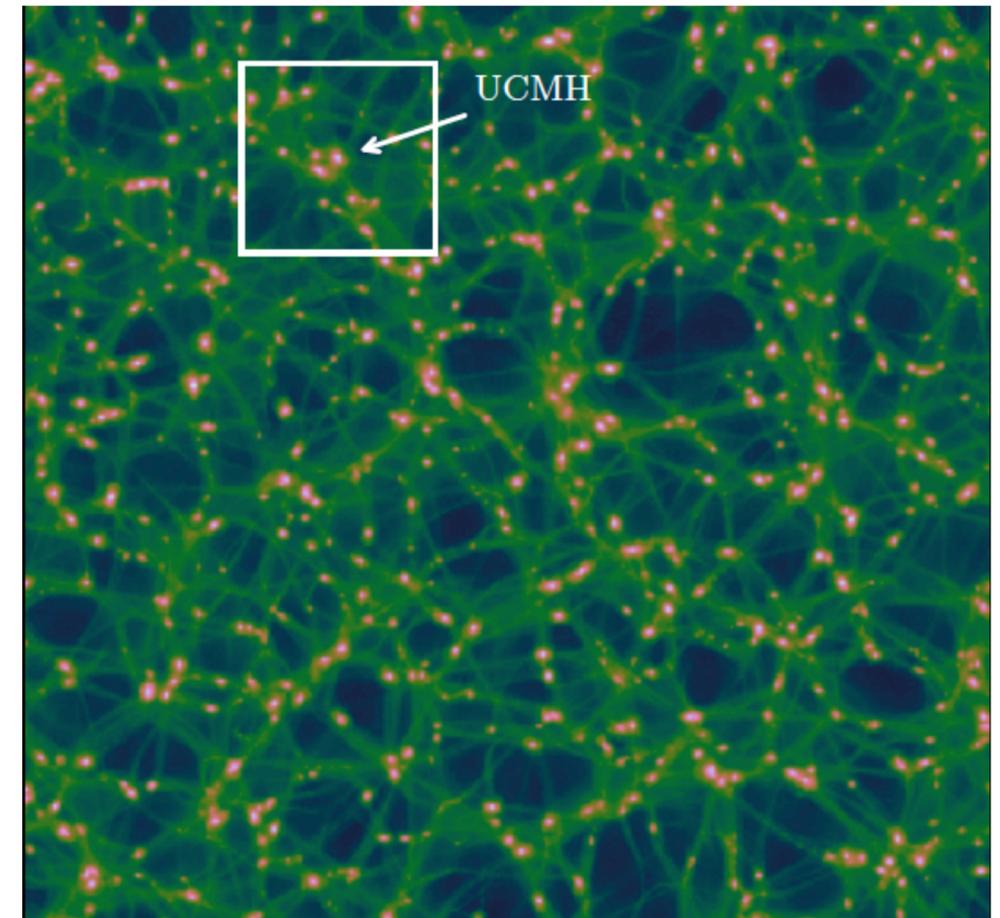
In the standard picture, DM density fluctuations of order $\delta_{\text{H}} \sim 10^{-5}$ at horizon entry will eventually **collapse** into halos at redshifts $z \sim 30 - 100$

N-body simulations show that these halos grow **hierarchically** from the smaller ones, and develop density profiles of **Navarro-Frenk-White (NFW)** form

$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

If fluctuations are of order $\delta_{\text{H}} \sim 10^{-3}$ then **Ultra Compact Mini Halos (UCMH)** can form much earlier, at **recombination time** $z \sim 1000$

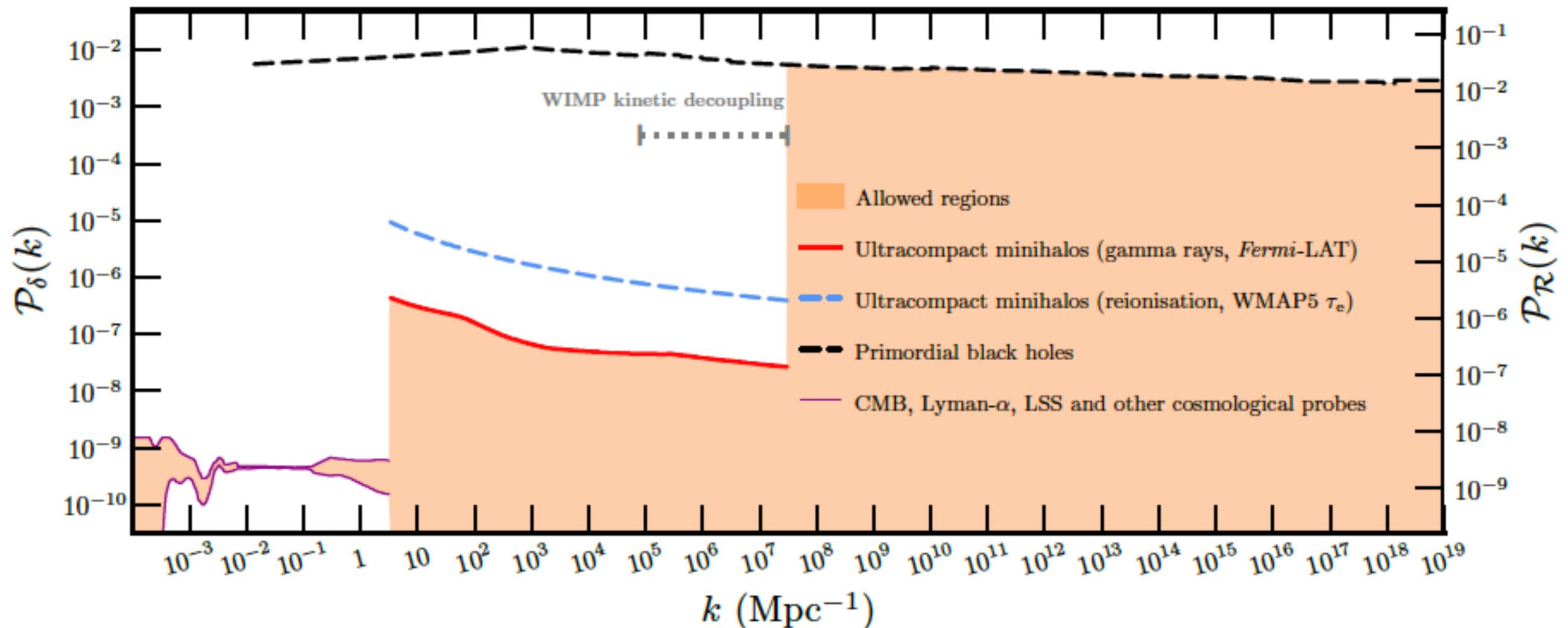
UCMHs are expected to have **denser profiles** than NFW halos



[Delos et al. \(2018\) 1806.07389v2](#)

Generic properties of Ultra Compact Mini Halos

UCMHs provide access to **very small scales**, and since the required amplitudes are not as high as those required to form PBHs ($\delta_H \sim 0.3$) **constraints are much stronger**



Bringmann et al. (2013) 1102.2484v3

Generic properties of Ultra Compact Mini Halos

Most of the constraining power of UCMHs comes from the **boost in the DM annihilation signal**

$$\left. \frac{dE}{dVdt} \right|_{\text{DM}}(z) = \langle \rho_{\text{DM}}^2 \rangle (1+z)^6 p_{\text{ann}} \quad \text{where} \quad \langle \rho_{\text{DM}}^2 \rangle \equiv B(z) \langle \rho_{\text{DM}} \rangle^2$$

Here $B(z) \equiv \langle (1 + \delta(z))^2 \rangle = 1 + \langle \delta^2(z) \rangle$ is the **cosmological boost factor**

This is already present for NFW halos, but it is expected to be much more important for UCMHs, since they are typically denser and they form earlier

How is $B(z)$ computed?

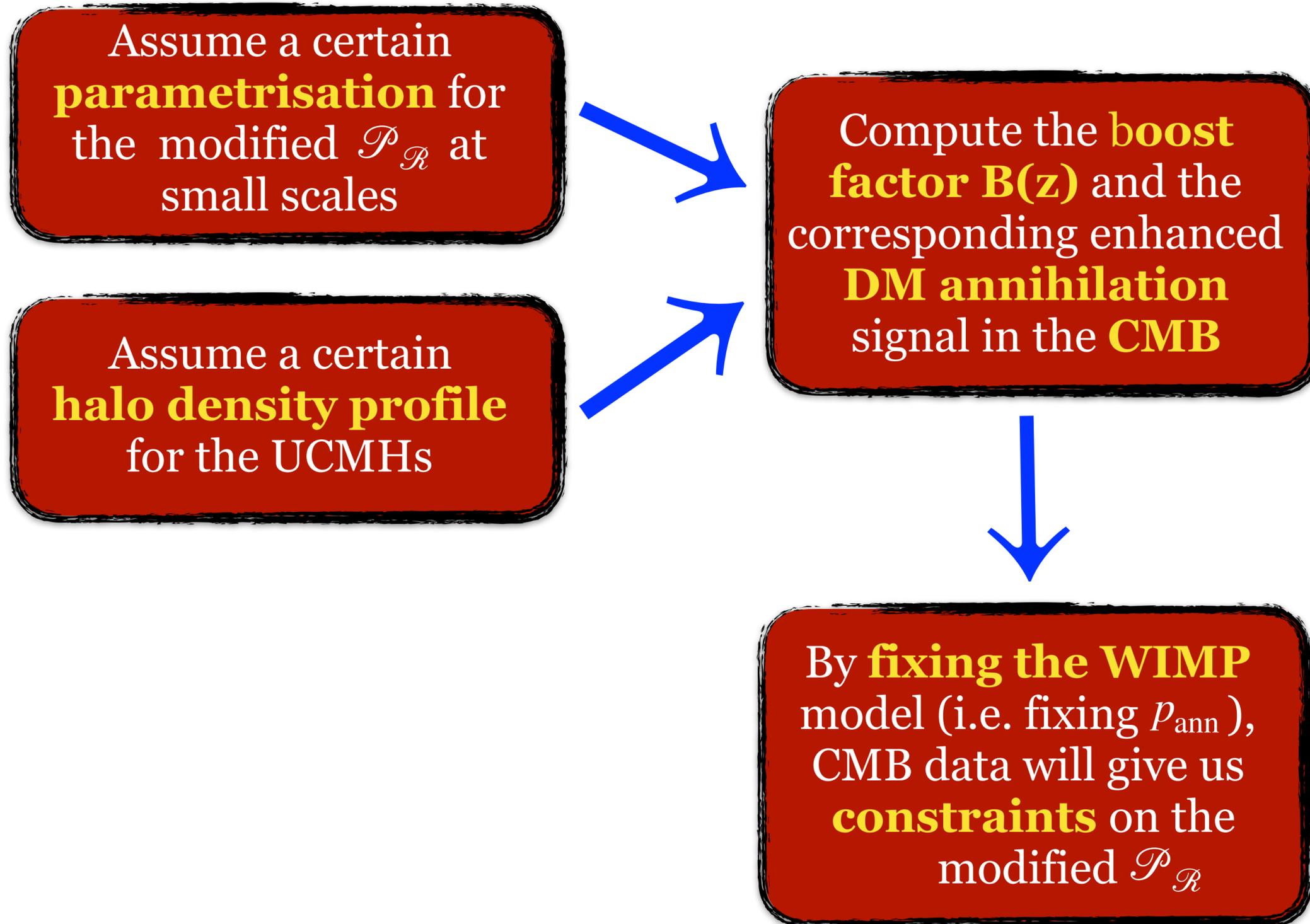
$$B(z) = (1+z)^{-3} (\bar{\rho}_m)^{-2} \int_{M_{\text{min}}}^{\infty} dM \frac{dn}{dM}(M, z) \int_0^{R(M)} dr 4\pi r^2 \rho^2(r)$$

Halo mass function
Depends on $\mathcal{P}_{\mathcal{R}}$!

Halo density profile

Generic properties of Ultra Compact Mini Halos

Recipe to get the constraints



Generic properties of Ultra Compact Mini Halos

Recipe to get the constraints

Assume a certain **parametrisation** for the modified $\mathcal{P}_{\mathcal{R}}$ at small scales

Assume a certain **halo density profile** for the UCMHs

Compute the **boost factor $B(z)$** and the corresponding enhanced **DM annihilation signal** in the **CMB**

By **fixing the WIMP** model (i.e. fixing P_{ann}), CMB data will give us **constraints** on the modified $\mathcal{P}_{\mathcal{R}}$

- Which density profile?
- How to compute the halo mass function?

Generic properties of Ultra Compact Mini Halos

Density profile of UCMHs?

- **Bringmann et al. (2013)**: They considered the $\rho \sim r^{-9/4}$ profile for UCMHs, based on the theory of self-similar secondary infall (**Bertschinger (1985)**). They obtained constraints on $\mathcal{P}_{\mathcal{R}}$ using galactic γ - rays
- **Nakama et al. (2017)**: They also constrained $\mathcal{P}_{\mathcal{R}}$ with γ - rays, but used a NFW profile for the UCMHs
- **Natarajan et al. (2015)**: They constrained $\mathcal{P}_{\mathcal{R}}$ using Planck CMB data, assuming again a NFW profile for the UCMHs
- **Delos et al. (2018)**: By means of N-body simulations, they claim that the correct inner halo profile is $\rho \sim r^{-3/2}$. They got new constraints using γ - rays

Generic properties of Ultra Compact Mini Halos

Density profile of UCMHs?

Following [Delos et al. \(2018\)](#), we assume the following density profile

$$\rho_{\text{UCMH}}(r) = \frac{\rho_s}{(r/r_s)^{3/2}(1+r/r_s)^{3/2}} \begin{cases} \nearrow \sim r^{-3/2} \text{ at small } r \\ \searrow \sim r^{-3} \text{ at big } r \end{cases}$$

arising from a spike in the small-scale primordial spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = A_0 k_s \delta(k - k_s)$$

According to the N-body of [Delos et al. \(2018\)](#):

$$\rho_s \simeq 30 \bar{\rho}_m (1 + z_F)^3 \quad \text{and} \quad r_s \simeq 0.7 k_s^{-1} (1 + z_F)^{-1}$$

Goal: Using this newly proposed profile, compute the boost factor and update Planck constraints on A_0 and k_s

The halo mass function and the Press-Schechter formalism

How to compute the halo mass function?

Assume a **smoothed** density field $\delta_R = \int W(\mathbf{x}; R)\delta(\mathbf{x})d^3\mathbf{x}$

where $W(\mathbf{x}; R)$ is a window function corresponding to a mass $M = (4\pi/3)\bar{\rho}_M R^3$

Main ansatz: The fraction of halos with mass greater than M is the same as the probability that δ_R will exceed the critical threshold $\delta_c = \frac{3}{5} \left(\frac{3\pi}{2}\right)^{2/3} \simeq 1.686$
(according to the spherical collapse model)

If δ_R is a **Gaussian** random field, then

$$P_{\delta_R > \delta_c} = \frac{1}{\sqrt{2\pi}\sigma} \int_{\delta_c}^{\infty} \exp\left[-\frac{\delta_R^2}{2\sigma^2}\right] d\delta_R = \frac{1}{2} \operatorname{erfc}\left[\frac{\delta_c}{\sqrt{2}\sigma}\right]$$

where $\sigma^2(M, z) = \int_0^{\infty} d\log k \mathcal{P}_m(k, z) \tilde{W}^2(kR)$ is the **mass variance** of δ_R

and $\mathcal{P}_m(k, z) \sim D^2(z)T(k)\mathcal{P}_{\mathcal{R}}(k)$ is the linear matter spectrum

The halo mass function and the Press-Schechter formalism

How to compute the halo mass function?

One can show that for $M \rightarrow 0$, then $P_{\delta_R > \delta_c} \rightarrow 1/2$ i.e., *only half of mass of the universe is in collapse objects!* Add fudge factor to account for that, $F(> M) = 2P_{\delta_R > \delta_c}$

Nota Bene: This can be rigorously proven using excursion set theory

Then the halo mass function is readily obtained (define $\nu \equiv \delta_c/\sigma$)

$$n(M, z)dM = \frac{\bar{\rho}_M}{M} \frac{\partial F(> M)}{\partial M} dM = \frac{\bar{\rho}_M}{M^2} f_{\text{PS}}(\nu) \frac{d \log \nu}{d \log M} dM$$

where $f_{\text{PS}}(\nu) = \sqrt{\frac{2}{\pi}} \nu \exp(-\nu^2/2)$

A more accurate treatment, that considers *ellipsoidal* collapse (rather than spherical) is the one by **Sheth and Tormen**

$$f_{\text{ST}}(\nu) = A \left(1 + \frac{1}{\tilde{\nu}^{2q}} \right) f_{\text{PS}}(\tilde{\nu}) \quad \text{with} \quad A \simeq 0.322, \quad \tilde{\nu} = 0.84\nu \quad \text{and} \quad q = 0.3$$

Computing the cosmological boost factor

Split the contribution from the two kind of halos

$$B(z) = 1 + B_{\text{NFW}}(z) + B_{\text{UCMH}}(z)$$

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1} \quad \mathcal{P}_{\mathcal{R}}(k) = A_0 k_s \delta(k - k_s)$$

$$B_i(z) = (1+z)^{-3} (\bar{\rho}_m)^{-2} \int_{M_{\min}}^{\infty} dM \frac{dn}{dM}(M, z) \int_0^{R(M)} dr 4\pi r^2 \rho_i^2(r)$$

Nota Bene: M_{\min} depends on the WIMP model, but it lies in the range $10^{-6} - 10^{-9} M_{\odot}$

Computing the cosmological boost factor

Calculation of the NFW part

A knowledge about $\mathbf{R(M)}$ is required for the integral over the density profile

A good estimator is

$$\frac{M}{(4\pi/3)R^3} = \Delta_c \bar{\rho}_M(z) \text{ with } \Delta_c \simeq 200$$

The integration over r then yields

$$B_{\text{NFW}}(z) = \frac{\Delta_c}{3\bar{\rho}_M^0} \int_{M_{\min}}^{\infty} dM M \frac{dn}{dM}(M, z) f(c(M, z))$$

where $f(c) = \frac{c^3}{3} \left[1 - \frac{1}{(1+c)^3} \right] \left[\log(1+c) - \frac{c}{1+c} \right]^{-2}$

and we defined the **concentration parameter** $c \equiv R/r_s$

For $c(M, z)$ we take the power-law fit from [Neto et al. \(2007\) 0706.2919](#)

Computing the cosmological boost factor

Calculation of the UCMH part

The integral over the UCMH density profile **diverges** at small r

This is solved by noting that *WIMP annihilations flatten the inner core*

$$\rho_{\text{UCMH}}(r \leq r_{\text{cut}}) \equiv \rho_{\text{max}} = \frac{m_{\text{DM}}}{\langle \sigma v \rangle (t - t_F)}$$

The integration over r gives

$$\int_0^R dr 4\pi r^2 \rho_{\text{UCMH}}^2(r) = 4\pi \rho_s^2 r_s^3 \left[\frac{1}{3} + \frac{2c+3}{2(c+1)^2} + \log\left(\frac{c}{1+c}\right) - \frac{2D^{-1}+3}{2(D^{-1}+1)^2} + \log(1+D) \right]$$

where $D \equiv (\rho_{\text{max}}/\rho_s)^{2/3}$

Previous expression can be highly simplified by noting that $D, c \gg 1$

Computing the cosmological boost factor

Calculation of the UCMH part

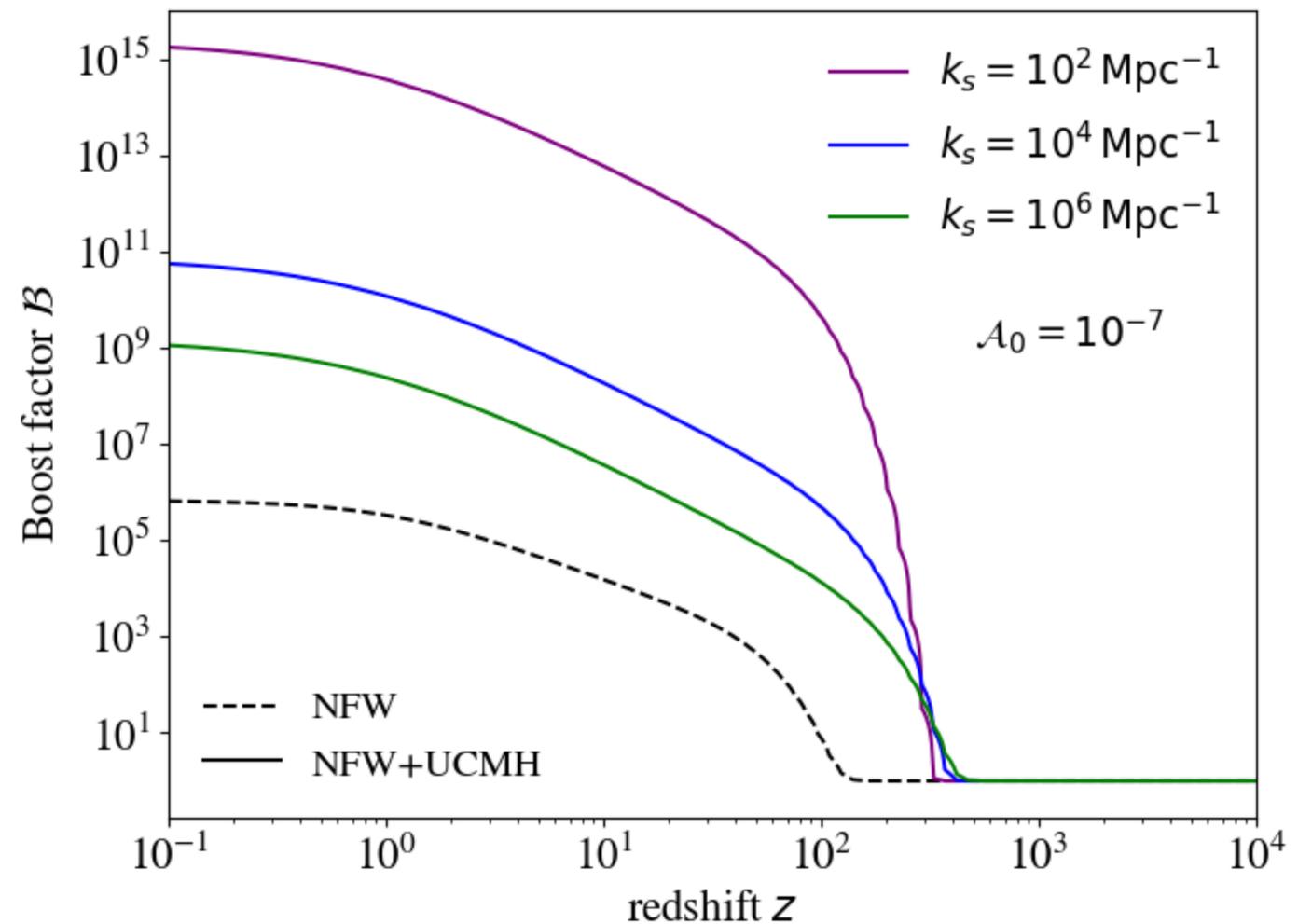
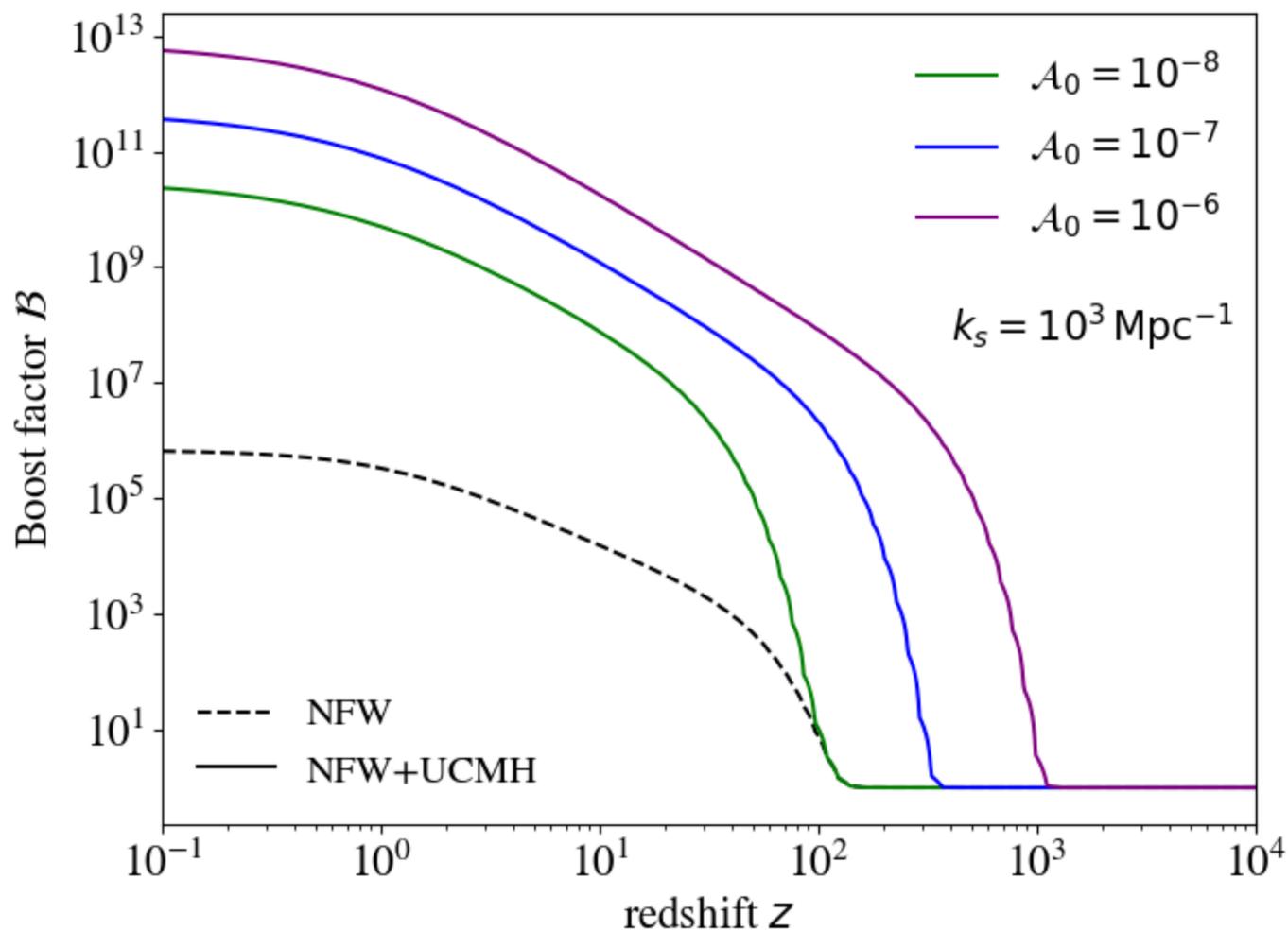
The final result reads

$$B_{\text{NFW}}(z) = \frac{c_1}{(1+z)^3 k_s^3} \int_{M_{\text{min}}}^{\infty} dM \frac{dn}{dM}(M, z) [1 + z_F(M)]^3 \log \left\{ \frac{c_2 m_{\text{DM}}}{\bar{\rho}_M \langle \sigma v \rangle [1 + z_F(M)]^3 t(z)} \right\}$$

where the effective redshift of halo formation is estimated from $\sigma(M, z_F) \sim 1$

Computing the cosmological boost factor

Main results: We take $m_{\text{DM}} = 100 \text{ GeV}$, $\langle\sigma v\rangle = 3 \times 10^{-26} \text{ cm}^3\text{s}^{-1}$, $M_{\text{min}} = 10^{-6} M_{\odot}$



The spiky spectrum **enhances** the boost factor and leads to an **earlier** halo formation

The more standard NFW scenario is recovered for $\mathcal{A}_0 \rightarrow 0$ and $k_s \rightarrow \infty$

Prospects and conclusions

- Include (at least in an heuristic way) the effects due to **halo mergers**
- Implement boost factor calculation in **CLASS** and get updated constraints on A_0 and k_s using the most recent Planck data
- Translate these into constraints on the early universe (EMDE or inflation)

Ultra Compact Mini Halos are excellent probes of the nature of Dark matter and can also give us hints about the Early universe

THANKS FOR YOUR ATTENTION