WIMP Minimal Dark Matter (and charged resonances) at a future muon collider

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Mainly based on:

- S. Bottaro, A. Strumia, NV, JHEP 06 (2021) 143
- NV, arXiv:2304.12362

Outline

- WIMP Minimal Dark Matter

  thermal freeze-out (Sommerfeld Enhancement and Bound State Formation)

- Status of the search for WIMPs
- Opportunities offered by a multi-TeV Muon Collider (MuCol)
- MDM bound state production at a MuCol

WIMP Minimal Dark Matter

Cirelli, Fornengo, Strumia,
Nucl.Phys.B 753 (2006) 178-194

The minimal solution to the DM puzzle:

simply add to the SM an EW multiplet

$$\mathscr{L} = \mathscr{L}_{\rm SM} + c \begin{cases} \bar{\mathcal{X}}(i\not{D} + M)\mathcal{X} & \text{when } \mathcal{X} \text{ is a spin 1/2 fermionic multiplet} \\ |D_\mu \mathcal{X}|^2 - M^2|\mathcal{X}|^2 & \text{when } \mathcal{X} \text{ is a spin 0 bosonic multiplet} \end{cases}$$

χ is an *n*-tuplet of the SU(2)L gauge group, $n=\{1,2,3,4,5, ...\}$

The neutral component is the lightest, is (automatically) stable and is a good DM candidate

WIMP Minimal Dark Matter

2006 status:

Cirelli, Fornengo, Strumia,
Nucl.Phys.B 753 (2006) 178-194

Quantum numbers $SU(2)_L$	$U(1)_Y$	Spin	DM can decay into	DM mass in TeV	$m_{DM^\pm} - m_{DM}$ in MeV	Events at LHC $\int \mathcal{L}\,dt = 100/\text{fb}$	σ_{SI} in $10^{-45}\,\text{cm}^2$
2	1/2	0	EL	0.54 ± 0.01	350	$320 \div 510$	0.2
2	1/2	1/2	EH	1.1 ± 0.03	341	$160 \div 330$	0.2
3	0	0	HH^*	2.0 ± 0.05	166	$0.2 \div 1.0$	1.3
3	0	1/2	LH	2.4 ± 0.06	166	$0.8 \div 4.0$	1.3
3	1	0	HH, LL	1.6 ± 0.04	540	$3.0 \div 10$	1.7
3	1	1/2	LH	1.8 ± 0.05	525	$27 \div 90$	1.7
4	1/2	0	HHH^*	2.4 ± 0.06	353	$0.10 \div 0.6$	1.6
4	1/2	1/2	(LHH^*)	2.4 ± 0.06	347	$5.3 \div 25$	1.6
4	3/2	0	HHH	2.9 ± 0.07	729	$0.01 \div 0.10$	7.5
4	3/2	1/2	(LHH)	2.6 ± 0.07	712	$1.7 \div 9.5$	7.5
5	0	0	(HHH^*H^*)	5.0 ± 0.1	166	$\ll 1$	12
5	0	1/2	–	4.4 ± 0.1	166	$\ll 1$	12
7	0	0	–	8.5 ± 0.2	166	$\ll 1$	46

Table 1: **Summary of the main properties of Minimal DM candidates**. *Quantum numbers are listed in the first 3 columns; candidates with $Y \neq 0$ are allowed by direct DM searches only if appropriate non-minimalities are introduced. The 4th column indicates dangerous decay modes, that need to be suppressed (see sec. 2 for discussion). The 5th column gives the DM mass such that the thermal relic abundance equals the observed DM abundance (section 4). The 6th column gives the loop-induced mass splitting between neutral and charged DM components (section 3); for scalar candidates a coupling with the Higgs can give a small extra contribution, that we neglect. The 7th column gives the 3σ range for the number of events expected at LHC (section 6). The last column gives the spin-independent cross section, assuming a sample vale $f = 1/3$ for the uncertain nuclear matrix elements (section 5).*

Thermal targets

- MDM thermal relic produced via freeze out: the relic abundance is calculable and depends on one parameter: the mass M
- Important corrections that must be taken into account:

  Sommerfeld enhancement (SE), bound state formation (BSF)

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↓

Enhancement of the annihilation cross section at low relative velocities

J. Hisano, S. Matsumoto, and M. M. Nojiri, Phys. Rev. Lett. 92, 031303 (2004)
J. Hisano, S. Matsumoto, M. Nagai, O. Saito, and M. Senami, Phys. Lett. B 646, 34 (2007),
N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D 79, 015014 (2009

Thermal targets

- MDM thermal relic produced via freeze out: the relic abundance is calculable and depends on one parameter: the mass M
- Important corrections that must be taken into account:

  Sommerfeld enhancement (SE), bound state formation (BSF)

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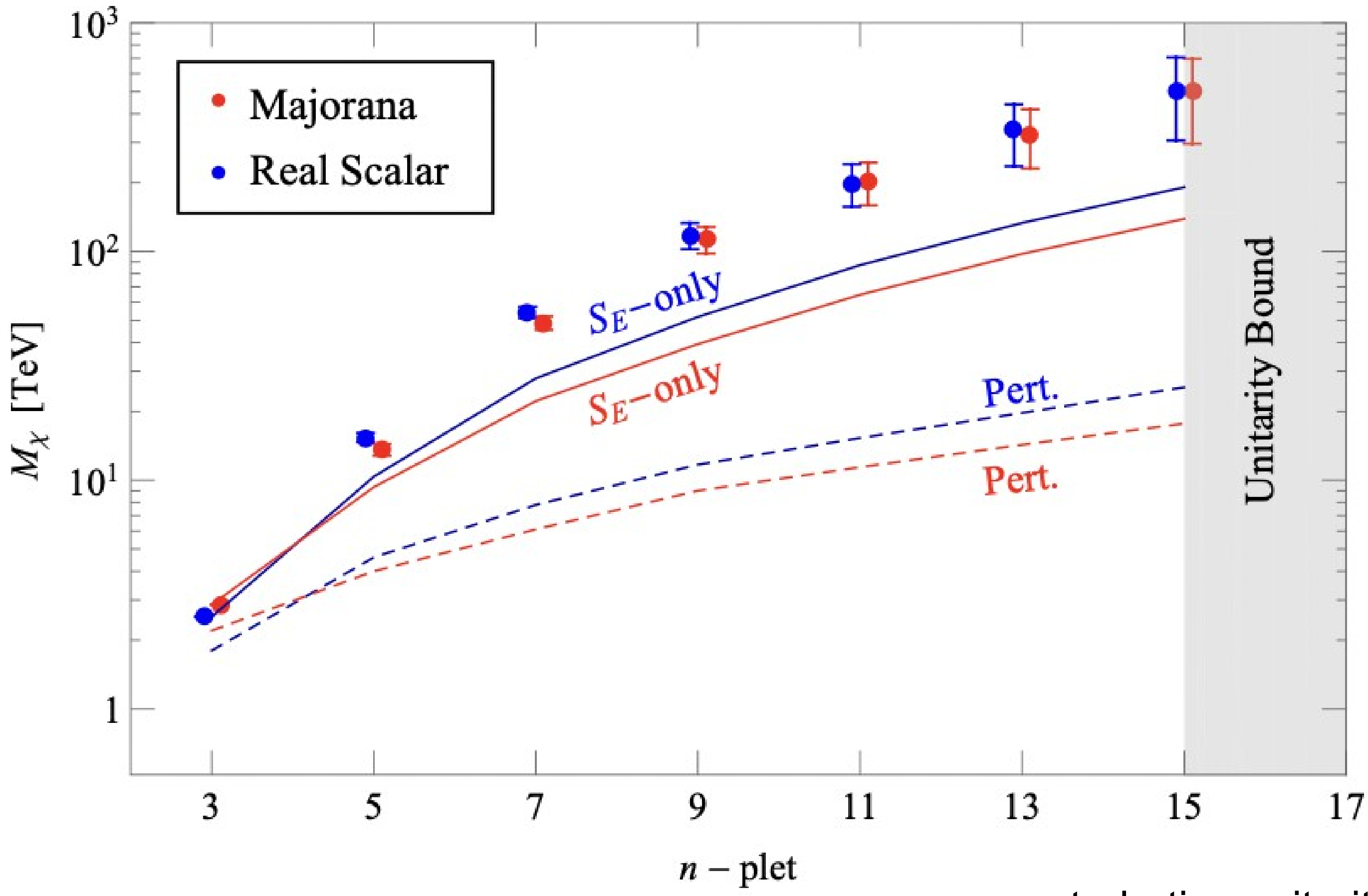
When $M \gtrsim M_{W,Z}/\alpha_W$ Coulomb-like attractive potential leads to the formation of MDM bound states

A. Mitridate, M. Redi, J. Smirnov, and A. Strumia, JCAP 05 (2017) 006

Thermal targets

S. Bottaro, D. Buttazzo, M. Costa, R. Franceschini, P. Panci, D. Redigolo, and L. Vittorio, Closing the window on WIMP Dark Matter, Eur. Phys. J. C 82, 31 (2022)

Majorana 5-plet is special, because it can be made **accidentally stable**. No need of specific UV completion, since the weak coupling stays perturbative up to very high scale, above the Planck scale

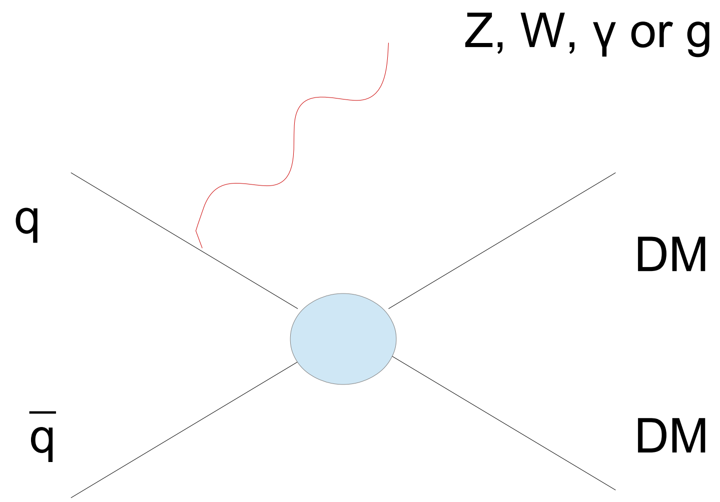


s-wave perturbative unitarity limit

Status of the search for WIMPs

At the LHC

conventional search strategies involve **mono-X** signatures with large missing energies, coming from the WIMP production



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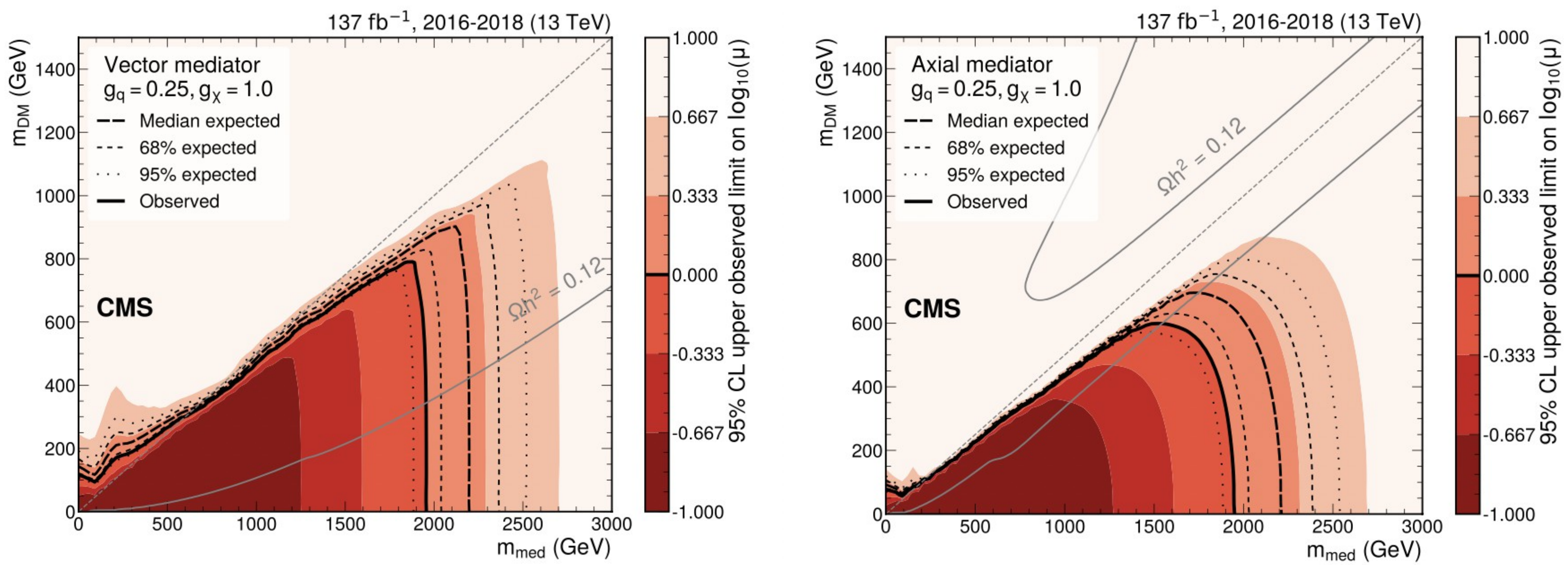
ATLAS: Phys. Rev. D 103, 112006 (2021)
CMS: JHEP 11 (2021) 153



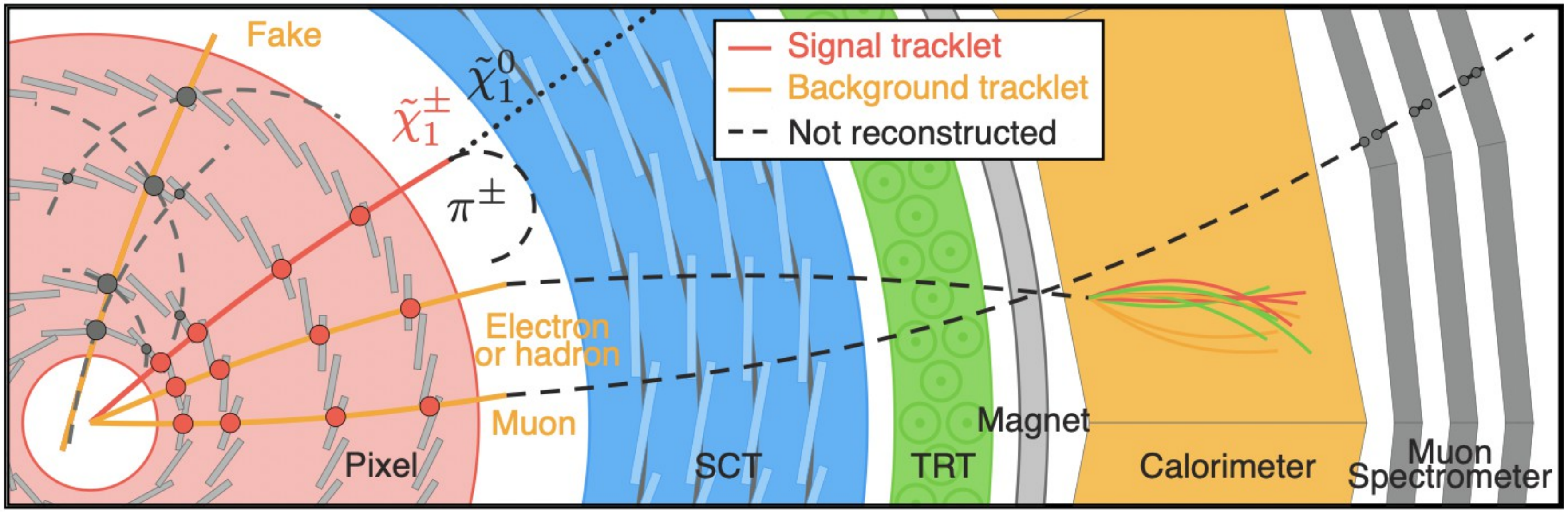
Status of the search for WIMPs

At the LHC

Charged (slightly) heavier components of the multiplet can leave distinctive signatures: they can travel finite distance in the detector before decaying, leaving a **disappearing track**. Searches for disappearing tracks have better sensitivities for the wino

For ex. M. Low and L.-T. Wang, JHEP 1408, 161 (2014)

ATLAS-CONF-2021-015



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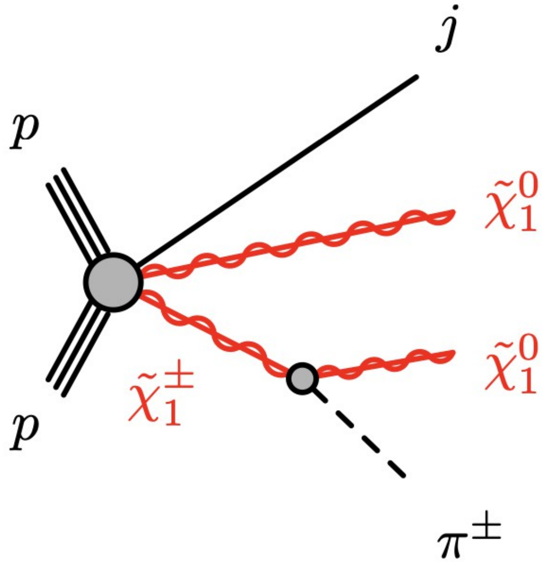
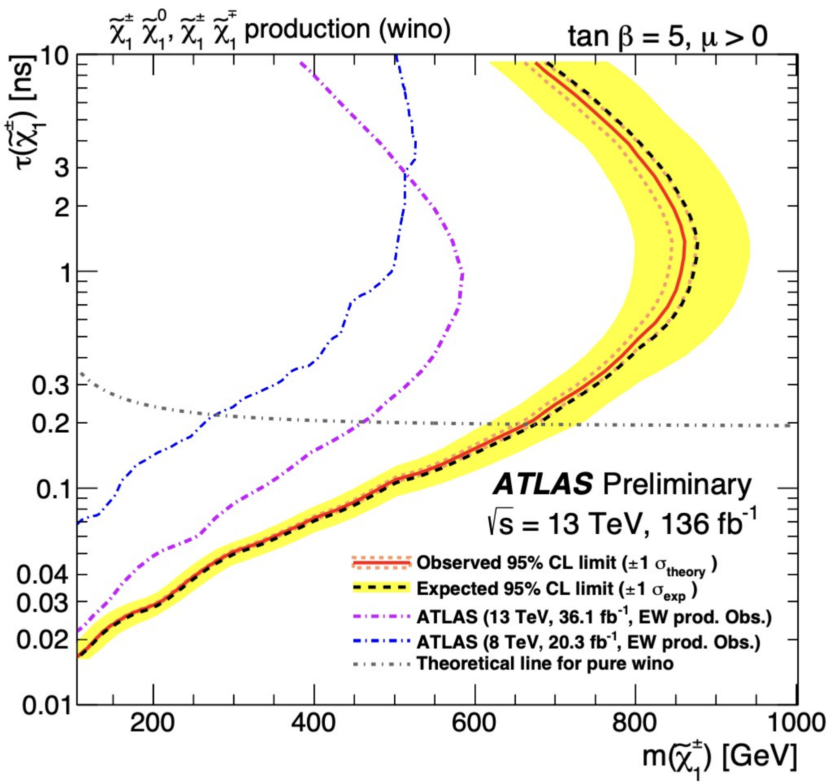
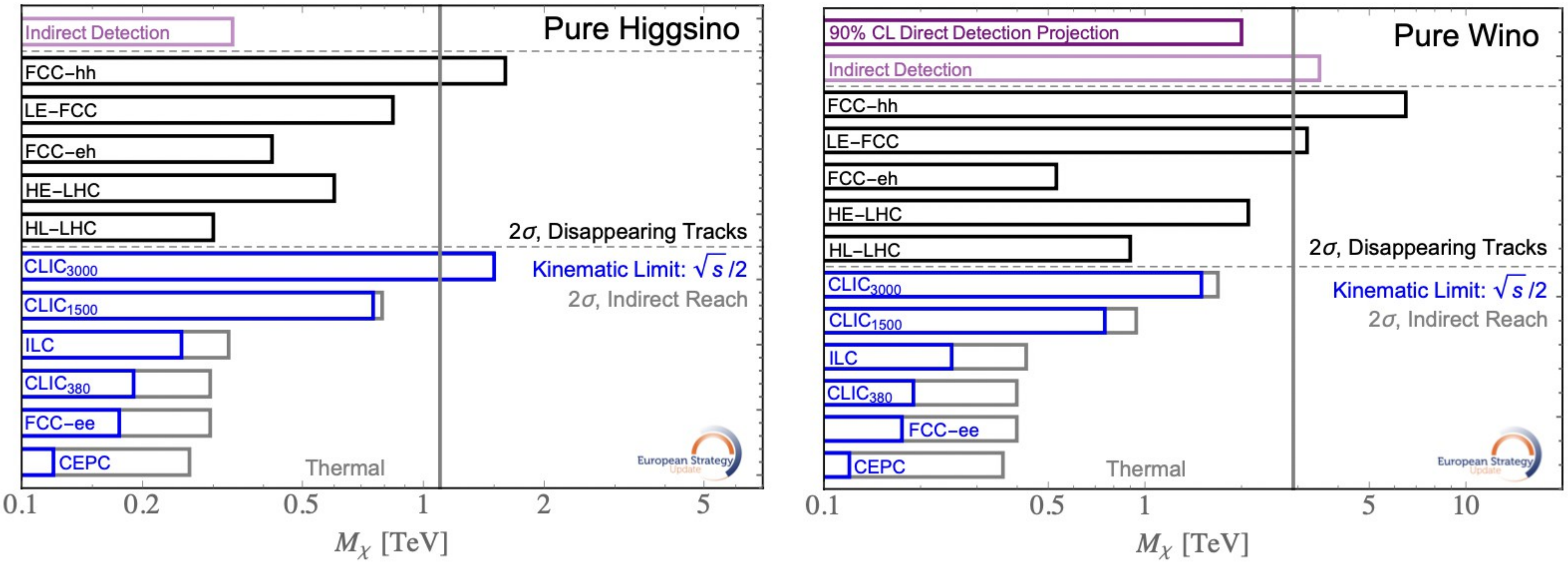




Status of the search for WIMPs

Prospects for FCC

physics Brief ing Book - Input for the European Strategy for Particle Physics Update 2020, arXiv:1910.11775v2 [hep-ex]



Wino target can be reached at the FCC-hh but no hopes for the 5-plet target

A Future Muon Collider

D. Stratakis et al. (Muon Collider), A Muon Collider Facility for Physics Discovery, (2022), arXiv:2203.08033

K. M. Black et al., Muon Collider Forum Report, (2022), arXiv:2209.01318 [hep-ex].

C. Accettura et al., Towards a Muon Collider, (2023), arXiv:2303.08533 [physics.acc-ph].

…………

mu+ mu- in a circular collider with a ring of the size of the LHC, 27 Km (possibly using the LHC ring ..)

Energy and Luminosity design targets:

$\sqrt{s}$ = 1, 3, 10, 30, 50 TeV $\quad$ L = 0.1, 0.9, 10, 90, 250 ab-1 $\quad$ $L = 10 \left(\frac{\sqrt{s}}{10\,\text{TeV}}\right)^2 \text{ab}^{-1}$

Advantages:

typically higher effective collision energy (hadron colliders pay PDFs, e+e- sinchrotron radiation effects)

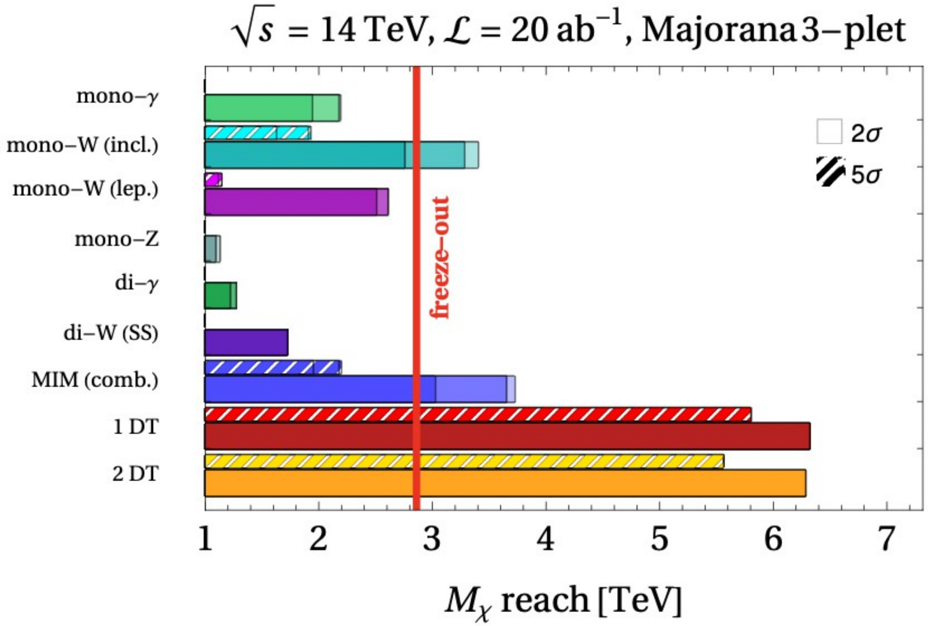
lower background (compared to hadron colliders)

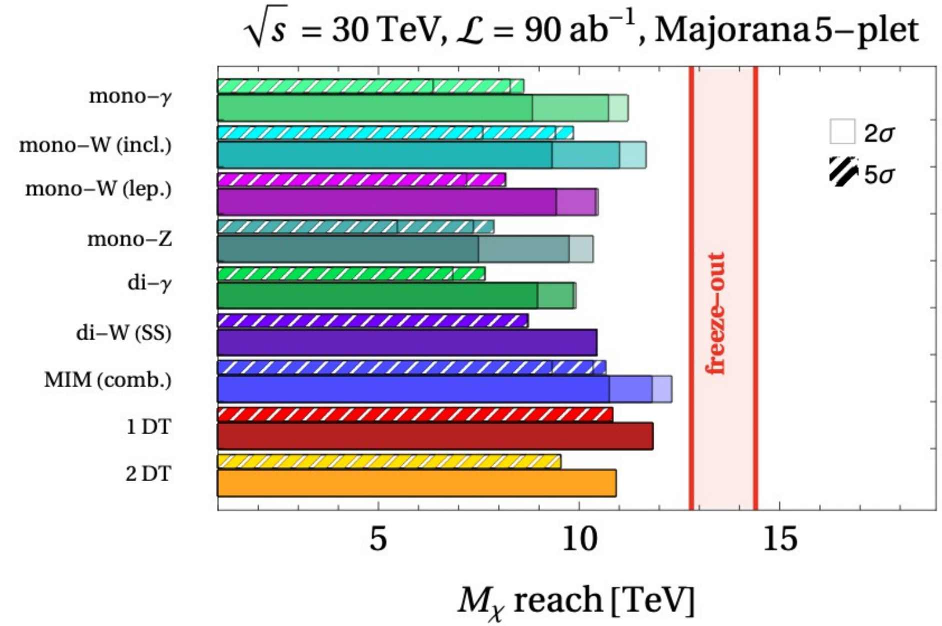
Main challenge: short life-time of muons

Status of the search for WIMPs

Missing energy, disappearing tracks and precision measurements at a future **muon collider**

Plots from S. Bottaro *et al.*, Eur. Phys. J. C 82, 31 (2022)



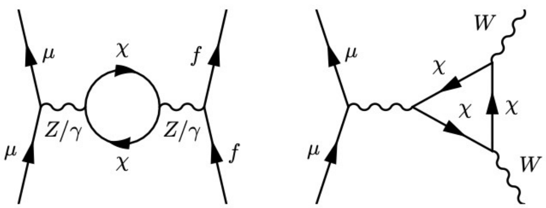


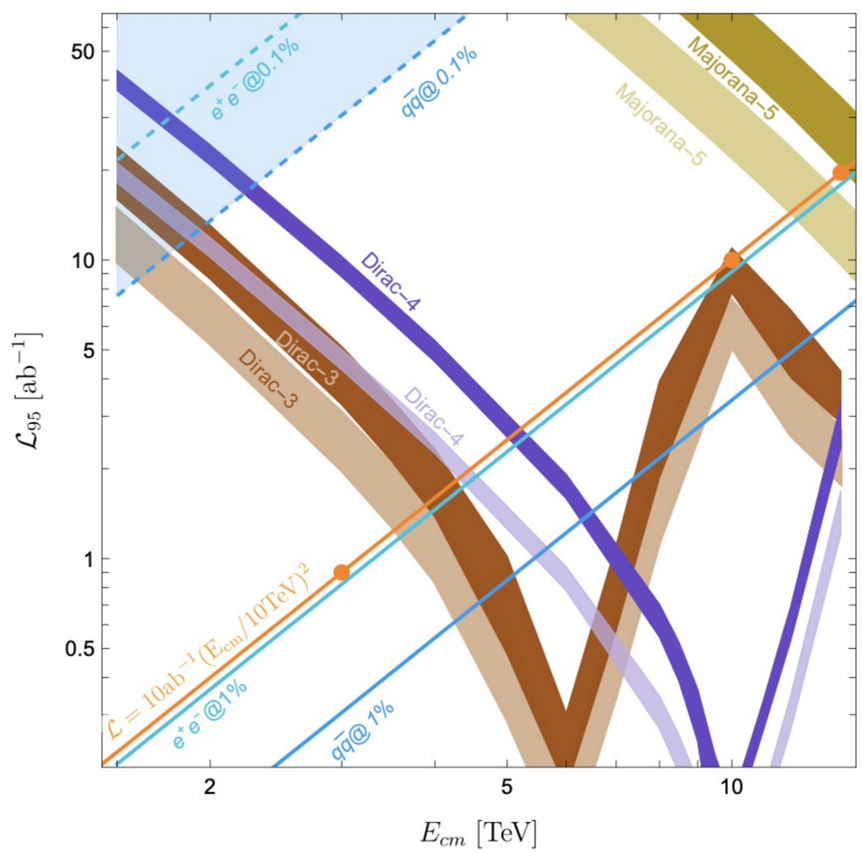
(For DT searches see also R. Capdevilla, F. Meloni, R. Simoniello, and J. Zurita, arXiv:2102.11292 [hep-ph])

Status of the search for WIMPs

Missing energy, disappearing tracks and <u>precision measurements</u> at a future **muon collider**

R. Franceschini and X. Zhao, arXiv:2212.11900





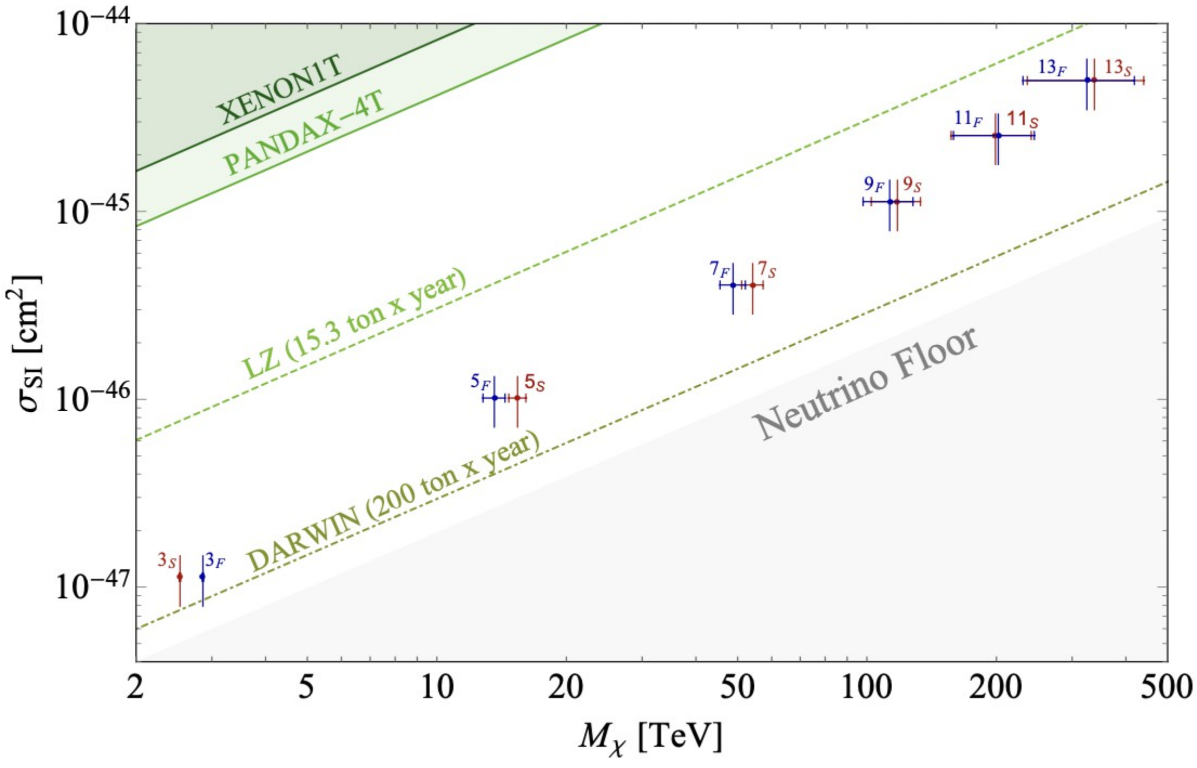
5-plet could be ecluded by a 14 TeV MuCol with about 20 ab-1

Important to take into account radiation effects at higher energies
(S. Chen, A. Glioti, R. Rattazzi, L. Ricci, and A. Wulzer, “Learning from radiation at a very high energy lepton collider,” JHEP 05 (2022) 180)

Status of the search for WIMPs

DM Direct searches prospects

Plots from S. Bottaro *et al.*, Eur. Phys. J. C 82, 31 (2022)



Status of the search for WIMPs

DM indirect searches

prospects

Plots from S. Bottaro *et al.*, Eur. Phys. J. C 82, 31 (2022)

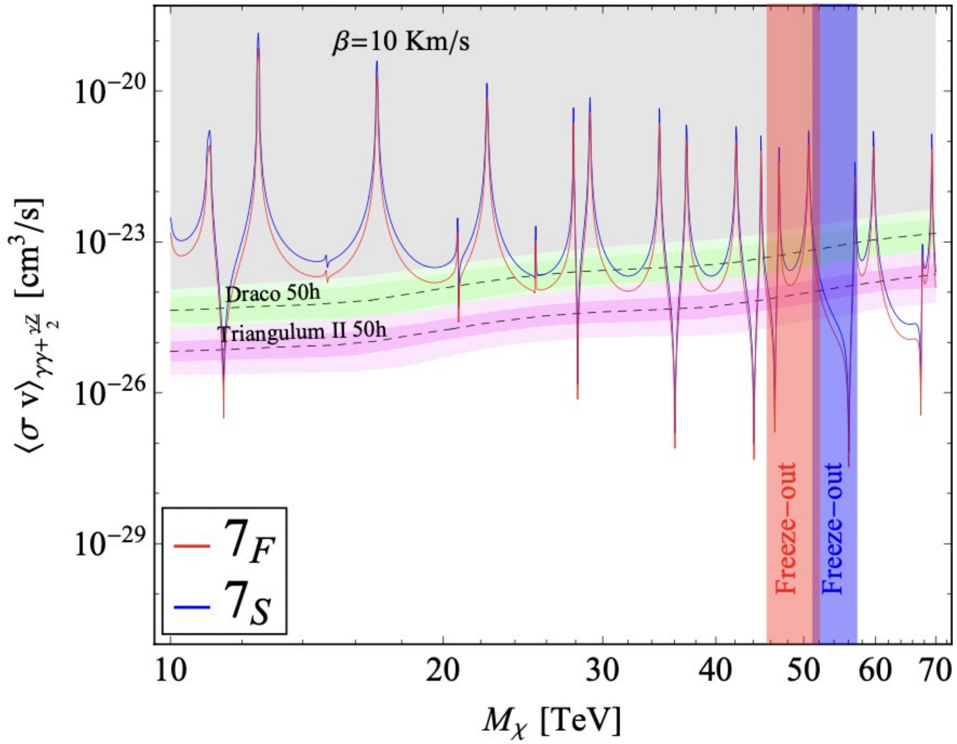


FIG. 7. Expected CTA sensitivities (dashed black lines) with 68% and 95% CL intervals derived as in Ref. [20] assuming 50 hours observation time towards Draco (green) and Triangulum II (magenta). We show the SE annihilation cross-section into the channels that contribute to the monocromatic gamma line signal (i.e. $\gamma\gamma$ an γZ) for a scalar 7-plet (blue) and a fermionic 7-plet (red). The vertical bands show the predicted thermal masses for the scalar 7-plet (blue) and the fermionic 7-plet (red), where the theory uncertainty is dominated by the neglected NLO contributions (see Table 1).

Status of the search for WIMPs

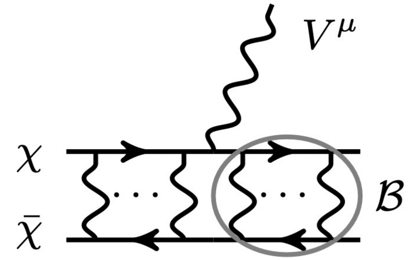
In Conclusions,

Even at a muon collider it is not possible to test the 5-plet *directly* by "conventional searches": mono-X, MIM, DT, ...

Indirect exclusion is possible by precision measurements with an energy of at least 14 TeV

DM direct detection experiments need a long exposure to test the MDM scenario: DARWIN 200 ton/year

MDM bound states



When $M \gtrsim M_{W,Z}/\alpha_W$ Coulomb-like attractive potential

$$V = -\alpha_{\rm eff}\,\frac{e^{-M_{W,Z}\,r}}{r}$$

$$5 \otimes 5 = 1 \oplus 3 \oplus 5 \oplus 7 \oplus 9$$

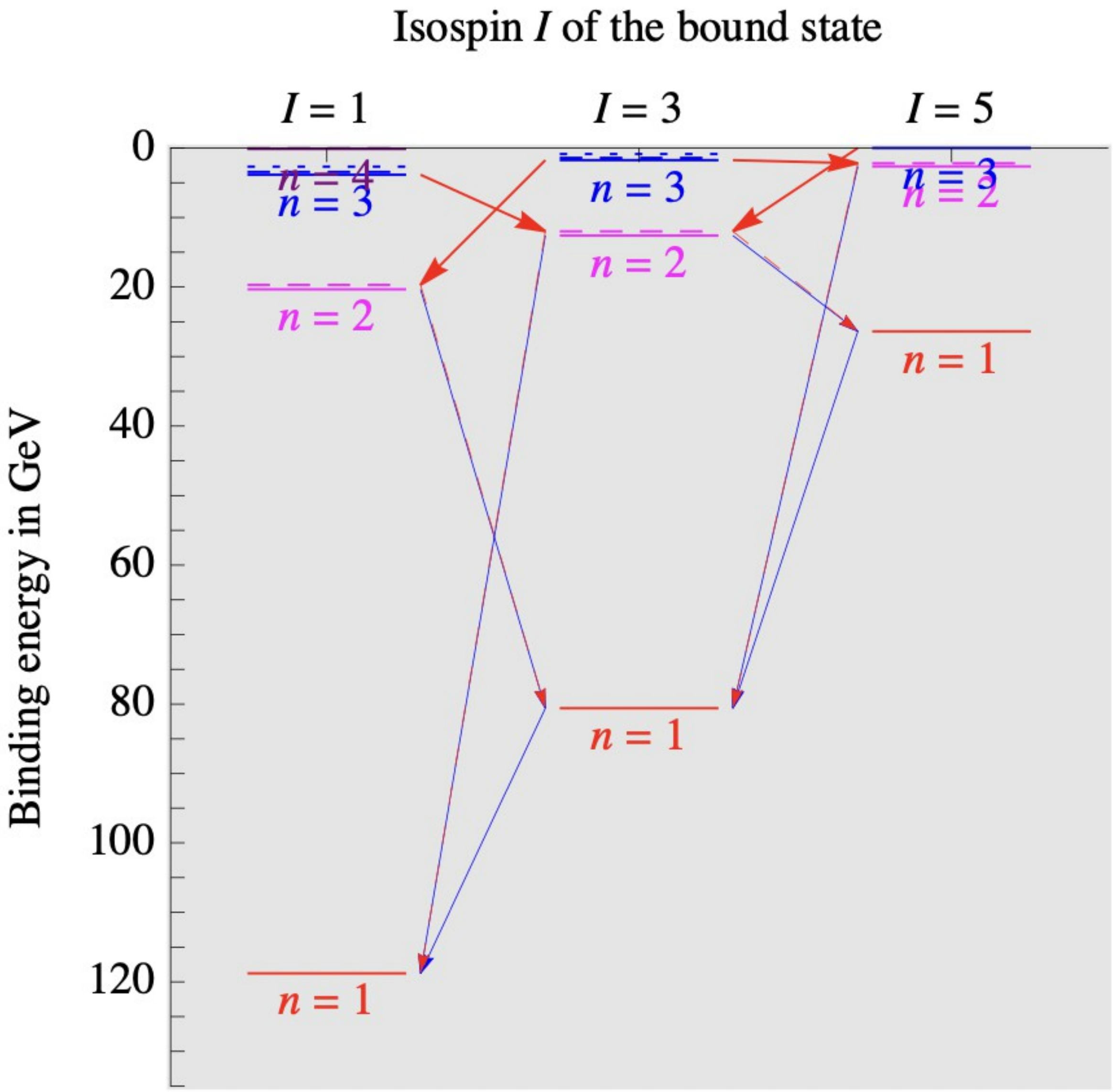
$$I = 1\ (\alpha_{\rm eff} = 6\alpha_2),\ I = 3\ (\alpha_{\rm eff} = 5\alpha_2),\ \text{and}\ I = 5\ (\alpha_{\rm eff} = 3\alpha_2)$$

leads to the formation of MDM bound states

$$E_B \approx \frac{\alpha_{\rm eff}^2 M}{4n^2}\left[1 - n^2 y - 0.53 n^2 y^2 \ell(\ell+1)\right]^2 \qquad \text{where} \qquad y \approx \frac{1.74 M_{W,Z}}{\alpha_{\rm eff} M}$$

Calculations (in $SU(2)_L$ symmetric approximation) first performed in A. Mitridate, M. Redi, J. Smirnov, A. Strumia, "Cosmological Implications of Dark Matter Bound States", JCAP 05 (2017) 006

MDM bound states

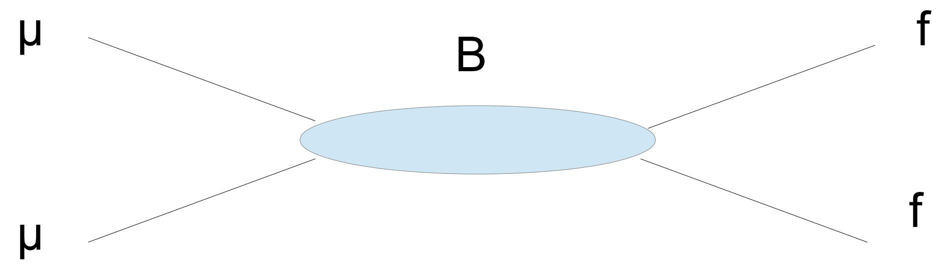


MDM bound states

name	Quantum numbers					Annihilation		Decay	
${}^{n}_{J}\ell_I^{PC}$	n	I	S	ℓ	E_B	$\Gamma_{\rm ann}$	into	$\Gamma_{\rm dec}$	into
${}^1_1s_1^{-+}$	1	1	0	0	118 GeV	$3240\,\alpha_2^5 M \approx 1.63\,{\rm GeV}$	$V\tilde{V}$	0	—
${}^1_1s_3^{--}$	1	3	1	0	81 GeV	$15625\,\alpha_2^5 M/48 \approx 0.17\,{\rm GeV}$	$f_L\bar{f}_L + HH^*$	$36\,\alpha_2^6\alpha_{\rm em} M \approx 4.6\,{\rm keV}$	${}^1s_1\gamma$
${}^1_1s_5^{-+}$	1	5	0	0	26 GeV	$567\,\alpha_2^5 M/4 \approx 0.07\,{\rm GeV}$	VV	$295\,\alpha_2^6\alpha_{\rm em} M \approx 38\,{\rm keV}$	${}^1s_3\gamma$
${}^2_1s_1^{-+}$	2	1	0	0	20.3 GeV	$405\alpha_2^5 M \approx 0.2\,{\rm GeV}$	$V\tilde{V}$	$13\,\alpha_2^6\alpha_{\rm em} M \approx 1.7\,{\rm keV}$	${}^1s_3\gamma$
${}^2_1s_3^{--}$	2	3	1	0	13 GeV	$15625\,\alpha_2^5 M/384 \approx 21\,{\rm MeV}$	$f_L\bar{f}_L + HH^*$	$(6.9\,\alpha_2 + 0.3\,\alpha_{\rm em})\alpha_2^6 M \approx 3.7\,{\rm keV}$	${}^1s_{1+5}V$
${}^2_1s_5^{-+}$	2	5	0	0	2.6 GeV	$567\,\alpha_2^5 M/32 \approx 9\,{\rm MeV}$	$V\tilde{V}$	$28.4\,\alpha_2^6\alpha_{\rm em} M \approx 3.6\,{\rm keV}$	${}^1s_3\gamma$
${}^2_Jp_1^{++}$	2	1	1	1	19.7 GeV	$\mathcal{O}(\alpha_2^7 M) \sim {\rm keV}$	VV	$20.4\,\alpha_2^4\alpha_{\rm em} M \approx 2.5\,{\rm MeV}$	${}^1s_3\gamma$
${}^2_1p_3^{+-}$	2	3	0	1	12 GeV	$\mathcal{O}(\alpha_2^8 M) \sim 10\,{\rm eV}$	VVV	$(30.2\,\alpha_2 + 0.3\,\alpha_{\rm em})\,\alpha_2^4 M \approx 15.3\,{\rm MeV}$	${}^1s_{1+5}V$
${}^2_Jp_5^{++}$	2	5	1	1	2.2 GeV	$\mathcal{O}(\alpha_2^7 M) \sim {\rm keV}$	VV	$4.7\,\alpha_2^4\alpha_{\rm em} M \approx 0.6\,{\rm MeV}$	${}^1s_3\gamma$
${}^3_1s_1^{-+}$	3	1	0	0	3.8 GeV	$120\alpha_2^5 M \approx 60\,{\rm MeV}$	$V\tilde{V}$	$0.34\,\alpha_2^4\alpha_{\rm em} M \approx 42\,{\rm keV}$	${}^2p_3\gamma$
${}^3_1s_3^{--}$	3	3	1	0	1.7 GeV	$15625\alpha_2^5 M/1296 \approx 6.0\,{\rm MeV}$	$f_L\bar{f}_L + HH^*$	$(0.003 + 0.005)\alpha_2^4\alpha_{\rm em} M \approx 1\,{\rm keV}$	${}^2p_{1+5}\gamma$
${}^3_1s_5^{-+}$	3	5	0	0	1.7 MeV	$21\alpha_2^5 M/4 \approx 2.7\,{\rm MeV}$	$V\tilde{V}$	$0.3\alpha_2^4\alpha_{\rm em} M \approx 36\,{\rm keV}$	${}^2p_3\gamma$
${}^3_Jd_3^{--}$	3	3	1	2	0.9 GeV	$\mathcal{O}(\alpha_2^9 M) \sim {\rm eV}$	$f_L\bar{f}_L$	$0.4\alpha_2^4\alpha_{\rm em} M \approx 52\,{\rm keV}$	${}^2p_{1+5}\gamma$

We are particularly ineterested in the states that can be directly produced at a muCol: the isospin triplets
Scalar bound state can be produced via VBF, but with lower cross sections

Resonant production at a MuCol



We can calculate (also numerically with Madgraph) the xsec based on the annihilation rate of the bound state (width)

We are in a narrow-width regime. Breit-Wigner:

$$\sigma(i_1 i_2 \to B \to f) \approx \mathrm{BW}(s)\sigma_{\mathrm{peak}}$$

$$\mathrm{BW}(s) = \frac{M_B^2\Gamma_B^2}{(s - M_B^2)^2 + M_B^2\Gamma_B^2} \simeq \Gamma_B M_B \pi\, \delta(s - M_B^2), \qquad \sigma_{\mathrm{peak}} = \frac{16\pi S_B}{M_B^2 S_{i_1} S_{i_2}} \mathrm{BR}_{i_1 i_2} \mathrm{BR}_f$$

Resonant production at a MuCol

Two important effects to take into account:

- The beam-energy spread
- The Initial State Radiation

Resonant production at a MuCol

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- The beam-energy spread
- The Initial State Radiation

Each beam energy is statistically distributed (we assume a Gaussian) around the nominal design √s

$$\wp(s) = \frac{1}{\sqrt{2\pi}\Delta_E} \exp\left[-\frac{(\sqrt{s}-M_B)^2}{2\Delta_E^2}\right] , \qquad \Delta_E = \sqrt{2}\sigma_E .$$

Important correction to the BW estimates in the very narrow width regime ($\sigma_E > \Gamma_B$), which is characteristic of the MDM bound states

$$\sigma(i \to B \to f) \simeq \epsilon \sigma_{\text{peak}}, \qquad \epsilon = \frac{\sqrt{\pi}\,\Gamma_B}{4\sigma_E} .$$

Resonant production at a MuCol

Two important effects to take into account:

- The beam-energy spread
- The Initial State Radiation

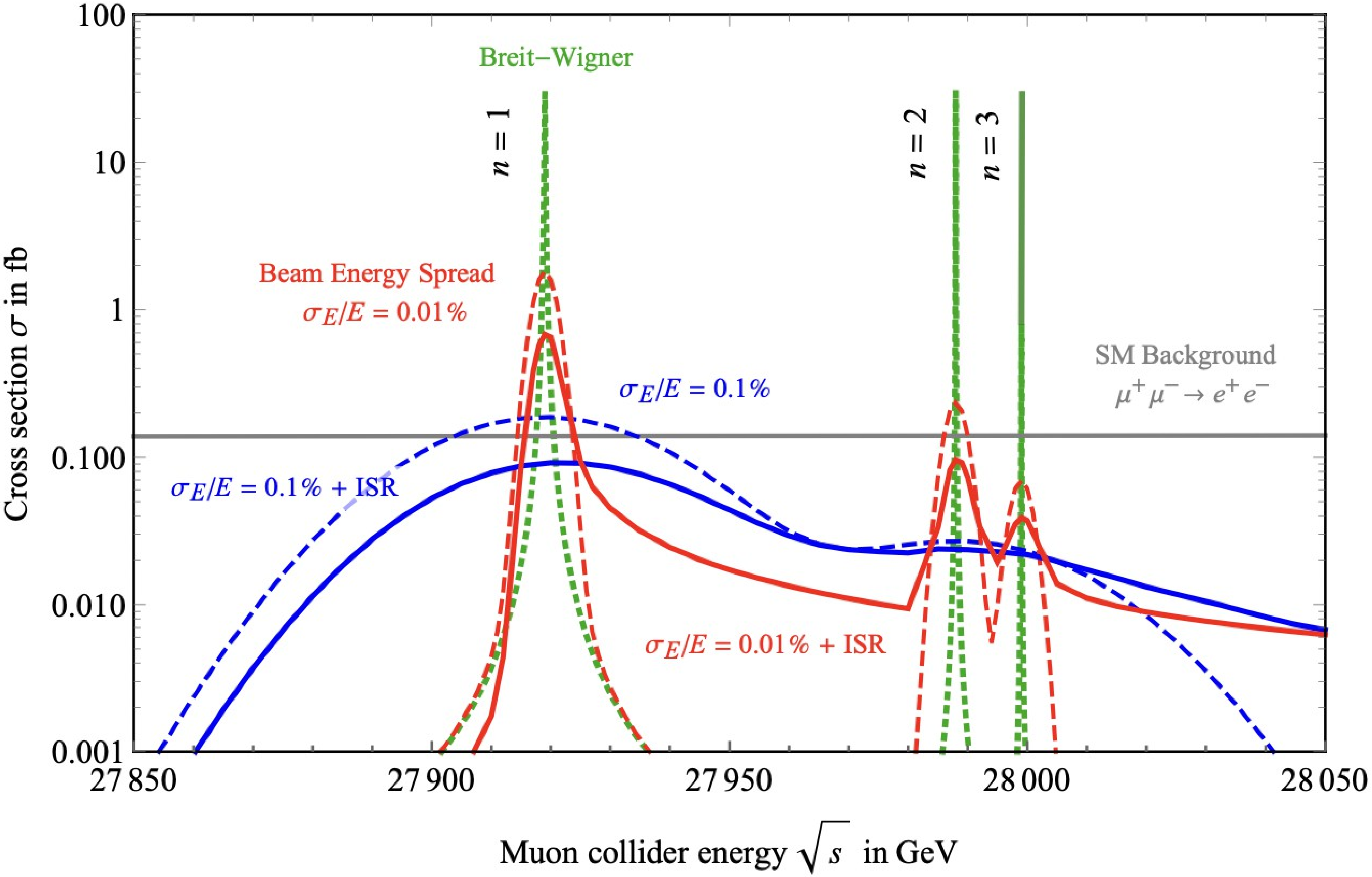
Initial muons irradiate (mainly) photons. This reduce their energy, such that the amount of muons with energy equal to the beam energy gets reduced by an order unity factor, analytically given by

$$\sim (\Gamma/M)^{4\alpha_{\rm em}\ln(E/m_\mu)/\pi}$$

And also leads to a ”radiative-return” effect

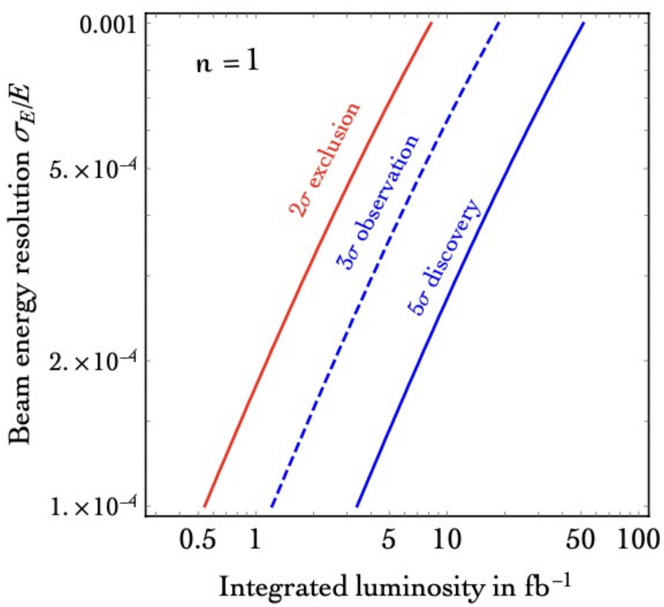
Precise calculation, that we use, in S. Jadach, B.F.L. Ward, Z. Was, Phys.Rev.D 63 (2001) 113009
See also M. Greco, T. Han, Z. Liu, “ISR effects for resonant Higgs production at future lepton colliders”, Phys.Lett.B 763 (2016) 409

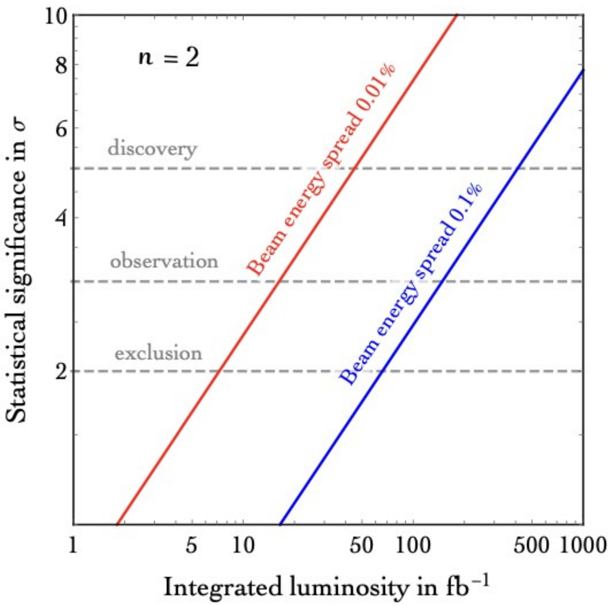
Resonant production at a MuCol

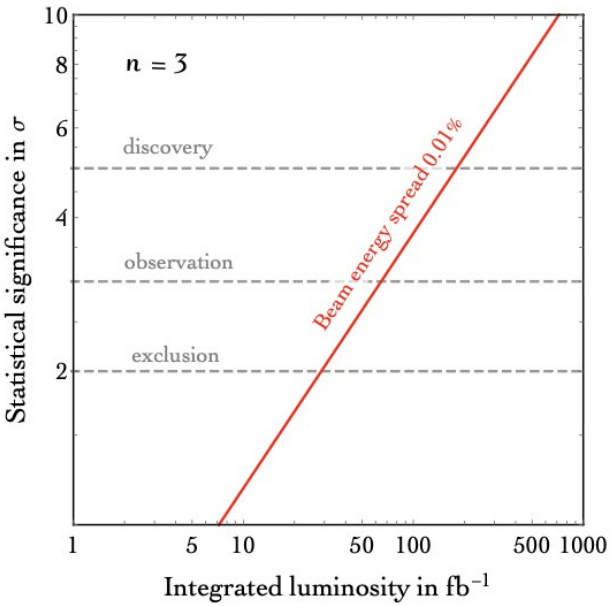


Resonant production at a MuCol

Reach







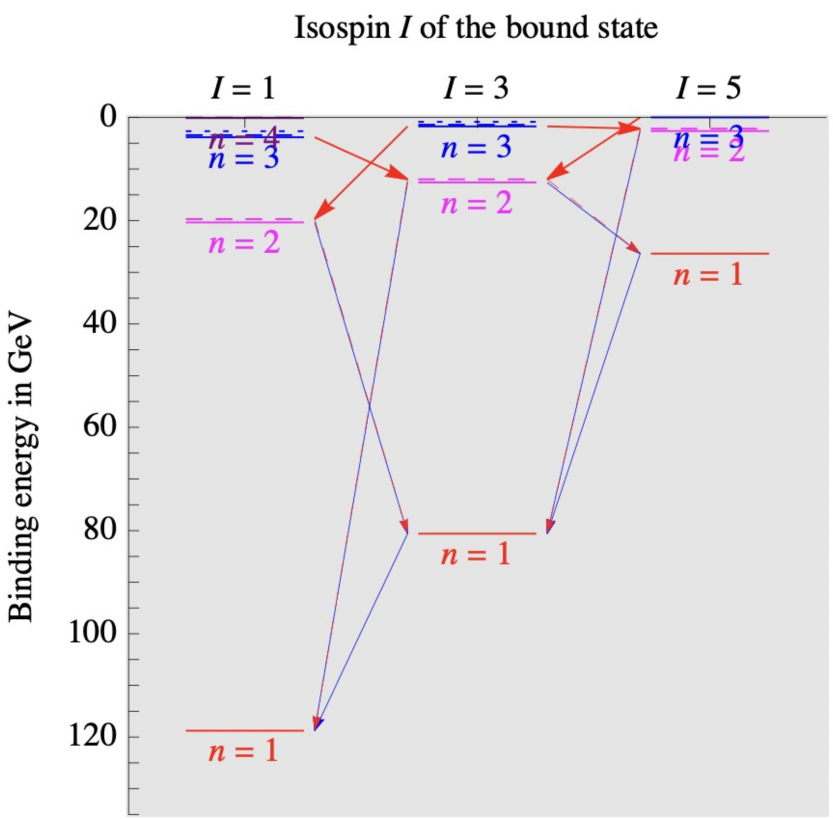
Resonant production at a MuCol

Possibility for <u>spectroscopy</u>

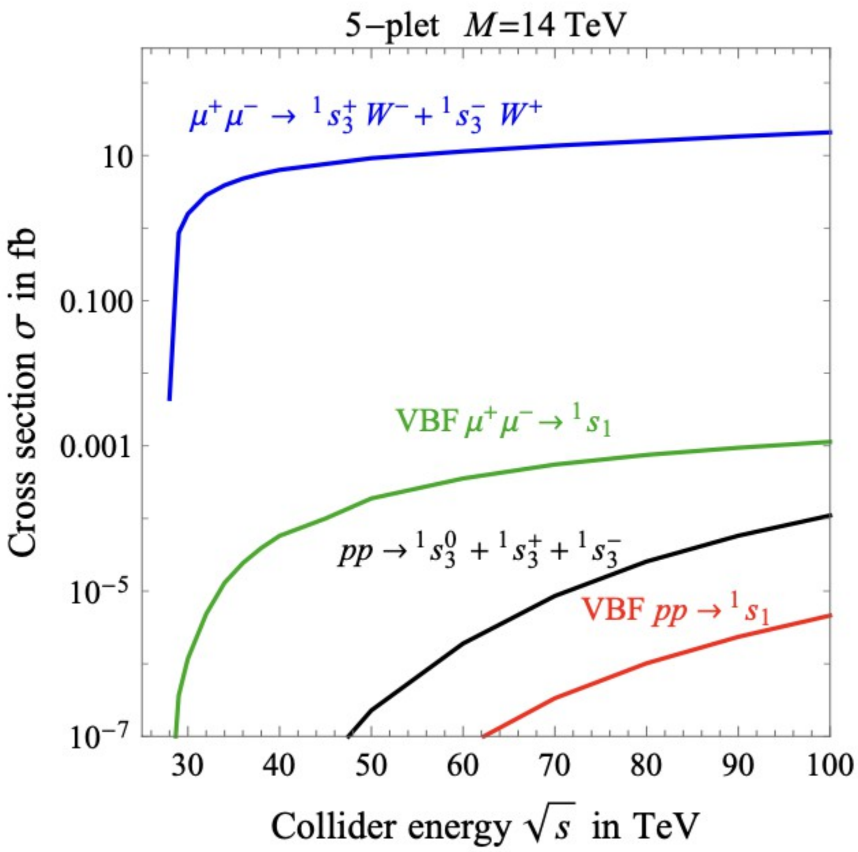
In particular,

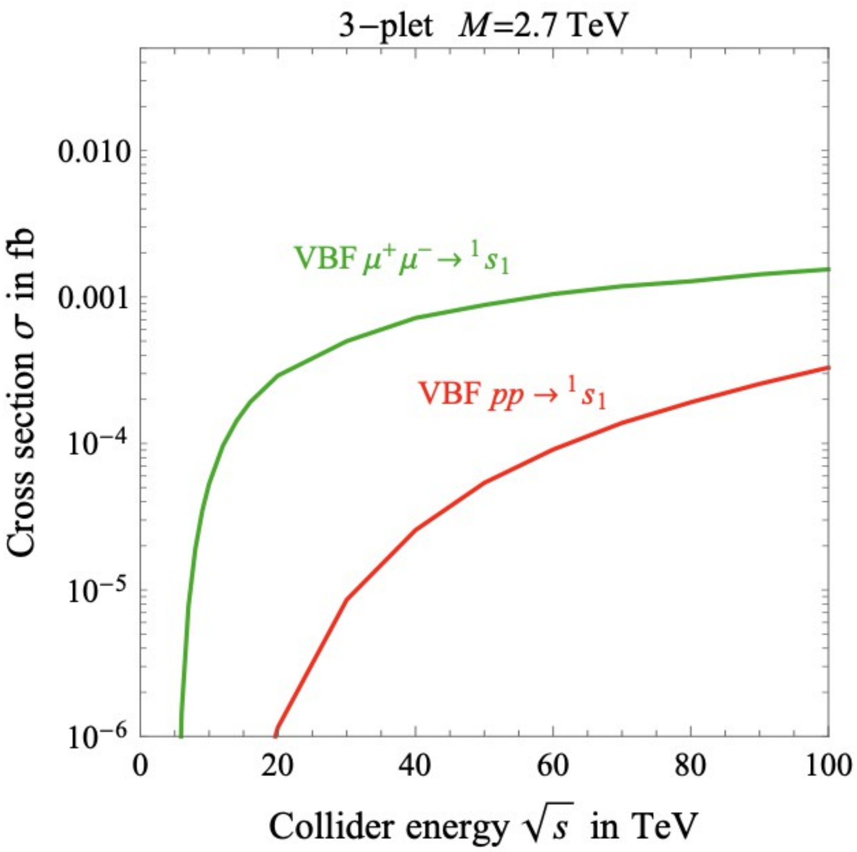
$^1s_3$ bound state decays (with magnetic transitions) into a monochromatic photon with energy of about 38 GeV and branching ratio 9 10^{-5}. This corresponds to 19 events in a run with resolution 10^{-3} and luminosity L = 90/ab

With a similar high statistics, it would be also possible to detect the $^2s_3$ state decaying into $^1s_1$ with a gamma emission of 105 GeV and in $^1s_5$ with a gamma emission of 13 GeV



Other productions at future colliders





MDM bound state at future colliders

(Bottaro, Strumia, NV, JHEP 06 (2021) 143)

Conclusions: possibility to discover the 5-plet with just few fb-1 (one day of run)

Drawback: on-peak

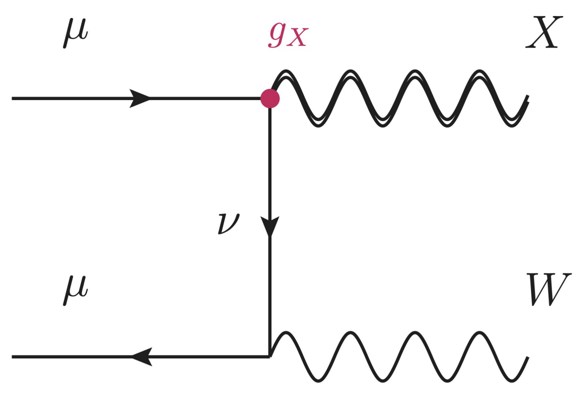
↓

it is possible to overcome it

W associated production

X generic spin-1 resonance of the SSM W-prime type

$$\mathcal{L}^{W'}_{eff} = \frac{g_X}{\sqrt{2}} \left[V^{CKM}_{ij} \bar{u}_i \gamma^\mu P_L d_j \right.$$
$$\left. + V^{PMNS}_{ij} \bar{\nu}_i \gamma^\mu P_L \ell_j \right] X_\mu + H.c.$$



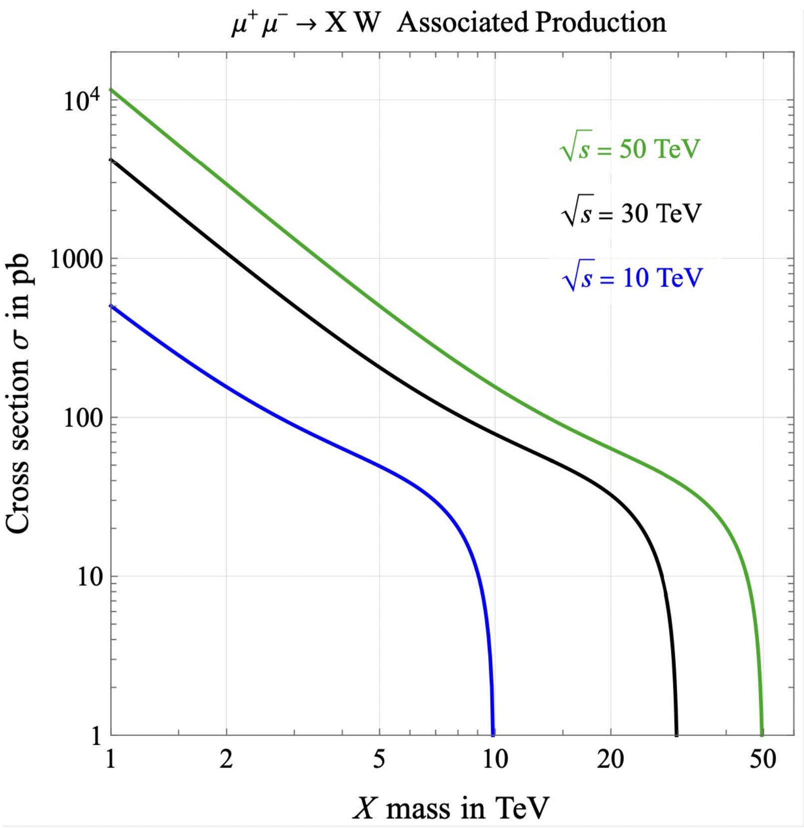
MDM bound state can be described by the effective W' description with:

$$g_X = g_{1_{s_3}} \simeq 0.014\, g_2$$

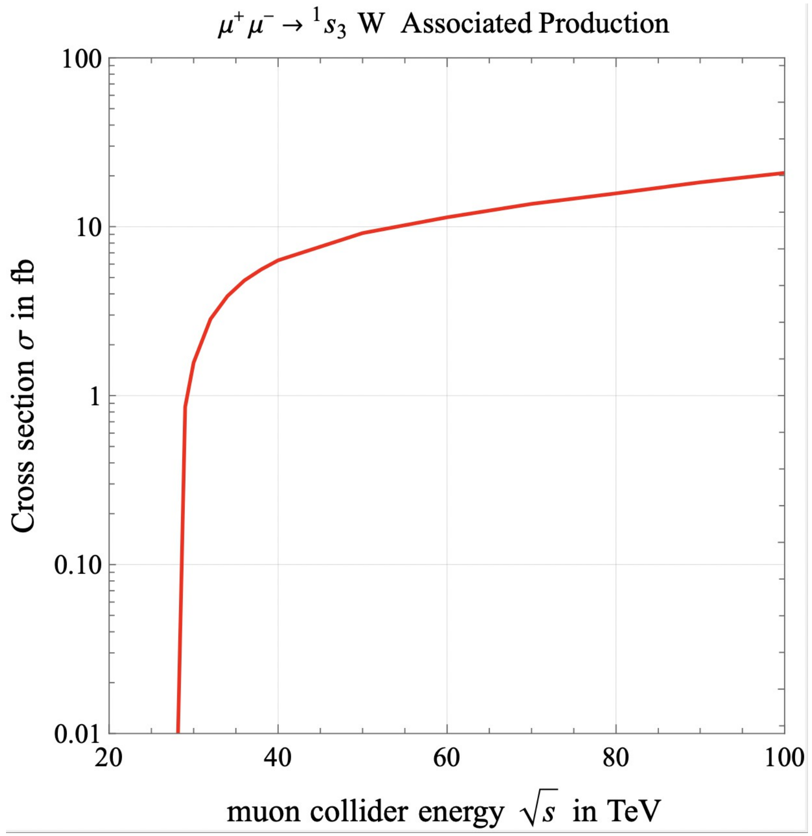
NV, arXiv:2304.12362

$$\sigma(\mu^+\mu^- \to X^+W^-) = \sigma(\mu^+\mu^- \to X^-W^+) \simeq$$
$$\frac{g_2^2\, g_X^2}{1536\, \pi\, s^2\, m_X^2\, m_W^2} \left[s^2 + 10\, m_X^2 s + m_X^4 + m_W^4 \right.$$
$$\left. + 10\, m_W^2 (s - 5 m_X^2) \right] \sqrt{(s - m_X^2)^2 - 2 m_W^2 (s + m_X^2) + m_W^4}$$

W associated production



SSM case, g_X=g_2



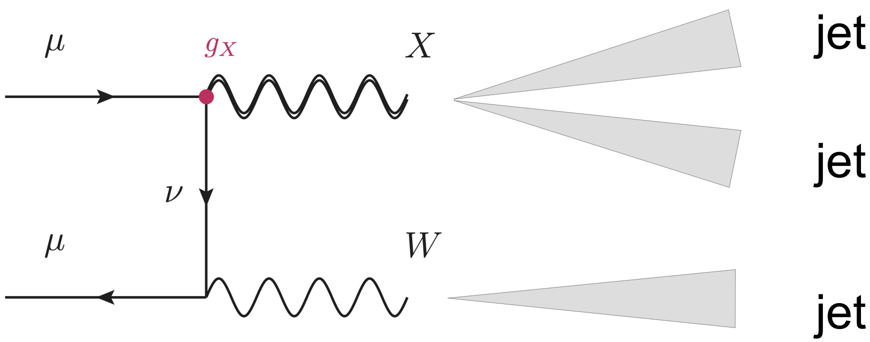
MDM bound state

Search strategy

- Fully hadronic final state

  Acceptance selection

  At least 3 jets with:



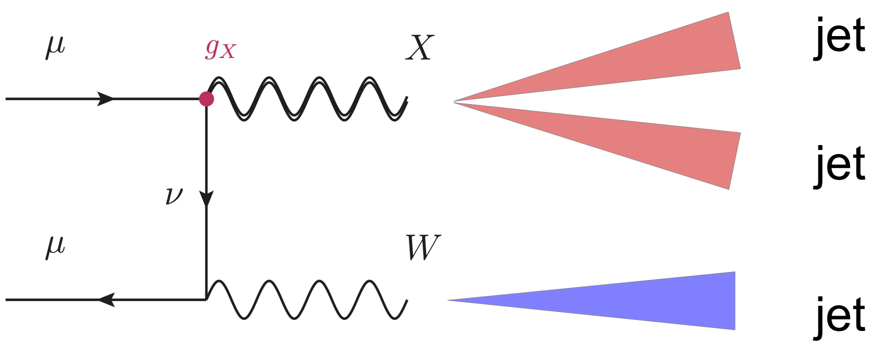
$$p_T\, j > 30 \text{ GeV}\,, \quad |\eta_j| < 2.5\,, \quad \Delta R_{jj} > 0.4$$

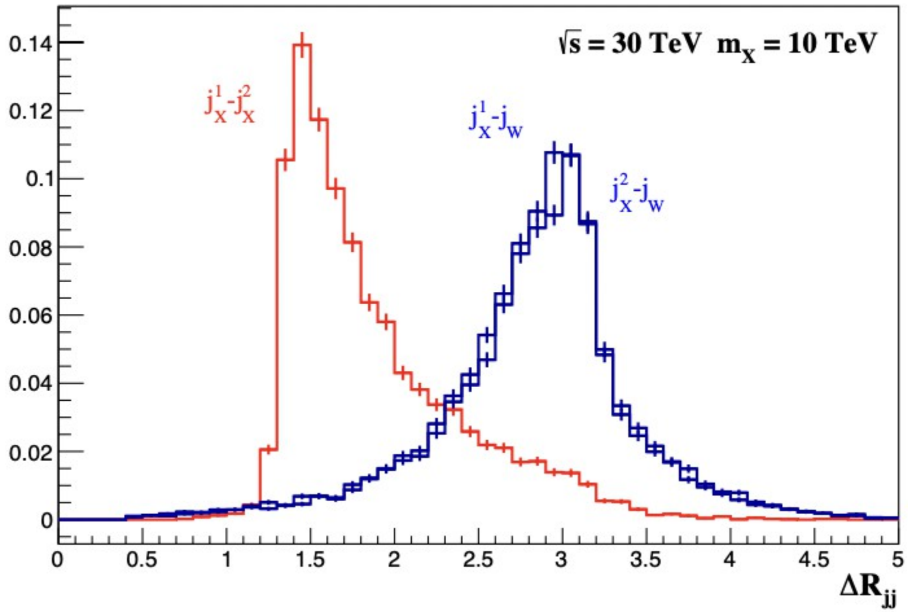
- Background is small, few fb before selection

  mainly given by $\gamma^* \rightarrow$ jets and (by a smaller component) VV $\rightarrow$ jets

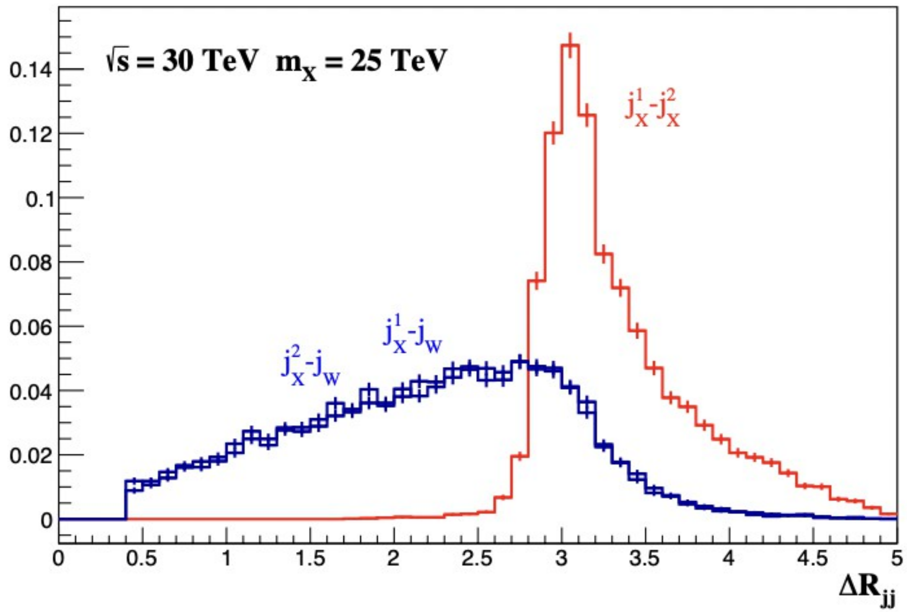
Search strategy

W, X reconstruction





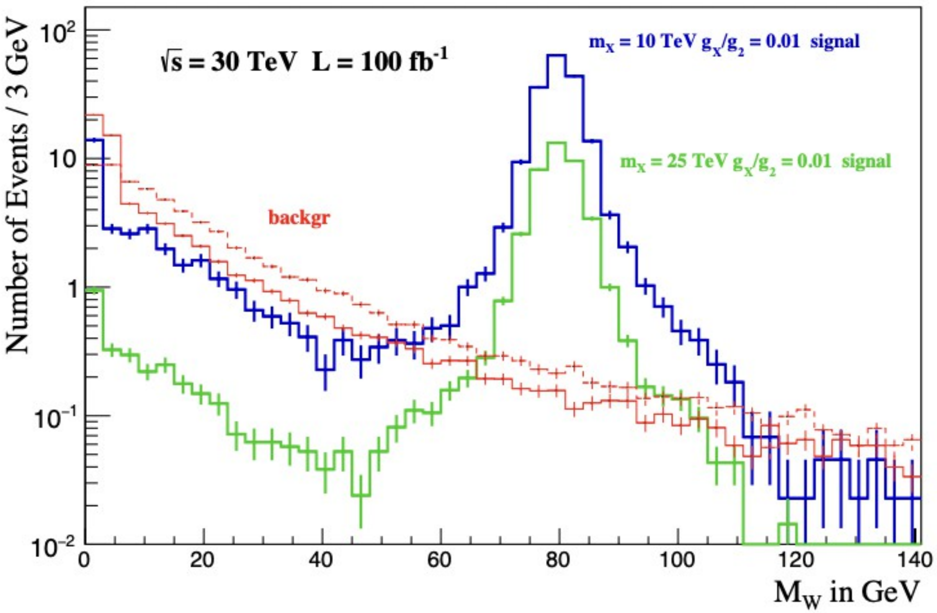
$m_X \leq \sqrt{s}/2$

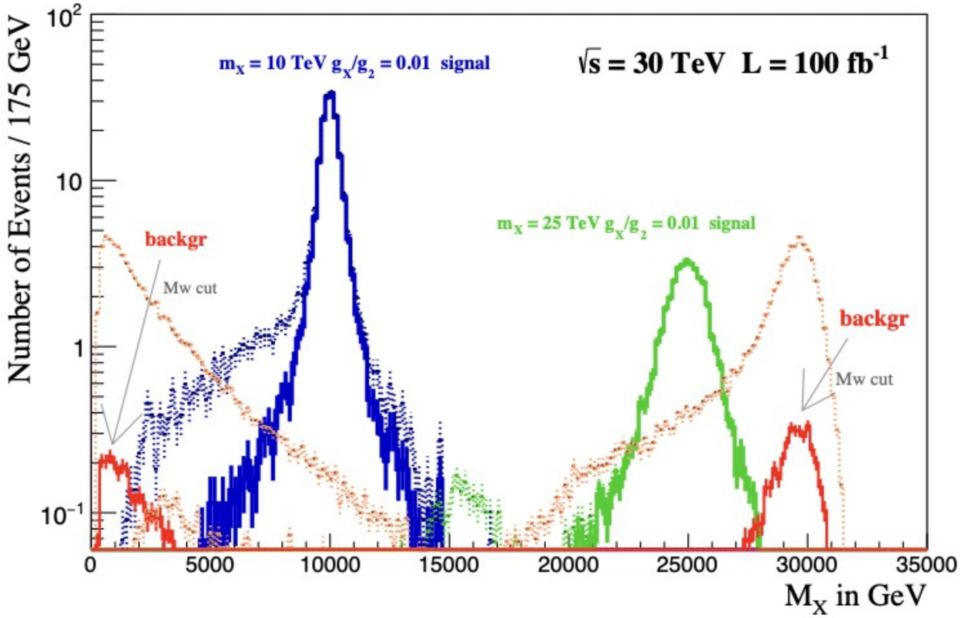


$m_X > \sqrt{s}/2$

Search strategy

W, X reconstruction

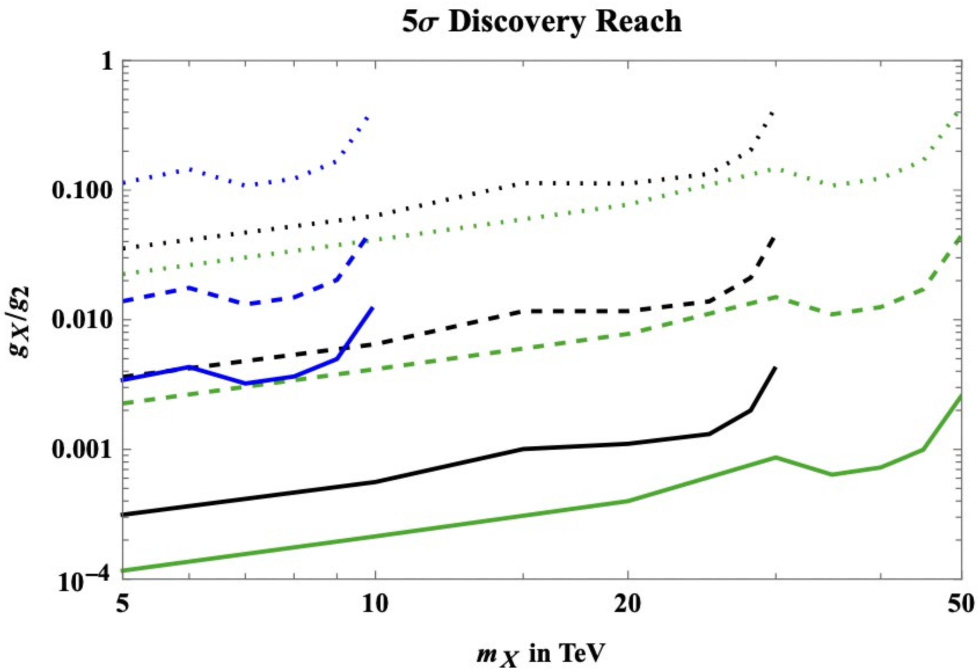


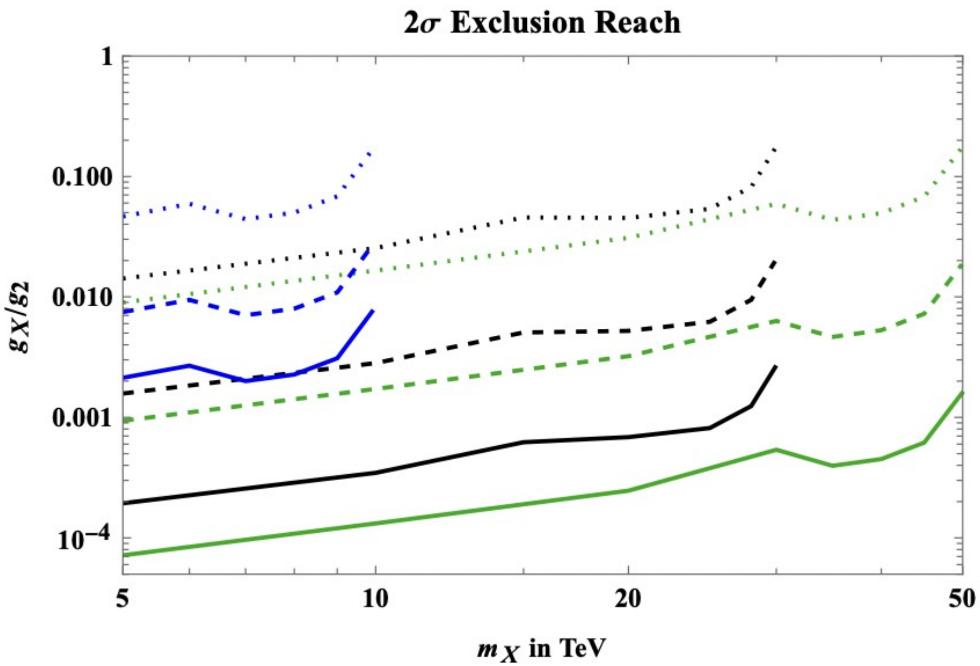


M_W cut:

$$50\,\mathrm{GeV} < M_W < 110\,\mathrm{GeV}$$

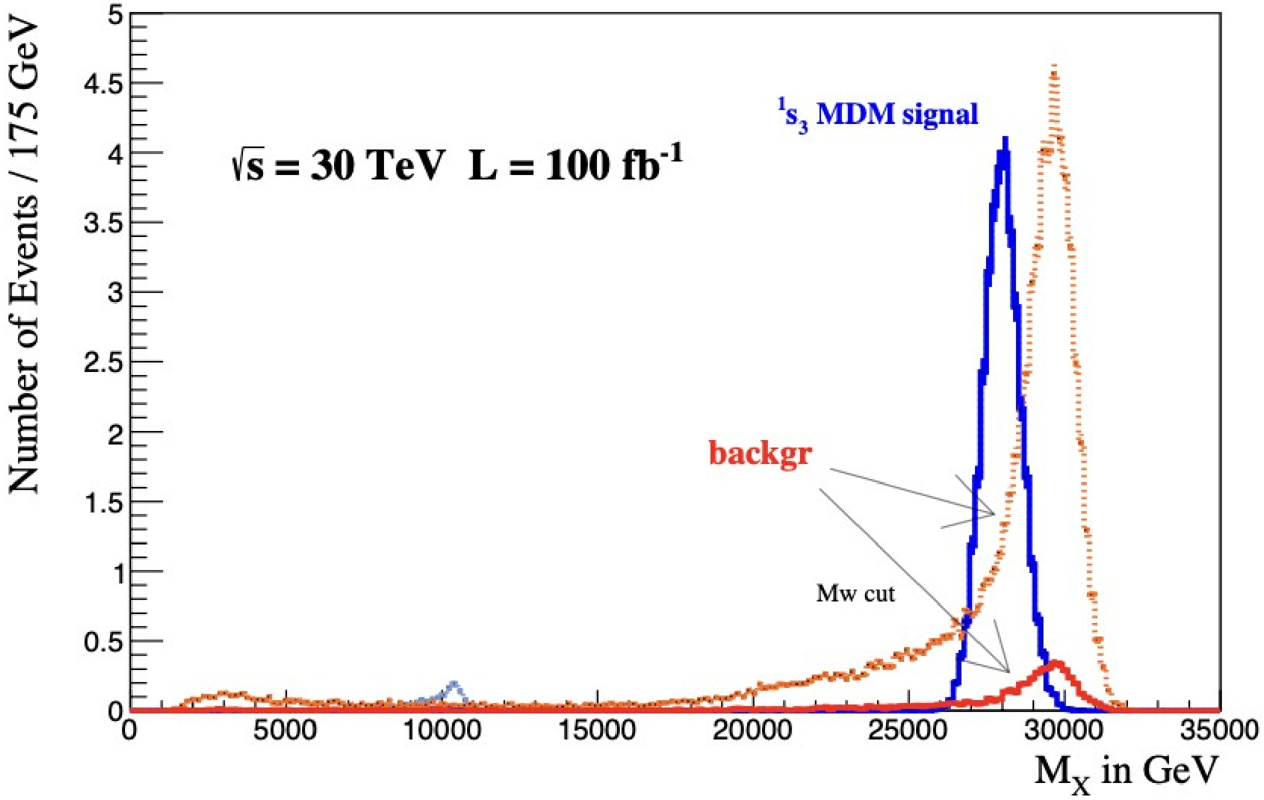
W+X Reach





A SSM W-prime ($g_X=g_2$) with a mass up to 9, 28 and 46 TeV can be discovered respectively by a 10, 30 and 50 TeV MuCol with just 50/pb

MDM 5-plet buond state in the W associated channel



A 5-plet MDM bound state can be excluded with about 34 fb^{-1} and discovered with 210 fb^{-1} by a 30 TeV muon collider

Conclusions

- WIMP MDM compelling solution to the DM puzzle
- Fermionic 5-plet is a particularly attracting hypothesis (neutral component automatically stable), the thermal target is at M~14 TeV and is beyond the reach of current and future hadronic colliders
- Some chance to test it would come from future direct and indirect search experiments (DARWIN, CTA) and especially from a future muon collider
- Mono-X and DT signatures could be however not sufficiently efficient to reach the 5-plet thermal target. Possibilities for indirect tests via precision mesurements
- But 5-plet MDM form Bound States which can be produced with large cross sections at a future multi-Tev muon collider
- The 5-plet MDM BS can be discovered directly via the resonant production of the neutral component (on-peak search) and/or the W associated production of the charged components (above threshold analysis) with few fb-1 at a 30 TeV MuCol

backup

from S. Bottaro *et al.*, Eur. Phys. J. C 82, 31 (2022)

DM spin	EW n-plet	M_χ (TeV)	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$	$\Lambda_{\rm Landau}/M_{\rm DM}$	$\Lambda_{\rm UV}/M_{\rm DM}$
Real scalar	3	2.53 ± 0.01	–	2.4×10^{37}	$4 \times 10^{24}*$
	5	15.4 ± 0.7	0.002	7×10^{36}	3×10^{24}
	7	54.2 ± 3.1	0.022	7.8×10^{16}	2×10^{24}
	9	117.8 ± 15.4	0.088	3×10^{4}	2×10^{24}
	11	199 ± 42	0.25	62	1×10^{24}
	13	338 ± 102	0.6	7.2	2×10^{24}
Majorana fermion	3	2.86 ± 0.01	–	2.4×10^{37}	$2 \times 10^{12}*$
	5	13.6 ± 0.8	0.003	5.5×10^{17}	3×10^{12}
	7	48.8 ± 3.3	0.019	1.2×10^{4}	1×10^{8}
	9	113 ± 15	0.07	41	1×10^{8}
	11	202 ± 43	0.2	6	1×10^{8}
	13	324.6 ± 94	0.5	2.6	1×10^{8}

TABLE I. Freeze-out mass predictions for WIMP DM in real EW multiplets with $Y = 0$. The annihilation cross-section includes both the contribution of SE and BSF. We provide a measure of how close the DM annihilation cross-section is to the unitarity bound for s-wave annihilation $(\sigma v)^{J=0}_{\rm max} = 4\pi/M^2_{\rm DM}v$. Approaching the unitarity bound, the error on the WIMP mass grows proportionally to the enhancement of the next-to-leading order (NLO) contributions estimated in Eq. (23). We derive the scale where EW gauge coupling will develop a Landau pole by integrating-in the WIMP multiplet at its freeze-out mass. The stability of both scalar and fermionic DM can always be enforced by requiring a $\mathbb{Z}_2$ symmetry in the DM sector to forbid DM decays. This symmetry forbids the scalar and fermionic 3-plets decay at renormalizable level as indicated by the *. The value of the UV cut-off $\Lambda_{\rm UV}$ gives an idea of the required *quality* for this symmetry to make DM stable and avoid stringent bounds on decaying DM ($\tau_{\rm DM} > 10^{28}$sec) [26]: a new physics scale lower than $\Lambda_{\rm UV}$ would require a $\mathbb{Z}_2$ to explain DM stability, while a cut-off higher than $\Lambda_{\rm UV}$ would make DM stability purely accidental.