

Gegenbauer's Twin

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Gegenbauer Goldstones, JHEP 01 (2022) 076, [2110.06941]
Gegenbauer's Twin, JHEP 05 (2022) 140, [2202.01228]
with Matthew McCullough and Ennio Salvioni



Ten years after its discovery, the Higgs remains a mystery.

Why does it look lighter than the SM cutoff?

Why do its couplings look SM-like?

Are symmetries sufficient to explain this?

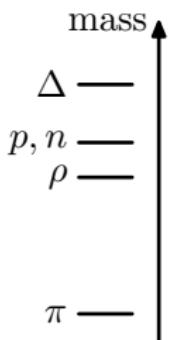
Lighter than the cutoff

Global spontaneous sym. breaking leads to massless scalars,
Nambu-Goldstone bosons (NGBs).

Small explicit symmetry breakings lead to small masses.
NGBs become pNGBs.

e.g. light pions

SSB: $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$
ESB: quark masses/charges



Same structure reproduced in composite pNGB Higgs models,
with the condensation of a new strong sector at the scale f .

SM-like couplings

In vanilla composite Higgs models,
Higgs coupling modifications are controlled by v^2/f^2 ,
and naturally of $\mathcal{O}(1)$.

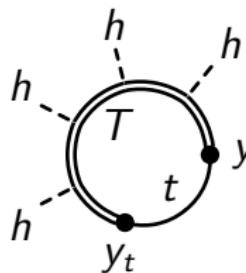
Experimental constraints require $v^2/f^2 \ll 1$,
which can only be achieved through fine-tuning.

Is this fine-tuning *generic*
or *specific* to vanilla models?

Vanilla composite Higgs

Minimal $\text{SO}(5) \rightarrow \text{SO}(4)$ spontaneous breaking

Minimal explicit breaking from top and gauge sectors

$$V(h) \sim$$

$$\sim \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \delta \sin^4 \frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad \text{vs.}$$

$$\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}} \gtrsim 0.94$$

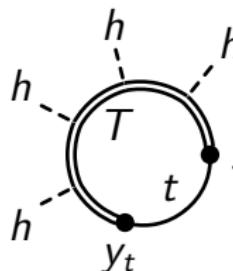
$$\rightarrow m_h^2 = 4\kappa \frac{y_t^2 N_c}{16\pi^2} M_T^2 \left(1 - \frac{1}{2\delta} \right)$$

$$\text{vs.} \quad M_T \gtrsim 1.3 \text{ TeV}$$

Vanilla composite Higgs

Minimal $\text{SO}(5) \rightarrow \text{SO}(4)$ spontaneous breaking

Minimal explicit breaking from top and gauge sectors

$$V(h) \sim$$

$$\sim y_t + \dots \sim \kappa \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \delta \sin^4 \frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad 1/\delta \lesssim 0.16 \quad \frac{vv}{M_{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}} \gtrsim 0.94$$

$$\rightarrow m_h^2 = 4\kappa \frac{y_t^2 N_c}{16\pi^2} M_T^2 \left(1 - \frac{1}{2\delta} \right) \quad \kappa \lesssim 0.13 \quad M_T \gtrsim 1.3 \text{ TeV}$$

Few percent tuning wrt $\delta \lesssim 1$, $\kappa \simeq 1$ expectation

Natural $v^2/f^2 \ll 1$ recipe

New non-minimal source of explicit breaking,
leading to a potential with structure at small field values,
radiatively stable, to avoid fine-tuning.

Radiatively stable pNGB potentials

Abelian example

Spontaneously broken global U(1)

One Nambu-Goldstone boson Π

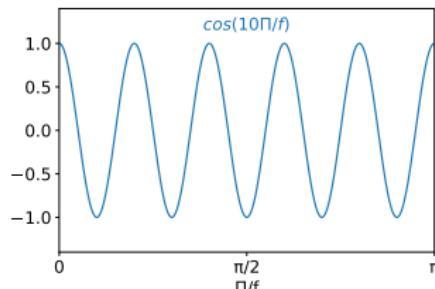
Non-linear field parameterisation: $\phi = e^{i\Pi/f}$

Small explicit breaking by a spurion K of charge $Q_K = -nQ_\phi$:

$$V \sim K \phi^n \sim \cos \frac{n\Pi}{f}$$

No other possible invariant, linear in K

Radiatively stable structure on $\frac{\Pi}{f} \sim \frac{1}{n}$ scale



Non-Abelian case

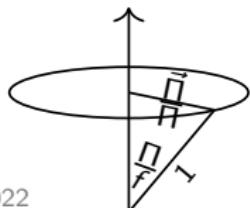
Spontaneously broken global $\text{SO}(N+1) \rightarrow \text{SO}(N)$

Same explicit breaking pattern

Unspecified strong sector in the UV

N Nambu-Goldstone bosons $\vec{\Pi}$ in the low-energy EFT

Field parameterisation: $\phi = (\frac{\vec{\Pi}}{\Pi} \sin \frac{\Pi}{f}, \cos \frac{\Pi}{f})$
with $\Pi \equiv |\vec{\Pi}|$



All-loop stability, at linear order

Explicit breaking by an irrep spurion (symmetric traceless)

$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n}$$

No other invariant, linear in K , can be constructed,
so all-loop linear renormalisation can only be multiplicative.

For $\text{SO}(N+1) \rightarrow \text{SO}(N)$, Gegenbauer polynomials:

$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n} = G_n^{(N-1)/2}(\cos \frac{\Pi}{f})$$

$$\phi = (\frac{\vec{\Pi}}{\Pi} \sin \frac{\Pi}{f}, \cos \frac{\Pi}{f})$$

One-loop stability, at linear order

Linear one-loop correction to $V(\frac{\Pi}{f})$:

$$\frac{\Lambda^2}{32\pi^2 f^2} \left(V'' + (N-1) \cot \frac{\Pi}{f} V' \right)$$

Radiative stability at one-loop and linear order if $\curvearrowright \propto V$

Differential equation of Gegenbauer polynomials

$$V\left(\frac{\Pi}{f}\right) \propto G_n^{(N-1)/2} \left(\cos \frac{\Pi}{f}\right)$$

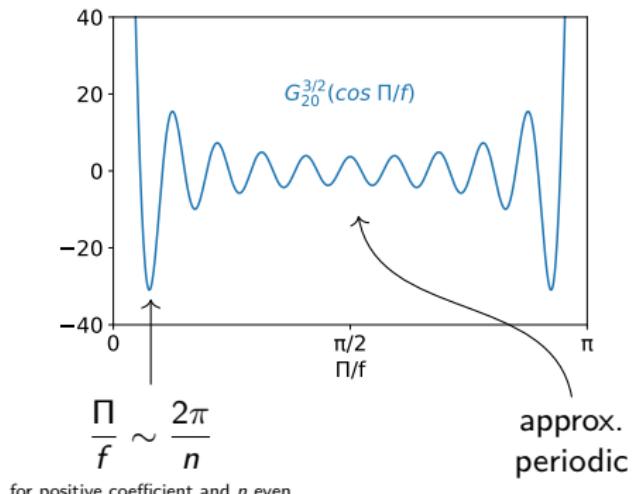
Gegenbauer polynomials

$G_n^{(N-1)/2}(\cos \frac{\Pi}{f})$ are eigenfunctions of linear renormalisation
for $\text{SO}(N+1) \rightarrow \text{SO}(N)$ pNGB potentials.[†]

Generalisation of $\cos(n\Pi/f)$ for $N > 1$
of Legendre polynomials for $N > 2$
i.e. partial wave/multipole expansion in *field* space



L.B. Gegenbauer
1849–1903



[†]Brezin, Zinn-Justin, Le Guillou '76: RGE eigenfunctions in 2D

Gegenbauer Higgs

relaxing v/f tuning

Pure Gegenbauer potential (unrealistic)

$N = 4$ for minimal $\text{SO}(5) \rightarrow \text{SO}(4)$ composite Higgs

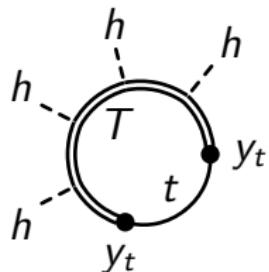
$$\frac{v}{f} = \sin \frac{\langle h \rangle}{f} \approx \frac{5.1}{n}$$
 is naturally small for sizeable n

→ small Higgs coupling modifications: $\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}}$

→ opposite trilinear self-coupling: $\frac{c_{hhh}}{c_{hhh}^{\text{SM}}} = -\sqrt{1 - \frac{v^2}{f^2}}$

Leading top-sector contribution

The top sector provides sizeable explicit breaking.



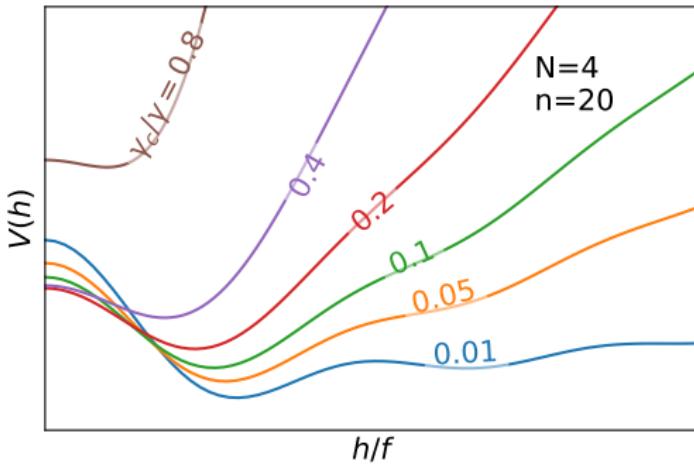
Leading contribution to potential of the form

$$V_{\text{top}}(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left(+ \sin^2 \frac{h}{f} \right)$$

positive sign now

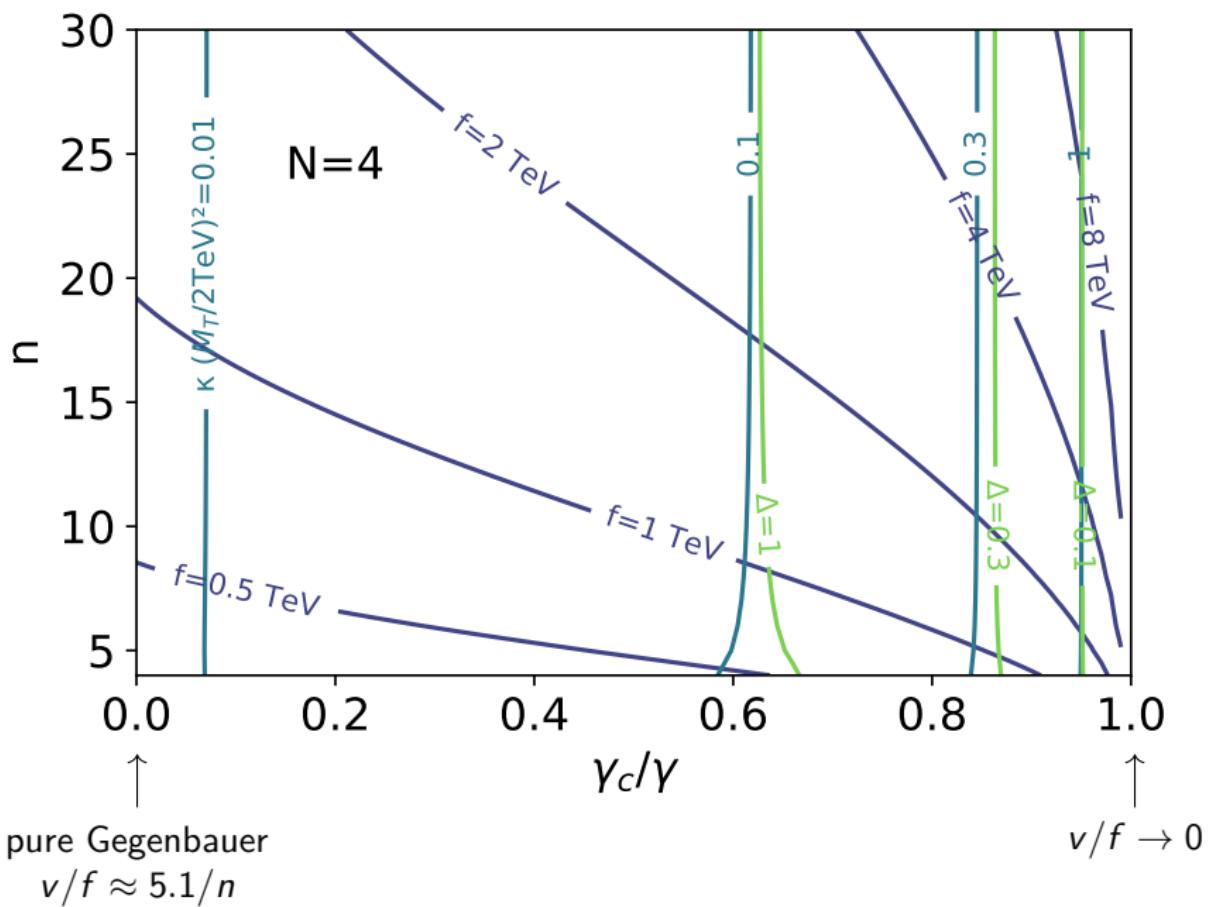
Top+Gegenbauer potential (more realistic)

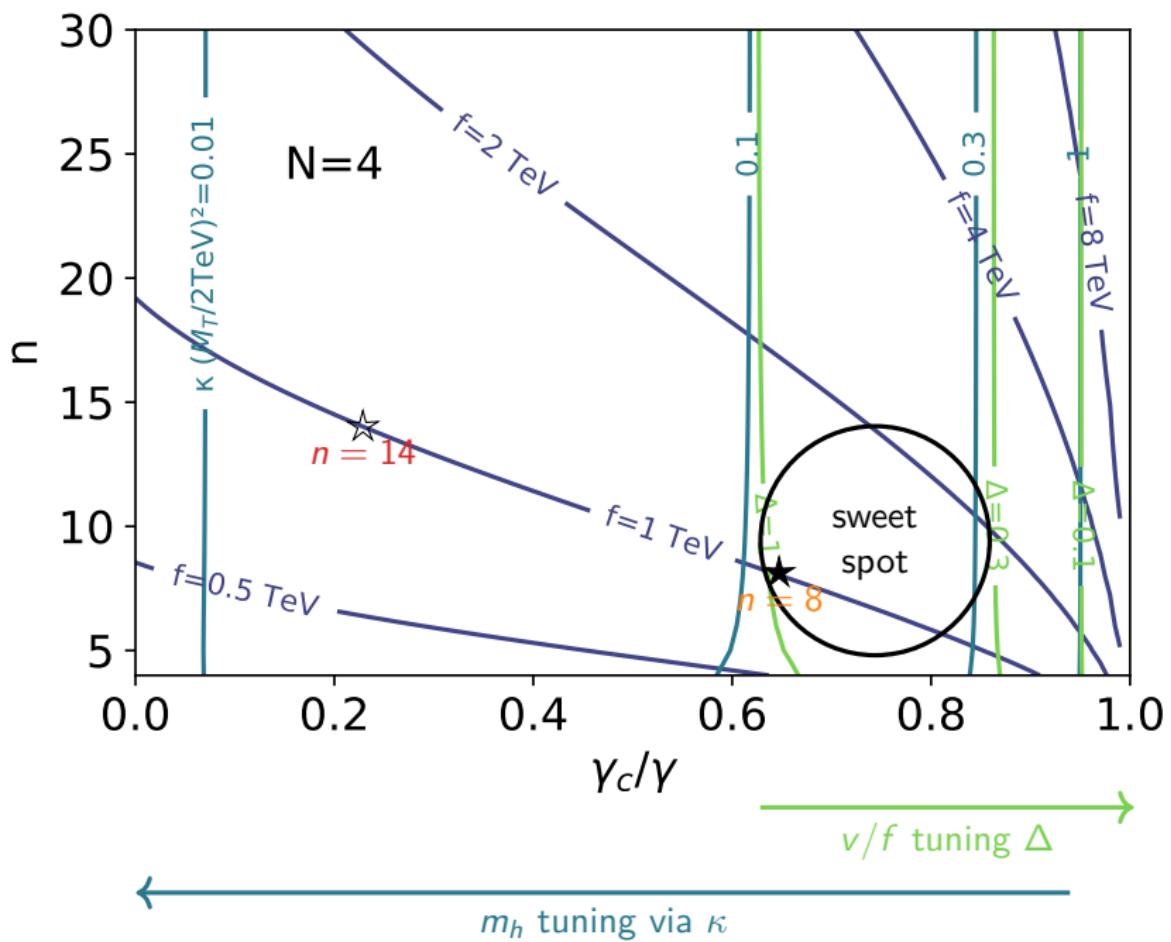
$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma G_n^{3/2} \left(\cos \frac{h}{f} \right) \right]$$



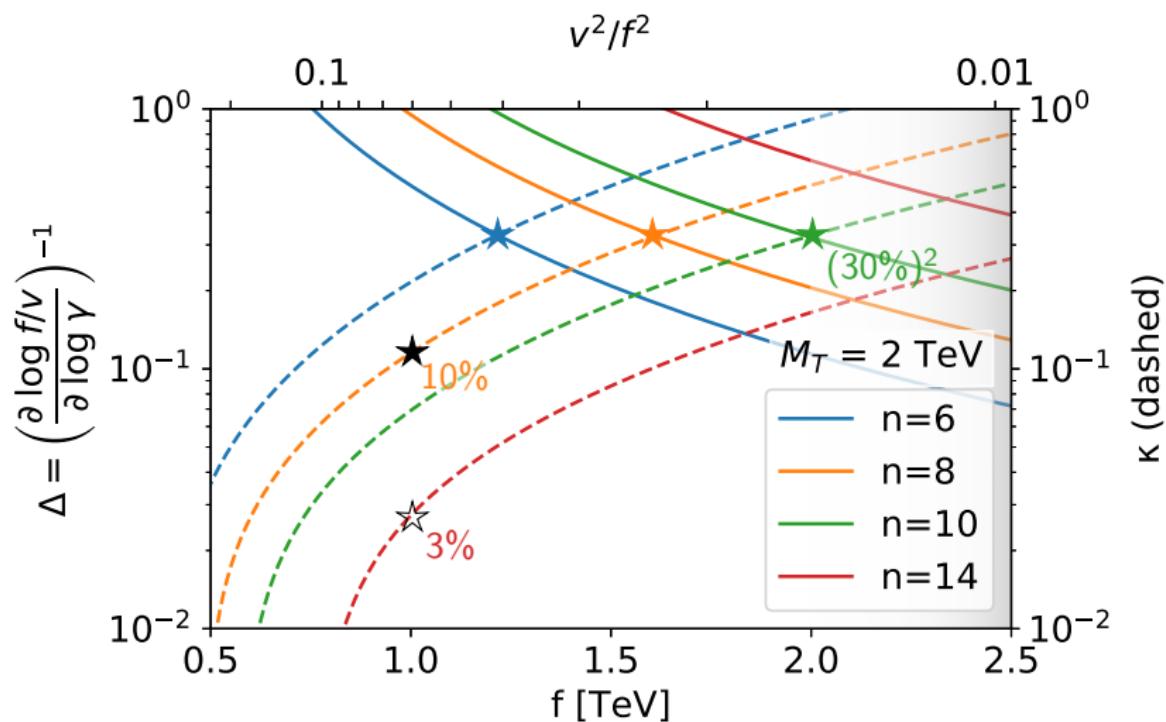
$v/f \rightarrow 0$ as $\gamma \rightarrow \gamma_c$

$\frac{m_h^2}{\kappa \frac{N_c y_t^2}{16\pi^2} M_T^2} \rightarrow$ as $\gamma \rightarrow \gamma_c$ relaxing $\kappa \rightarrow 1$





v/f and m_h tunings

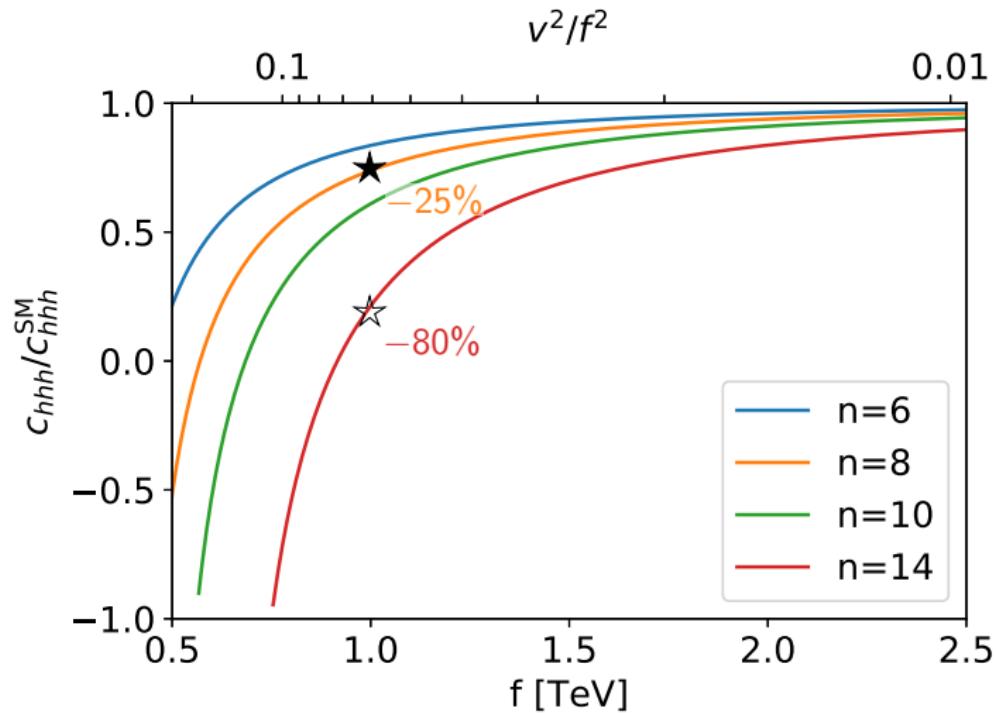


$$\Delta \approx 30\% \left(\frac{f}{4v} \frac{5.1}{n} \right)^{-2.1}$$

$$\kappa \approx 30\% \left(\frac{f}{4v} \frac{5.1}{n} \frac{2 \text{ TeV}}{M_T} \right)^2$$

$(M_T \gtrsim y_t f / \sqrt{2}$ since $m_t \sim M_T v/f$ for $y_t f \gg M_T$)

Trilinear Higgs self-coupling



$$\frac{c_{hhh}}{c_{hhh}^{SM}} \approx 1 - 1.2 \left(\frac{f}{v} \frac{5.1}{n + \lambda} \right)^{-2} \quad \text{when close to 1}$$

Gegenbauer Higgs benchmark phenomenology

$n = 8$ and $f \sim 1 \text{ TeV}$, $M_T \sim 2 \text{ TeV}$

- v/f and m_h totalising $\sim 10\%$ tuning
- top partners just escape HL-LHC searches
- Higgs coupling modifications $v^2/2f^2 \lesssim 3\%$ $\lesssim 2.6\% @\text{HL-LHC}, 2\sigma$
- Higgs self-coupling modification $\lesssim 25\%$ $\lesssim 100\% @\text{HL-LHC}, 2\sigma$
[ECFA '19]

Gegenbauer Higgs benchmark phenomenology

$n = 8$ and $f \sim 1 \text{ TeV}$, $M_T \sim 2 \text{ TeV}$

→ v/f and m_h totalising $\sim 10\%$ tuning

→ top na

Natural composite Higgs
still probed at the LHC and beyond!

$\tau \lesssim 3\%$

$\lesssim 2.6\% @\text{HL-LHC}, 2\sigma$

→ Higgs self-coupling modification $\lesssim 25\%$

$\lesssim 100\% @\text{HL-LHC}, 2\sigma$

[ECFA '19]

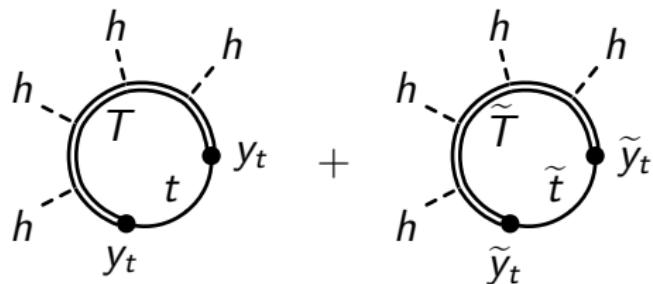
Gegenbauer's Twin

relaxing v/f and m_h tunings

Adding twins

[Chacko, Goh, Harnik '05]

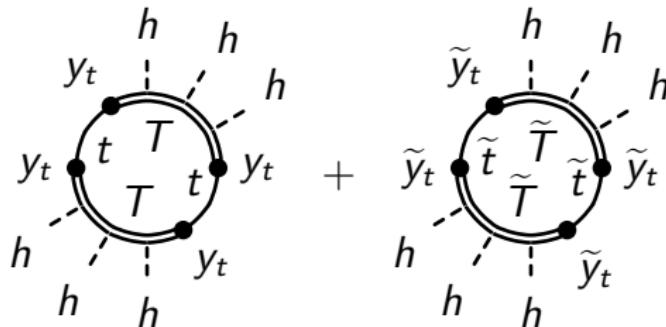
\tilde{T} EW charged, \tilde{t} neutral
(both charged under a $SU(3)_{\tilde{c}}$)



$$\frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \sin^2 \frac{h}{f} + \frac{N_c \tilde{y}_t^2}{16\pi^2} f^2 M_{\tilde{T}}^2 \cos^2 \frac{h}{f}$$

if twin parity enforces $y_t = \tilde{y}_t$ and $M_T = M_{\tilde{T}}$
no $\mathcal{O}(M_T^2)$ potential contribution

Adding twins



$$\frac{N_c y_t^4}{16\pi^2} f^4 \sin^4 \frac{h}{f} \log M_T + \frac{N_c \tilde{y}_t^4}{16\pi^2} f^4 \cos^4 \frac{h}{f} \log M_{\tilde{T}}$$

retaining $\mathcal{O}(\log M_T)$ dependence only

additional breaking required for viable EWSB

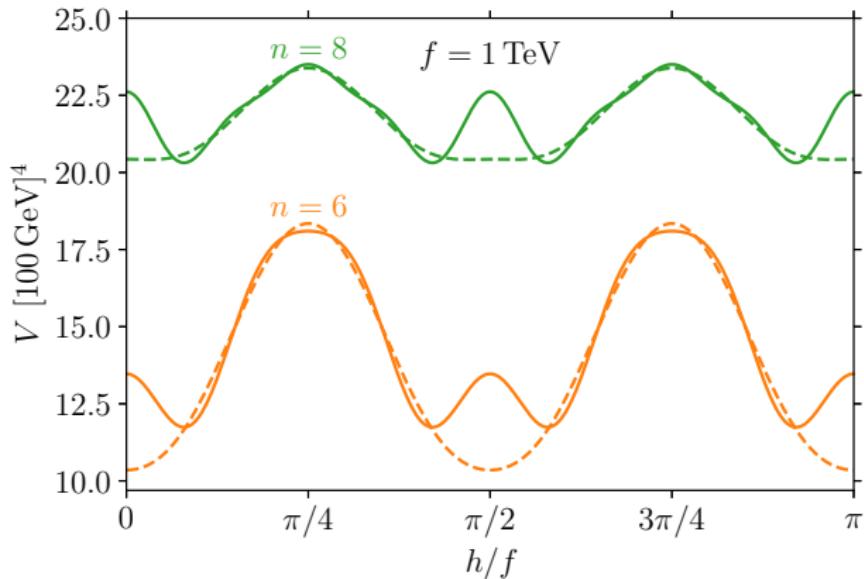
Realisation

- global spontaneous SB: $\text{SO}(8) \supset \text{SO}(4) \times \widetilde{\text{SO}(4)}$
 $\rightarrow \text{SO}(7)$
- 7 NGB generated
6 eaten by gauged $\text{SU}(2)_L \times \widetilde{\text{SU}(2)}_L$
leaving $\phi = (\vec{0}_3, \sin h/f, \vec{0}_3, \cos h/f)^T$ in the unitary gauge

[cf. Barbieri, Greco, Rattazzi, Wulzer '15]

- additional explicit SB: $\text{SO}(8) \rightarrow \text{SO}(4) \times \widetilde{\text{SO}(4)}$
by $2n$ -index irrep generating $G_n^{3/2}(\cos 2h/f)$ potential
- EWSB and twin parity spontaneously broken for n even

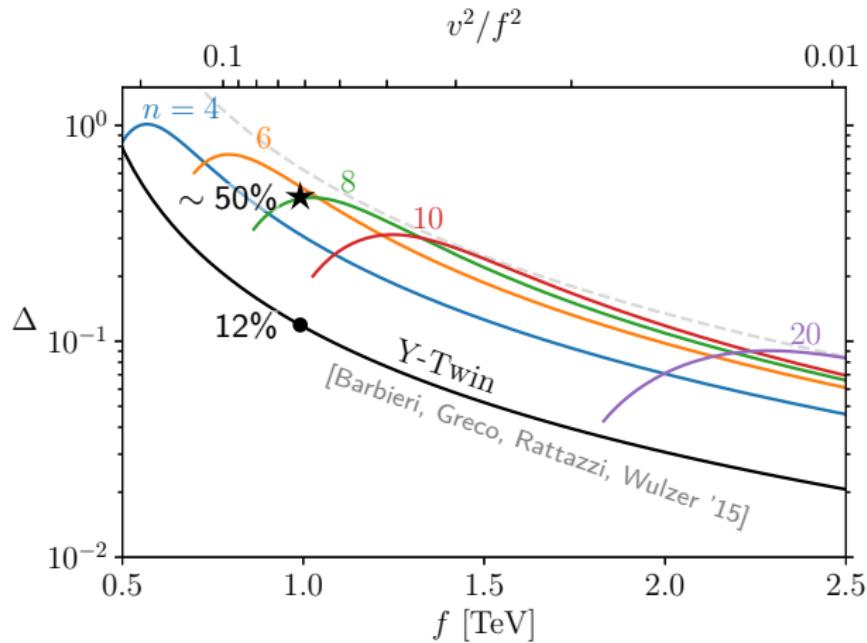
Top+Gegenbauer potential



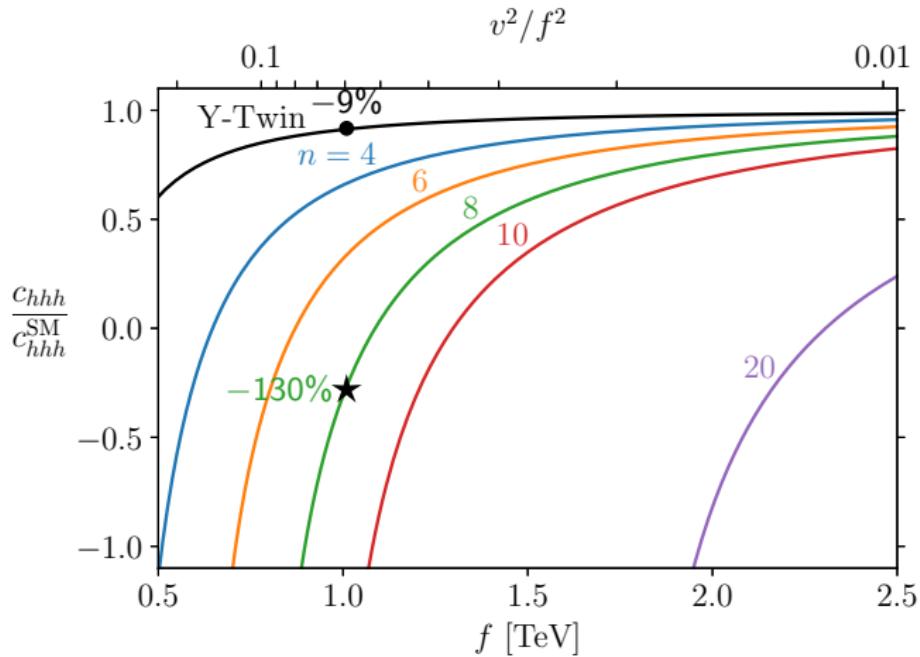
dashed: top only
solid: full

v/f and m_h tunings

- conservative definition RMS(eig. log-derivative matrix)
- dominated by top-sector dependence of v/f
- about 4 times better than usual $\Delta \approx 2v^2/f^2$ minimum

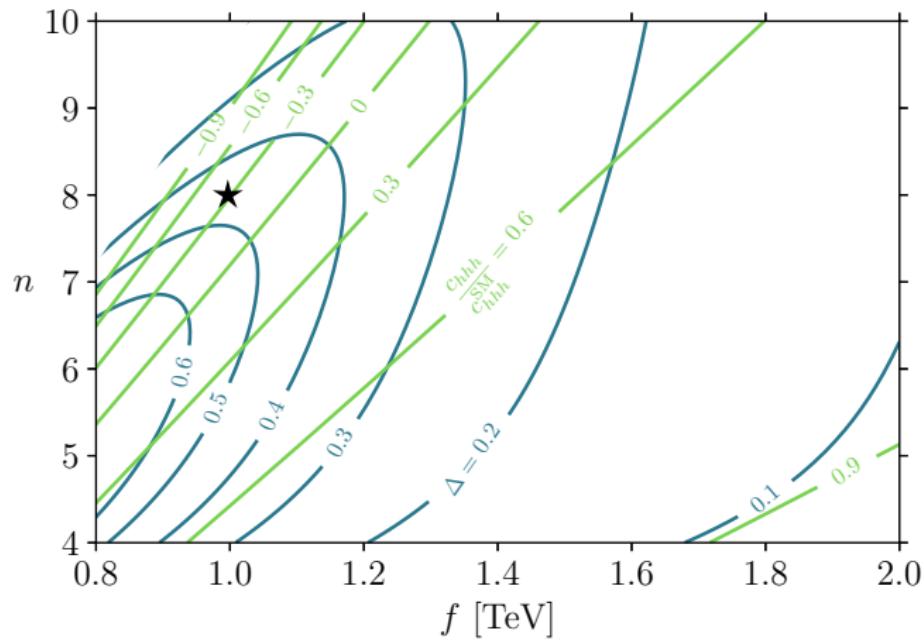


Trilinear Higgs self-coupling



Irreducible deviations

in the **natural** parameter space
for c_{hVV} and c_{hhh}



Gegenbauer's Twin benchmark phenomenology

$n = 8$ and $f \sim 1 \text{ TeV}$

- total tuning $\sim 50\%$
- M_T in the multi-TeV (unitarity violated in H scattering towards 6 TeV)
- single Higgs couplings $\sim 3\%$ $\lesssim 2.6\% @\text{HL-LHC}, 2\sigma$
- Higgs self-coupling $\sim 130\%$ $\lesssim 100\% @\text{HL-LHC}, 2\sigma$
[ECFA '19]

Gegenbauer's Twin benchmark phenomenology

$n = 8$ and $f \sim 1 \text{ TeV}$

→ total tuning $\sim 50\%$

→ M_T in the range $[1.5 \text{ TeV}, 6 \text{ TeV}]$ could be the first signal of new physics!

→ Higgs self-coupling $\lesssim 2.6\% @ \text{HL-LHC, } 2\sigma$

→ Higgs self-coupling $\sim 130\%$

$\lesssim 100\% @ \text{HL-LHC, } 2\sigma$

[ECFA '19]

Gegenbauer's Twin

Gegenbauer potentials are eigenfunctions of linear renorm.
for $\text{SO}(N+1) \rightarrow \text{SO}(N)$ pNGBs.

They naturally suppress v/f ,
resulting in $\mathcal{O}(1\%)$ single Higgs coupling deviations,
if the explicit breaking occurs in a high irrep.

The trilinear Higgs self-coupling receives $\mathcal{O}(100\%)$
modifications in the natural parameter space.

Symmetry-based naturalness is still being probed at the LHC,
and could survive it!