

# A NEW PARADIGM FOR QCD IN THE INFRARED?

Urko Reinosa\*

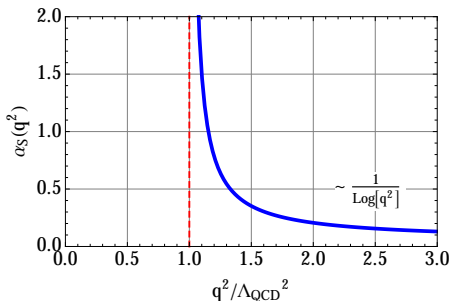
(based on various collaborations with N. Barrios, D. M. van Egmond,  
J. A. Gracey, M. Peláez, M. Tissier, J. Serreau and N. Wschebor)

\*Centre de Physique Théorique, Ecole Polytechnique,  
CNRS & Institut Polytechnique de Paris



May 10, 2023, Laboratoire Charles Coulomb, Montpellier.

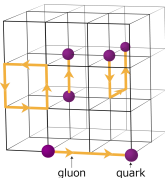
QCD is weakly coupled in the UV and strongly coupled in the IR:



Any serious account of its low energy properties  
requires non-perturbative methods.

This view will not be challenged in this talk  
but refined in some sense.

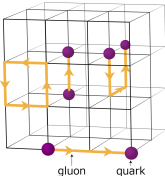
To date, the only really first-principle non-perturbative approach is based on **lattice QCD**. The (Euclidean) QCD functional integral

$$\int \mathcal{D}[A\psi\bar{\psi}] e^{-S_{\text{QCD}}[A,\psi,\bar{\psi}]} \longrightarrow$$


The diagram illustrates a 3D lattice structure, representing a discretized spacetime volume. Purple dots are placed at some of the lattice vertices. Orange arrows connect some of these dots, forming a path. One arrow is labeled 'gluon' and another 'quark'.

is discretized and evaluated via Monte-Carlo importance-sampling which **relies on a probabilistic interpretation**.

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The method **loses its predictive power whenever the probabilistic interpretation fails**: finite baryonic density, real-time processes, ...

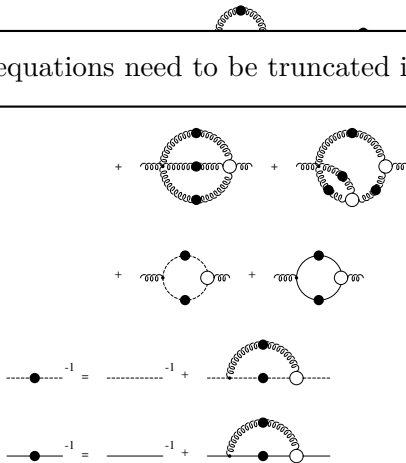
To overcome these limitations, **non-perturbative continuum methods** have been developed. The QCD functional integral is replaced by infinite hierarchies of equations (DS, FRG, NPI, ...):

The image displays two sets of Dyson-Schwinger equations represented by Feynman diagrams. The top set of equations describes the quark propagator, showing a bare quark line (wavy line with a black dot) equal to the sum of a dressed quark line (wavy line with a black dot) and two loop diagrams: a gluon loop and a ghost loop. The bottom set of equations describes the gluon propagator, showing a bare gluon line (dashed line with a black dot) equal to the sum of a dressed gluon line (dashed line with a black dot) and a ghost loop diagram.

$$\begin{aligned}
 \text{Quark Propagator: } \text{wavy line with black dot}^{-1} &= \text{wavy line with black dot}^{-1} + \text{gluon loop} + \text{ghost loop} \\
 &+ \text{gluon loop with quark self-energy} + \text{ghost loop with quark self-energy} \\
 &+ \text{gluon loop with quark self-energy} + \text{ghost loop with quark self-energy} \\
 \text{Gluon Propagator: } \text{dashed line with black dot}^{-1} &= \text{dashed line with black dot}^{-1} + \text{ghost loop} \\
 \text{Gluon Propagator: } \text{solid line with black dot}^{-1} &= \text{solid line with black dot}^{-1} + \text{ghost loop}
 \end{aligned}$$

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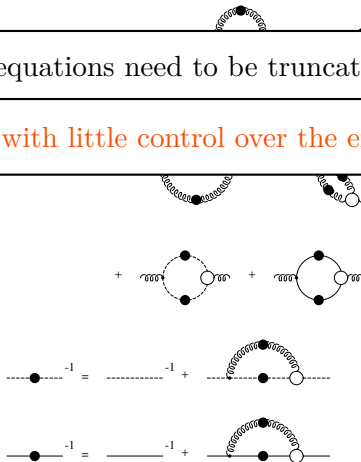
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These equations need to be truncated in practice

with little control over the error ...



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These equations need to be truncated in practice

with little control over the error ...




Not fully first principle since

relying on Faddeev-Popov gauge-fixing



$$\text{---}\bullet\text{---}^{-1} = \text{---}^{-1} + \text{---}\bullet\text{---}$$



Could one imagine a **third possible way into infrared QCD** that allows one to circumvent some of the limitations of the lattice while providing a systematic control over the error?

We believe that some of the results obtained over these past 20 years within **Landau gauge-fixed lattice simulations** point at that possibility.

This talks aims at reporting our progresses towards this goal ...

[M. Peláez, U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Rept. Prog. Phys. 84 (2021)]

# OUTLINE

I. Motivation ✓

II. Quarks and Gluons in the infrared

III. The Curci-Ferrari (CF) model

IV. Benchmarking the CF model:

- a. Pure glue case;
- b. Glue + Heavy quarks;
- c. Glue + Light quarks;

V. Probing the QCD phase structure from the CF model

I. Motivation ✓

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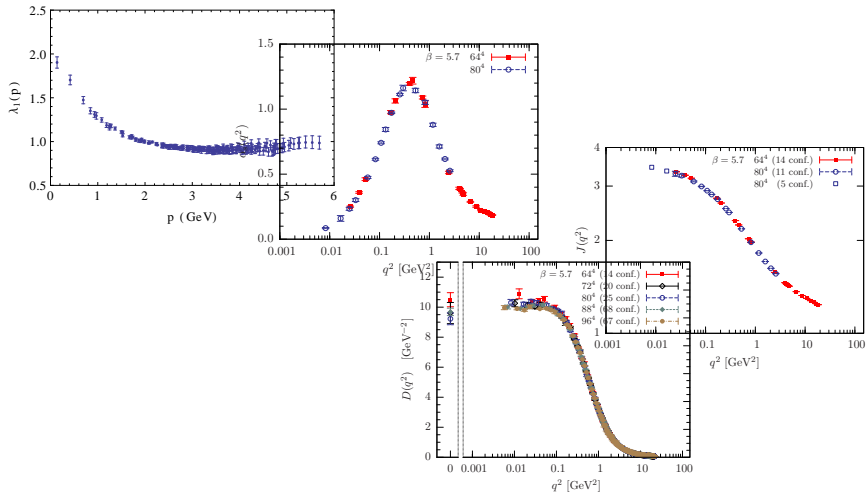
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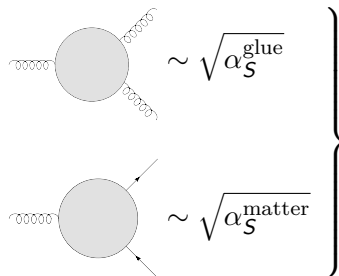
# LANDAU-GAUGE-FIXED LATTICE SIMULATIONS

Over the past 20 years, Landau-gauge-fixed lattice simulations have allowed us to refine our understanding of the dynamics of colored fields in the infrared while revealing unexpected features:



# NON-UNIVERSALITY OF THE STRONG INTERACTION IN THE INFRARED

We all learn that the strong interaction is universal:

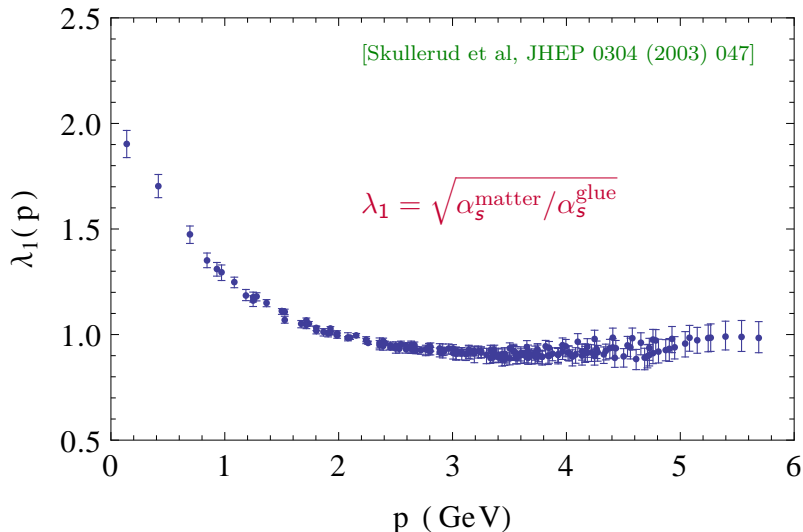


The diagram shows two Feynman diagrams for strong interaction processes, grouped by a large right curly brace. The top diagram shows a gluon (wavy line) entering a grey circular vertex from the left, and two gluons (wavy lines) exiting to the right. Next to it is the expression  $\sim \sqrt{\alpha_S^{\text{glue}}}$ . The bottom diagram shows a gluon (wavy line) entering a grey circular vertex from the left, and two quarks (straight lines) exiting to the right. Next to it is the expression  $\sim \sqrt{\alpha_S^{\text{matter}}}$ . To the right of the brace is an implication arrow  $\Rightarrow$  pointing to a red-bordered box containing the equation  $\alpha_S^{\text{glue}} = \alpha_S^{\text{matter}}$ .

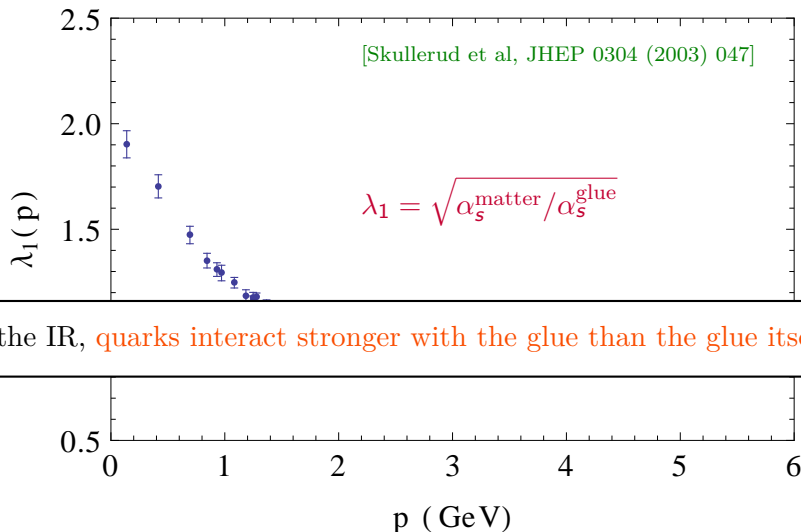
$$\left. \begin{array}{l} \sim \sqrt{\alpha_S^{\text{glue}}} \\ \sim \sqrt{\alpha_S^{\text{matter}}} \end{array} \right\} \Rightarrow \alpha_S^{\text{glue}} = \alpha_S^{\text{matter}}$$

However, this is a UV result which is **not true anymore in the IR.**

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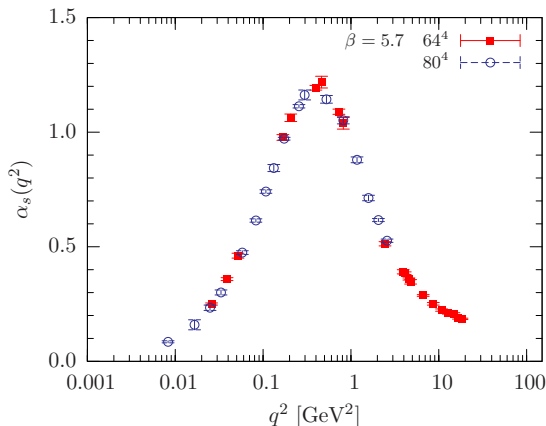
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Since there is a hierarchy of couplings in the infrared, it is interesting to look at the smallest of them,  $\alpha_s^{\text{glue}}$ . Here comes a second surprise:



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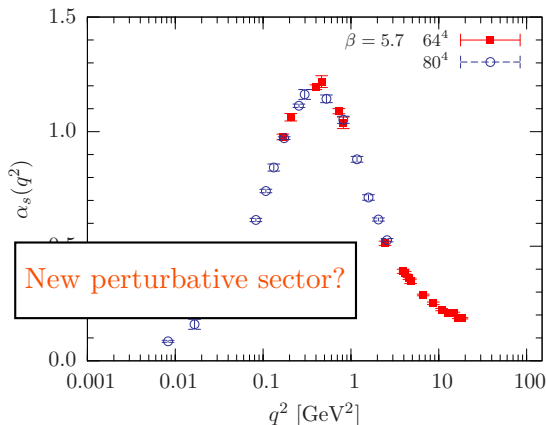
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[I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, PLB 676, 69 (2009)]

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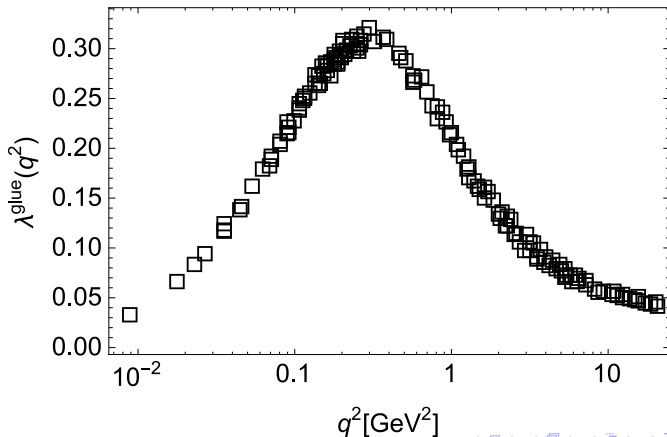


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## GLUE COUPLING

In fact, the natural expansion parameter of perturbation theory in the glue sector is not  $\alpha_s^{\text{glue}}$  but rather

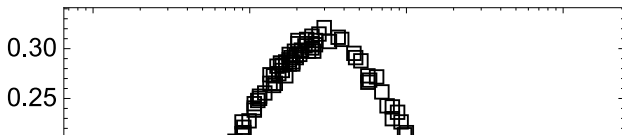
$$\lambda^{\text{glue}} \equiv \frac{g^2 N_c}{16\pi^2} = \frac{N_c}{4\pi} \alpha_s^{\text{glue}}$$



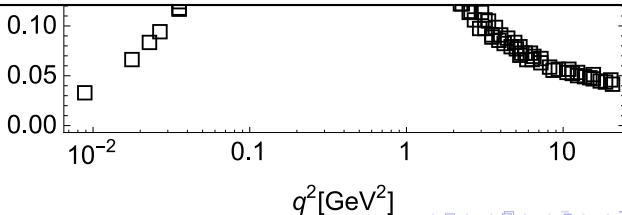
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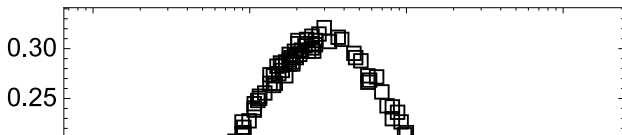
Could part of the IR glue-dynamics be captured perturbatively?



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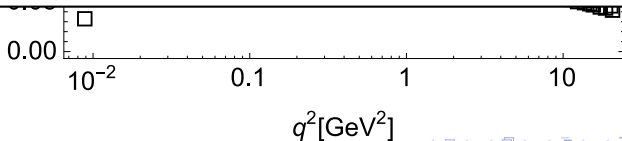
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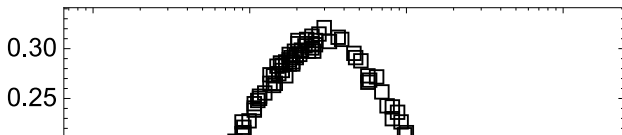
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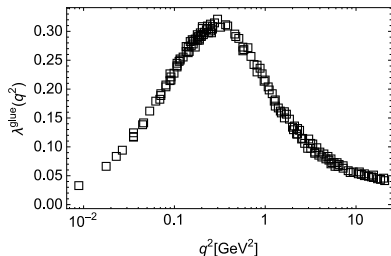
QCD would remain non-perturbative since  $\lambda^{\text{matter}} \simeq 4\lambda^{\text{glue}}$

but with “perturbative glue” at its core

$q^2 [\text{GeV}^2]$

## BUT WAIT ...

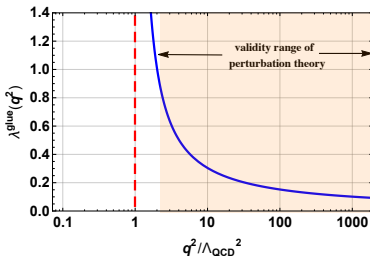
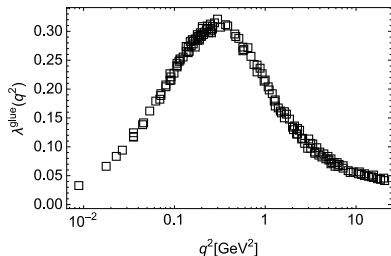
We have two seemingly contradictory pictures for the glue sector:



According to the first, **perturbation theory is valid over all scales.**

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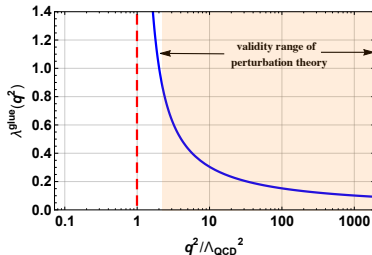
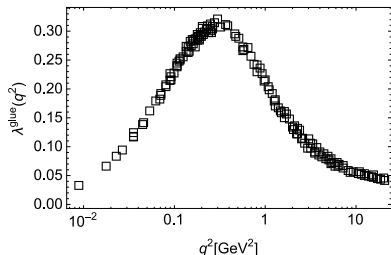
According to the first, **perturbation theory is valid over all scales.**

According to the second, **perturbation theory predicts its own failure.**



# BUT WAIT ...

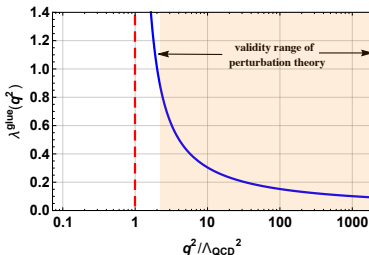
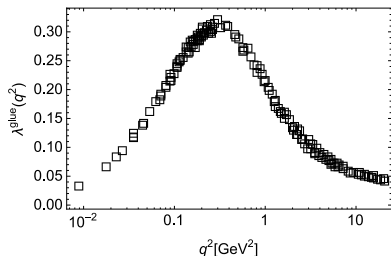
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Yet, the first is the outcome of a **first-principle lattice calculation**.

# BUT WAIT ...

We have two seemingly contradictory pictures for the glue sector:



Yet, the first is the outcome of a **first-principle lattice calculation**.

The second results instead from an **(uncontrolled) extrapolation of the Faddeev-Popov procedure**.

# GAUGE FIXING AND FADDEEV-POPOV ACTION

To set up a perturbative expansion we should in principle consider:

$$S_{YM}[A] \quad \text{with} \quad \partial_\mu A_\mu^a = 0 \quad [\text{Landau gauge}]$$

In practice, however, we use:

$$S_{FP} = S_{YM} + \int_x \left\{ i \hbar^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu c^a \right\} \quad [\text{Faddeev-Popov}]$$

These two ways of proceeding are often thought to be equivalent.

# GAUGE FIXING AND FADDEEV-POPOV ACTION

However, the equivalence is known to rely on a mathematically incorrect assumption (“Gribov copy problem”).

In fact:

- At high energies, the equivalence is seen to hold.
- At low energies, we have tangible evidence that it does not.

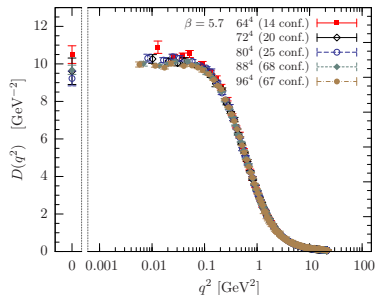
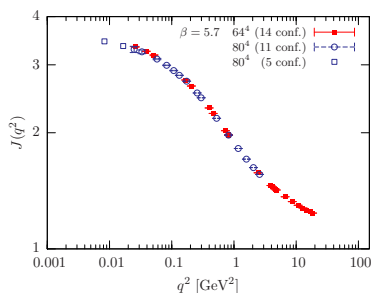
# SCALING VS DECOUPLING SOLUTIONS

**Kugo and Ojima:** when the FP action is taken seriously at all scales, one deduces a specific behavior of the correlation functions in the IR

$$\Rightarrow \text{“scaling” solution} \left\{ \begin{array}{l} J(q^2) \equiv q^2 \langle c(-q) \bar{c}(q) \rangle \rightarrow \infty \text{ as } q \rightarrow 0 \\ D(q^2) \equiv P_{\mu\nu}^{\perp}(q) \langle A_{\mu}(-q) A_{\nu}(q) \rangle \rightarrow 0 \end{array} \right.$$

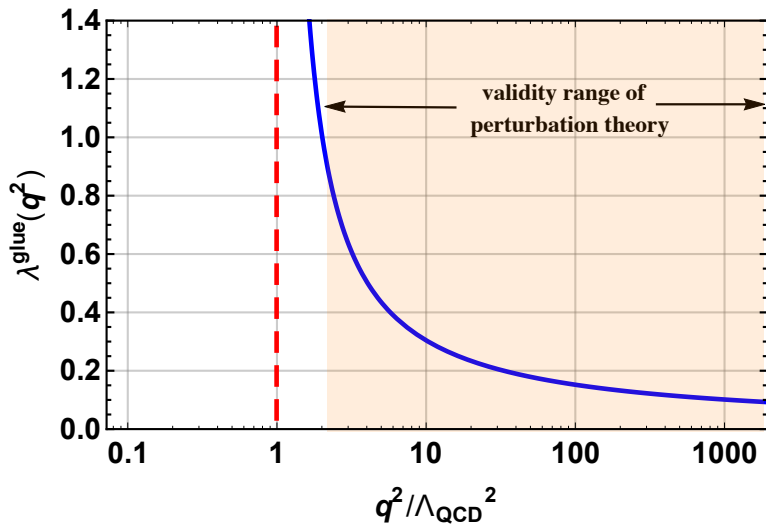
# SCALING VS DECOUPLING SOLUTIONS

At odds with the “decoupling” solution found on the lattice (which does not rely on FP):

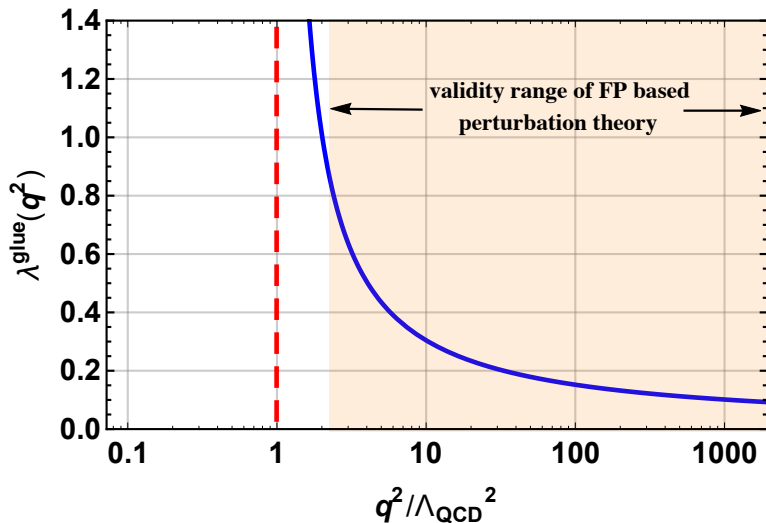


[I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, PLB 676, 69 (2009)]

# PERTURBATIVE GLUE SCENARIO

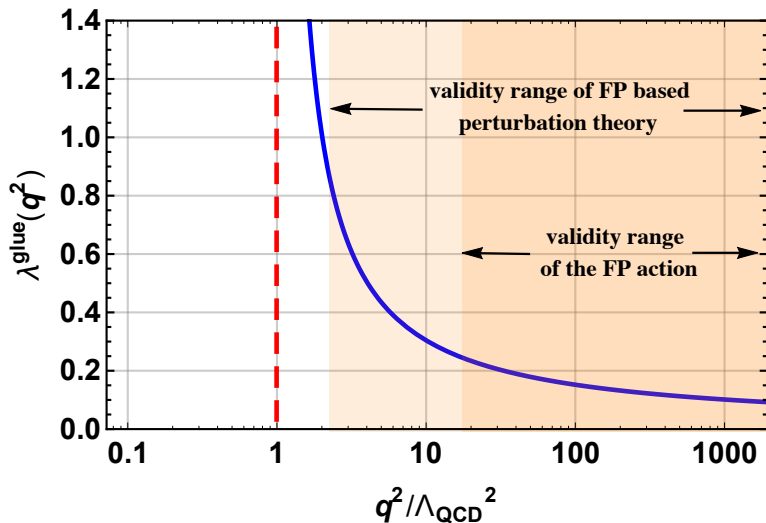


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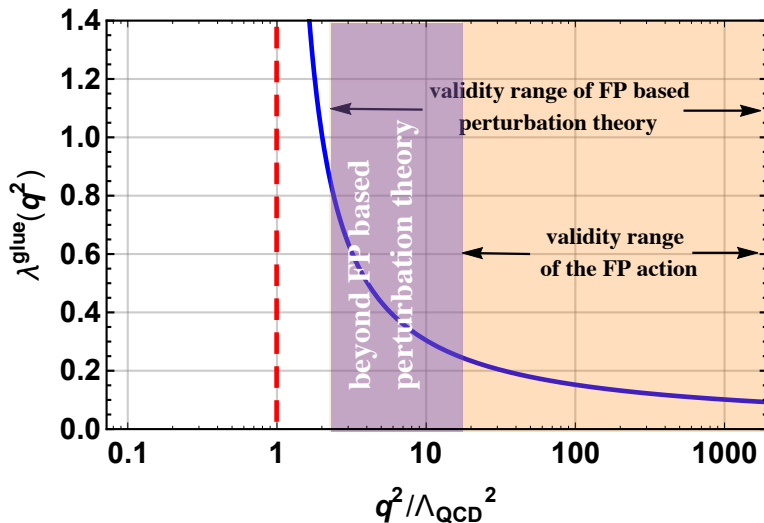




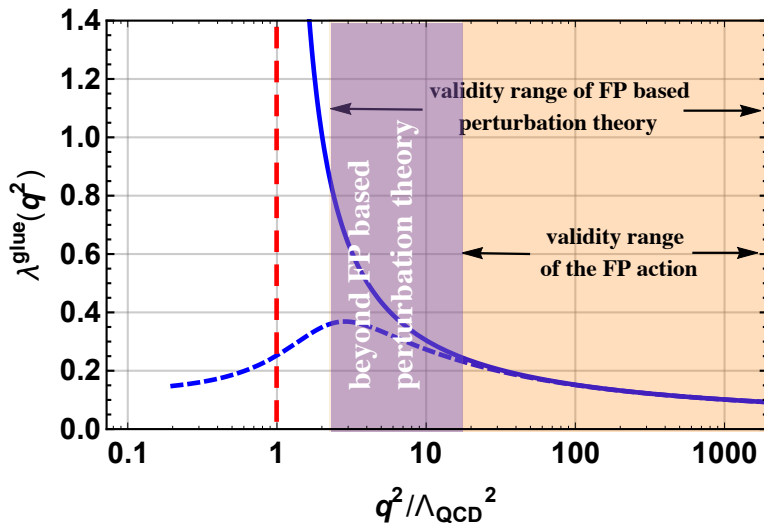
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# BEYOND THE FADDEEV-POPOV ACTION

In order to implement this perturbative glue scenario, we need to find how the FP action is modified in the infrared

$$S_{FP} \longrightarrow S_{FP} + \delta S$$

How to find the appropriate extension  $\delta S$ ?

- first-principle approach: not known;
- semi-first-principle approach: Gribov-Zwanziger;
- phenomenological approaches: add new operators to  $S_{FP}$  and try to constrain their couplings, or even discard them, using experiments/lattice simulations.

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# THE CURCI-FERRARI MODEL

The **Curci-Ferrari (CF) model** is one example of such an extension:

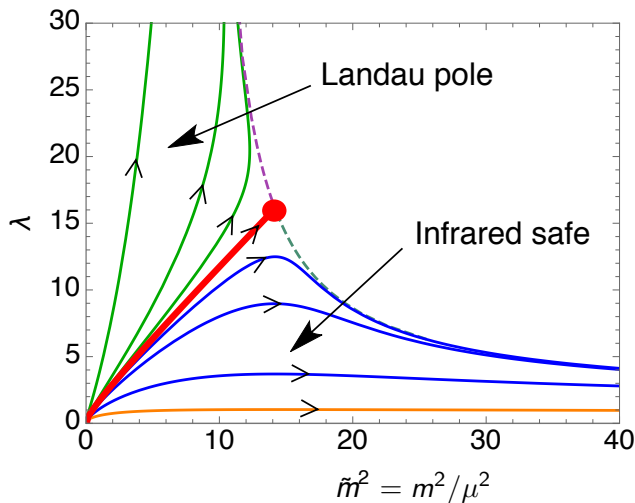
$$S_{CF} = \underbrace{\int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu \bar{c}^a D_\mu c^a + i h^a \partial_\mu A_\mu^a \right\}}_{\text{incomplete FP gauge-fixing, valid in the UV only a priori}} + \underbrace{\int_x \frac{m^2}{2} A_\mu^a A_\mu^a}_{\text{IR pheno term}}$$

Please, bear in mind that this is a **phenomenological approach** motivated by the decoupling behavior in the Landau gauge. No claim that our approach is first principle.

However: **the model is renormalizable**. So it relies on only one additional parameter  $m^2$  that can be fixed by comparison to gauge-fixed lattice simulations in the Landau gauge.

# FLOW DIAGRAM OF THE CF MODEL

The main interest of the CF model lies in its flow diagram :



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II. Quarks and Gluons in the infrared ✓

III. The Curci-Ferrari (CF) model ✓

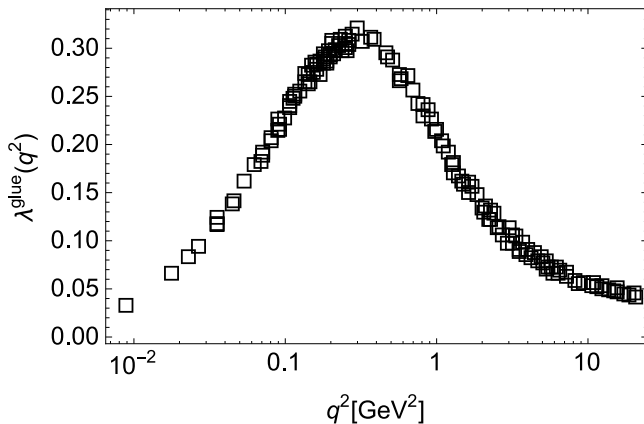
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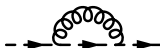
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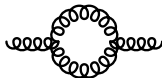
# PURE GLUE EXPANSION PARAMETER



A perturbative expansion within the CF model should suffice.

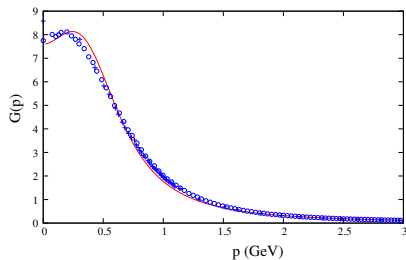


## ONE-LOOP TWO-POINT FUNCTIONS

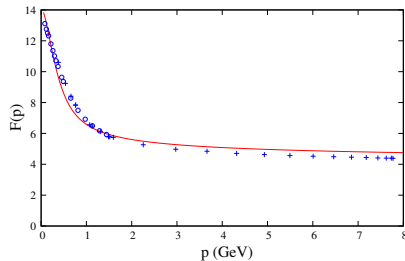


# FITS TO THE LATTICE

$$G(p) \equiv D(p)$$



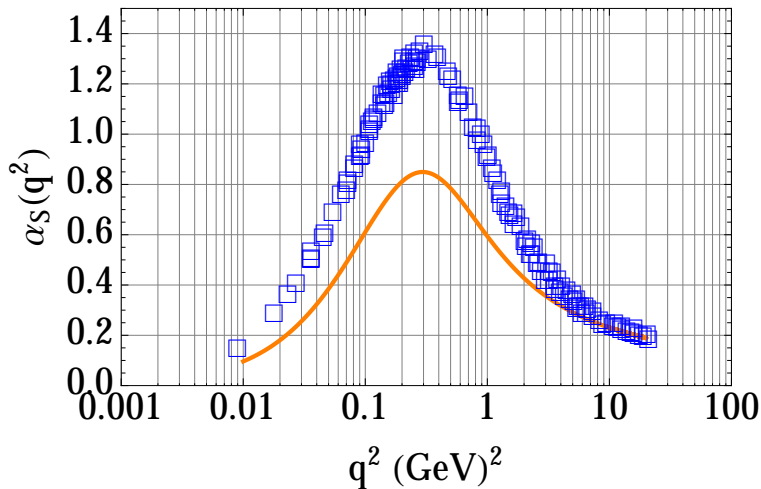
$$F(p) \equiv J(p)$$

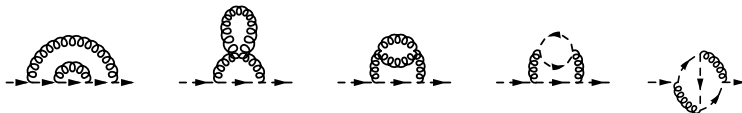


Tissier and Wschebor, Phys. Rev. D82 (2010) & Phys. Rev. D84 (2011).

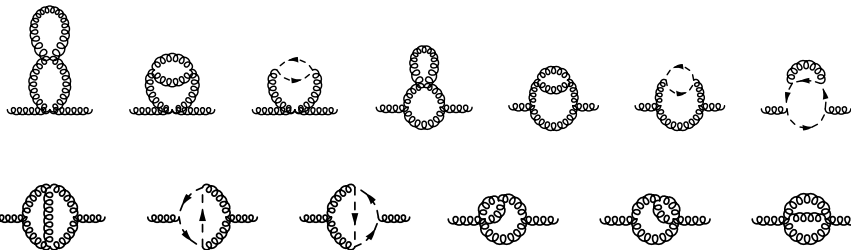
$$m_0 \simeq 500 \text{ MeV}$$

# RUNNING COUPLING





## TWO-LOOP TWO-POINT FUNCTIONS



# REDUCTION TO MASTER INTEGRALS

1. We use **Laporta algorithm** to decompose the two-loop two-point functions into a basis of (scalar) **master integrals**

$$\Gamma_{AA}^{(2)}(p) = p^2 + m^2 + \sum_{D \in \mathcal{M}} \mathcal{R}_{AA}(D) \mathcal{I}(D)$$

$$\Gamma_{c\bar{c}}^{(2)}(p) = p^2 + \sum_{D \in \mathcal{M}} \mathcal{R}_{c\bar{c}}(D) \mathcal{I}(D)$$

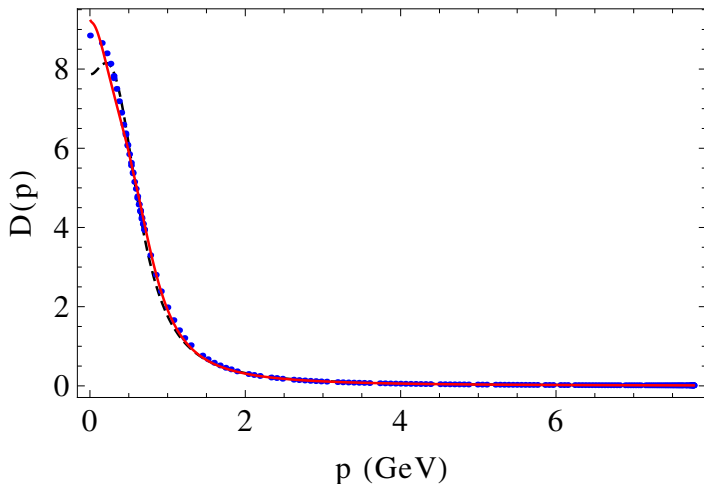
where  $\mathcal{R}_{AA}(D)$  and  $\mathcal{R}_{c\bar{c}}(D)$  are rational functions of  $p^2$  and  $m^2$ , and  $\mathcal{I}(D)$  is a master Feynman integral, with  $D$  among

$$D \in \mathcal{M} = \left\{ \text{bubble}, \text{triangle}, \text{triangle with internal line}, \text{triangle with internal line and dot}, \text{triangle with internal line and dot and external line}, \text{triangle with internal line and dot and external line and loop}, \text{triangle with internal line and dot and external line and loop and external line} \right\}$$

2. We then evaluate each of the masters using the **TSIL package**.

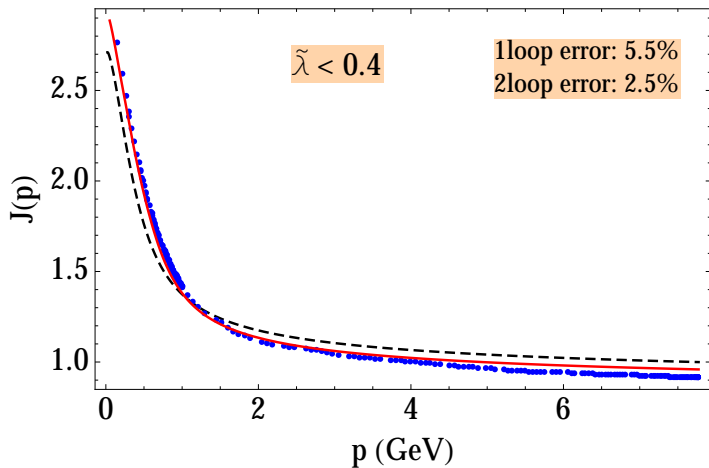
[<https://www.niu.edu/spmartin/TSIL/>]

# IMPROVED FITS TO THE LATTICE



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, Phys. Rev. D100 (2019)]

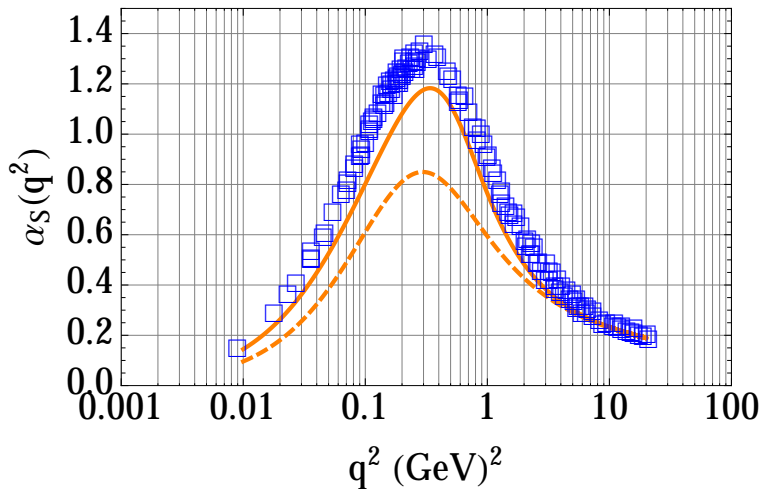
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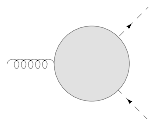
# IMPROVED RUNNING COUPLING



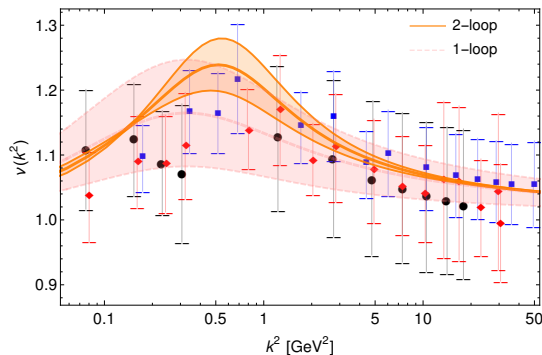
# ONE-LOOP AND TWO-LOOP THREE-POINT FUNCTIONS

# GHOST-ANTIGHOST-GLUON VERTEX

We have evaluated the ghost-antighost-gluon vertex



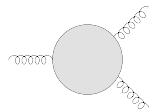
- at one-loop for any configuration of the external momenta;
- at two-loop in the vanishing gluon momentum configuration.



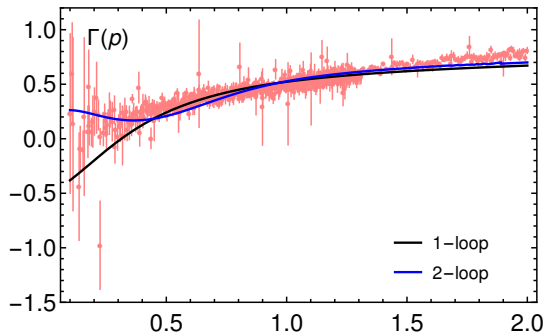
[N. Barrios, M. Peláez, U. Reinosa, N. Wschebor, Phys. Rev. D102 (2020)]

# THREE-GLUON VERTEX

Similarly, we have evaluated the three-gluon vertex



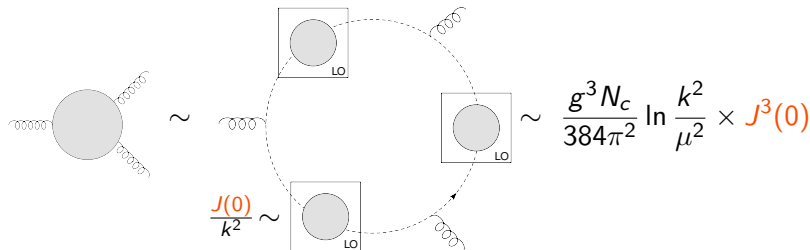
- at one-loop for any configuration of the external momenta;
- at two-loop in the one-vanishing-momentum configuration.



[N. Barrios, M. Peláez, U. Reinosa, Phys. Rev. D106 (2022)]

# ZERO-CROSSING?

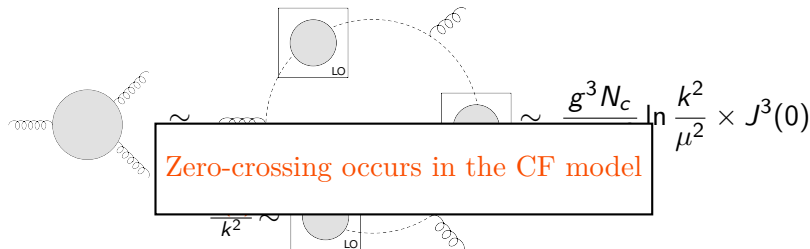
Using the decoupling of gluons and Smirnov's IR expansion (analog of Weinberg's UV expansion), one finds that the leading behavior is given by an effective one-ghost-loop:


$$\sim \frac{J(0)}{k^2} \sim \frac{g^3 N_c}{384 \pi^2} \ln \frac{k^2}{\mu^2} \times J^3(0)$$

[N. Barrios, M. Peláez, U. Reinosa, Phys. Rev. D106 (2022)]

# ZERO-CROSSING?

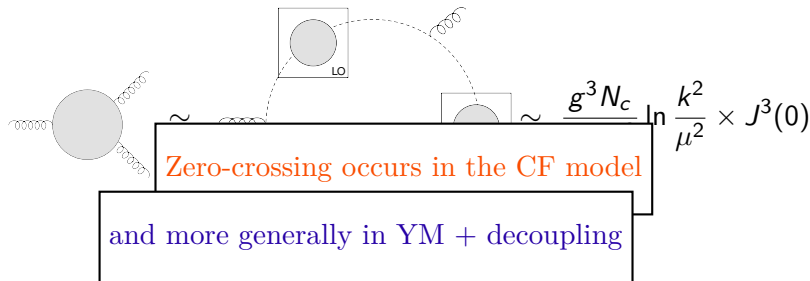
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[N. Barrios, M. Peláez, U. Reinosa, Phys. Rev. D106 (2022)]

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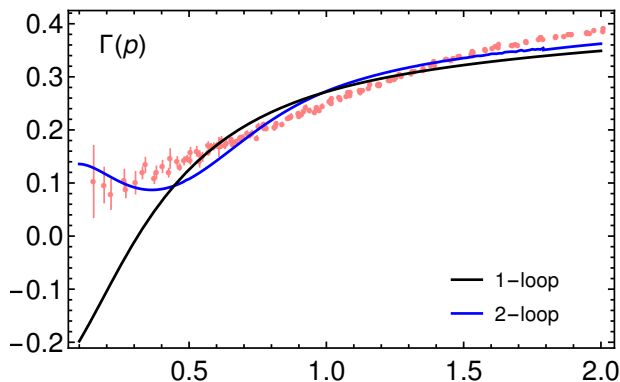
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[N. Barrios, M. Peláez, U. Reinosa, Phys. Rev. D106 (2022)]

# ZERO-CROSSING?

We find a small zero-crossing scale at two-loop order (a few MeV) compatible with some recent lattice data in the IR:



[N. Barrios, M. Peláez, U. Reinosa, Phys. Rev. D106 (2022)]



I. Motivation ✓

II. Quarks and Gluons in the infrared ✓

III. The Curci-Ferrari (CF) model ✓

IV. Benchmarking the CF model:

- a. Pure glue case; ✓
- b. Glue + Heavy quarks;
- c. Glue + Light quarks;

V. Probing the QCD phase structure from the CF model

# HEAVY QCD

Before heading to QCD, it is interesting to investigate a formal regime where all quarks are considered heavy (although not infinitely massive).

This “heavy QCD” regime is a good testing ground for any new method on the market.

The expansion parameter is similar to that of pure glue, so the perturbative CF should work here as well.

# THE QUARK PROPAGATOR

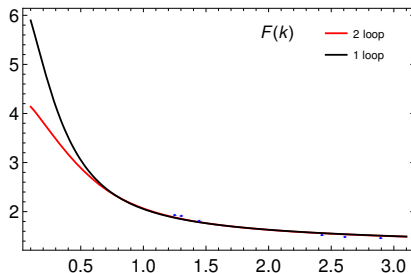
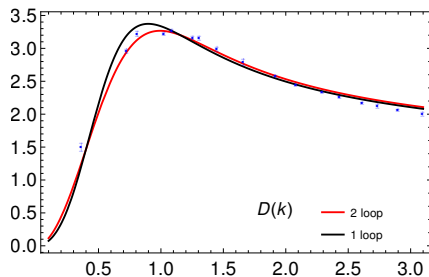
In addition to the ghost and gluon propagators, we have now the form factors of the **quark propagator**:

$$S(q) = \langle \psi \bar{\psi} \rangle = \frac{Z(q^2)}{i\not{q} + M(q^2)}$$

We evaluate the quark dressing function  $Z(q^2)$  and the quark mass function  $M(q^2)$  at one- and two-loop order of the perturbative CF expansion.

# RESULTS

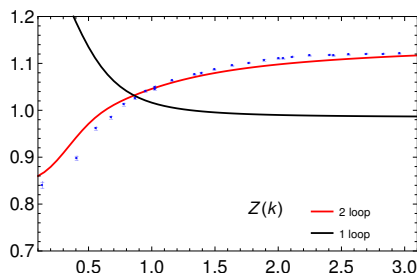
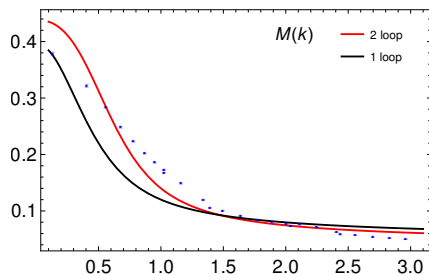
The pure glue sector is still very well described by the pert. CF model:



[N. Barrios, J. A. Gracey, M. Peláez, U. Reinosa, Phys. Rev. D104 (2021)]

# RESULTS

The pert. CF model also accounts for the quark form factors:



[N. Barrios, J. A. Gracey, M. Peláez, U. Reinosa, Phys. Rev. D104 (2021)]

N.B.: the quark dressing  $Z$  is completely off at one-loop. This is due to an accidental symmetry that makes the one-loop correction abnormally small in the UV.

I. Motivation ✓

II. Quarks and Gluons in the infrared ✓

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# QCD WITH LIGHT QUARKS

The **perturbative CF model is doomed to fail** for at least two reasons:

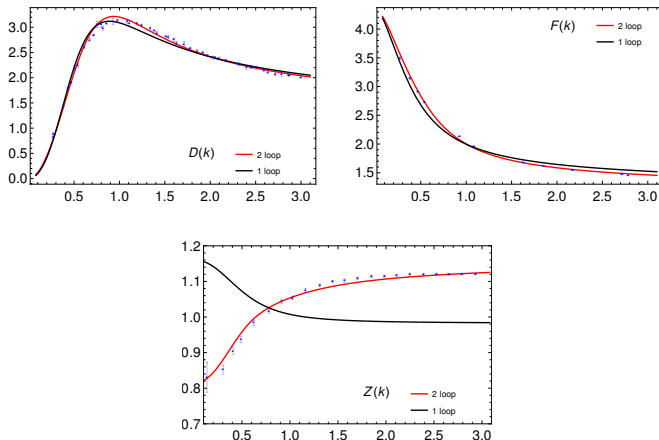
- no perturbative treatment can account for chiral symmetry breaking (responsible for most of the quark mass function);
- even though  $\lambda^{\text{glue}} < 0.3$  is perturbative,  $\lambda^{\text{matter}} \sim 4\lambda^{\text{glue}} \lesssim 1.2$  and thus reaches non-perturbative values.

This **does not mean that the CF model should be abandoned**, however, since:

- quantities that are little sensitive to chiral symmetry breaking could still be correctly accounted by the perturbative CF model;
- quantities that are governed by chiral symmetry breaking could still be accounted by the CF model beyond perturbation theory.

# PERTURBATIVE RESULTS

The perturbative CF model is still quite good at describing quantities that are not directly impacted by chiral symmetry breaking:

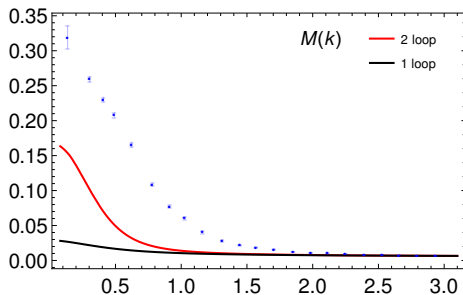


[N. Barrios, J. A. Gracey, M. Peláez, U. Reinosa, Phys. Rev. D104 (2021)]



# THE QUARK MASS FUNCTION

On the other hand, the perturbative CF model performs poorly on the quark mass function (as expected):



[N. Barrios, J. A. Gracey, M. Peláez, U. Reinosa, Phys. Rev. D104 (2021)]

Need to go beyond perturbation theory. But then, what is difference with the standard continuum non-perturbative approaches?

# CONTROLLED NON-PERTURBATIVE SET-UP

The problem with the **non-perturbative** approaches based on **FP** is that the truncations are ad-hoc, with **little control over the error**.

One can try invoking an expansion in  $1/N_c$  but the calculations are prohibitively difficult.

Within the CF model, however, one can invoke a **second small expansion parameter**  $\lambda^{\text{glue}} < 0.3$ .

The combination of both expansions in  $1/N_c$  and  $\lambda^{\text{glue}}$  seems to be the winning horse.

# CONTROLLED NON-PERTURBATIVE SET-UP

Example of the quark propagator:

$$\begin{aligned}
 (\text{thick line})^{-1} &= (\text{thin line})^{-1} \left[ \begin{array}{l} \text{gluon loop} \\ + \text{ghost loop} \\ + \text{quark loop} \\ + \text{quark-gluon loop} \\ + \text{quark-gluon-gluon loop} \\ + \text{quark-gluon-quark loop} \\ + \dots \end{array} \right]
 \end{aligned}$$

# CONTROLLED NON-PERTURBATIVE SET-UP

Example of the quark propagator:

$$\begin{aligned} (\text{thick arrow})^{-1} &= (\text{thin arrow})^{-1} - \left[ \begin{array}{c} \text{gluon loop} \\ \text{gluon self-energy} \\ \text{gluon exchange} \\ \text{gluon exchange} \\ \text{gluon exchange} \\ \text{gluon exchange} \\ \text{gluon exchange} \end{array} \right] \end{aligned}$$

The diagrams in the brackets represent various gluon corrections to the quark propagator. Some diagrams are crossed out with a red diagonal line, indicating they are suppressed by  $\lambda^{\text{glue}}$ .

suppressed by  $\lambda^{\text{glue}}$

# CONTROLLED NON-PERTURBATIVE SET-UP

Example of the quark propagator:

$$\begin{aligned}
 (\overrightarrow{\hspace{1.5cm}})^{-1} &= (\overrightarrow{\hspace{1.5cm}})^{-1} - \left[ \begin{array}{c} \text{diagram 1} \\ + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \\ + \text{diagram 5} + \text{diagram 6} \\ + \text{diagram 7} + \text{diagram 8} \dots \end{array} \right]
 \end{aligned}$$

The diagrams in the brackets represent various gluon loop corrections to the quark propagator. Diagrams with a red diagonal slash are suppressed by  $\lambda^{\text{glue}}$ , and diagrams with a green diagonal slash are suppressed by  $1/N_c$ .

suppressed by  $\lambda^{\text{glue}}$

suppressed by  $1/N_c$

# RAINBOW EQUATION

At leading-order, the double expansion in  $\lambda^{\text{glue}}$  and  $1/N_c$  leads to the family of diagrams

$$(\text{thick arrow})^{-1} = (\text{thin arrow})^{-1} \left[ \text{thin arrow with 1 loop} + \text{thin arrow with 2 loops} + \text{thin arrow with 3 loops} + \dots \right]$$

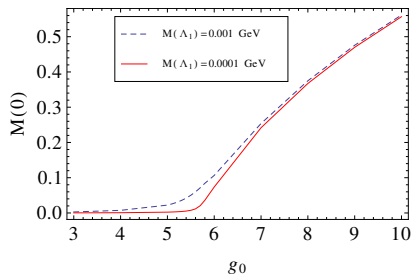
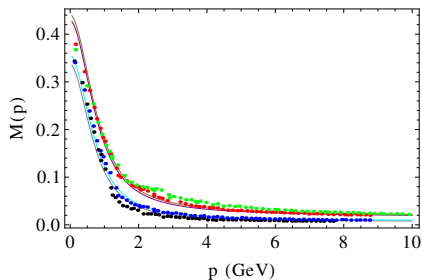
which can be resummed into

$$(\text{thick arrow})^{-1} = (\text{thin arrow})^{-1} - \text{thick arrow with 1 loop}$$

This is nothing but the well known **Rainbow equation** derived not from ad-hoc approximations but from a systematic expansion **controlled by two small parameters**.

# BACK TO THE QUARK MASS FUNCTION

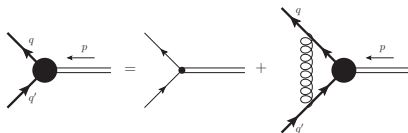
Good account of chiral symmetry breaking:



[M. Peláez, U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys. Rev. D96 (2017)]

# THE PION DECAY CONSTANT

Other vertex functions sensible to chiral symmetry breaking can be computed in a similar way. At LO the quark-antiquark-pion vertex is given by the well known **Rainbow-Ladder equation**:



As a first application, we were able to predict a value for the **pion decay constant in the chiral limit** in agreement with the expected value of 86 MeV.

[M. Peláez, U. Reinosa, J. Serreau, N. Wschebor, Phys. Rev. D107 (2023)]



I. Motivation ✓

II. Quarks and Gluons in the infrared ✓

III. The Curci-Ferrari (CF) model ✓

IV. Benchmarking the CF model: ✓

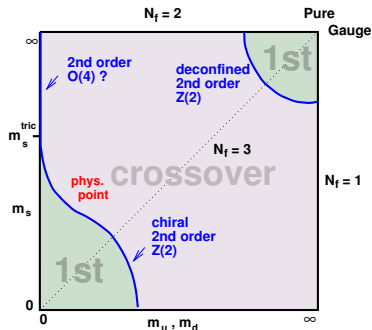
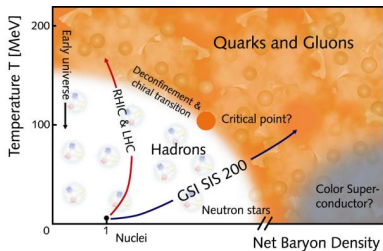
- a. Pure glue case; ✓
- b. Glue + Heavy quarks; ✓
- c. Glue + Light quarks; ✓

V. Probing the QCD phase structure from the CF model

# QCD PHASE STRUCTURE

What are the predictions of the CF model regarding the **confinement/deconfinement transition** and **chiral symmetry breaking**?

[U. Reinosa, Lecture Notes in Physics (2023)]

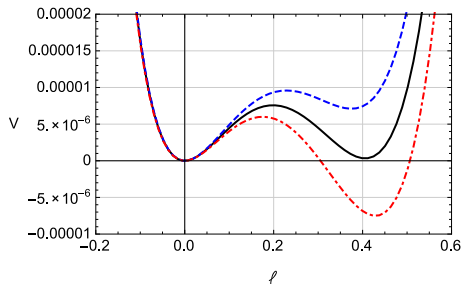


Order parameters:

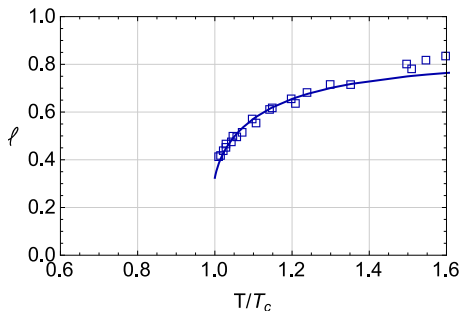
$$\ell \equiv \frac{1}{N_c} \left\langle \text{tr } P \exp \left\{ ig \int_0^\beta d\tau A_0 \right\} \right\rangle \quad \text{and} \quad \sigma \equiv \langle \bar{\psi} \psi \rangle$$

# PURE GLUE RESULTS

We have computed the Polyakov loop potential at one-loop order of the perturbative CF expansion. It does already a pretty good job in reproducing known features of the YM phase structure:



$T_c^{CF} \sim 268 \text{ MeV}$   
(vs 270 MeV on the lattice)

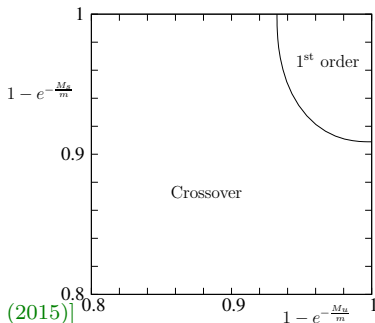


[D. M. van Egmond, U. Reinosa, J. Serreau, M. Tissier, SciPost Phys. 12 (2022).]

# HEAVY QUARK QCD

It does also a pretty good job in retrieving the **phase structure in the heavy quark sector**:

$M_c/T_c$	$N_f = 1$	$N_f = 2$	$N_f = 3$
Lattice	7.23	7.92	8.33
CF	6.74	7.59	8.07
Matrix	8.04	8.85	9.33
DSE	1.42	1.83	2.04



[U. Reinosa, J. Serreau, M. Tissier, Phys. Rev. D92 (2015)]

Two-loop corrections improve the results further.

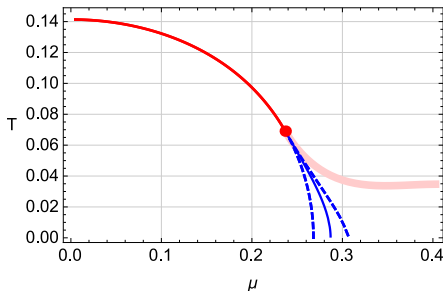
[J. Maelger, U. Reinosa, J. Serreau, Phys. Rev. D97 (2018)]

# QCD WITH LIGHT QUARKS

The light quark sector calls again for the use of the **Rainbow** equation at finite temperature/density:

$$(\longrightarrow)^{-1} = (\longrightarrow)^{-1} - \text{rainbow diagram}$$

A preliminary (qualitative) study leads to the presence of a **CEP** in the phase diagram:



[J. Maelger, U. Reinosa, J. Serreau, Phys. Rev. D101 (2020)]

# CONCLUSIONS

- Over the past 20 years, lattice simulations of Landau-gauge correlation functions have revealed unexpected aspects of the dynamics of quarks and gluons in the infrared.
- This allows one to contemplate a new path into QCD that treats the pure glue interactions perturbatively, while dealing with the remaining interactions via a well tested  $1/N_c$ -expansion.
- These ideas cannot be put in practice via the standard perturbative set-up since the latter relies on the FP Landau gauge-fixed action, valid only in the ultraviolet.

# CONCLUSIONS

- Lattice results for the gluon propagator suggest to model the unknown part of the Landau gauge-fixed action in the infrared via the Curci-Ferrari model.
- Within this model, the new strategy appears to be well under control and allows one to reproduce a number of lattice QCD results (correlators, phase structure, ...).
- These results point to the idea that a better understanding of the gauge fixing in the infrared could open new pathways into infrared QCD.

THANK YOU!



# BACKUP

# A FREQUENT CONFUSION

The **Curci-Ferrari model** is often confused with **massive Yang-Mills a.k.a. Proca theory** which amounts to adding a mass term prior to fixing any gauge:

$$S_{Proca} \equiv S_{YM} + \int_x \frac{m^2}{2} A_\mu^a A_\mu^a \quad \text{vs} \quad S_{CF} \equiv S_{FP} + \int_x \frac{m^2}{2} A_\mu^a A_\mu^a$$

Quite different models actually:

- $S_{Proca}$  is non-renormalizable while  $S_{CF}$  is renormalizable;
- $S_{Proca}$  breaks gauge invariance while in  $S_{CF}$  it is already explicitly broken by the gauge fixing provided by  $S_{FP}$ ;
- $S_{Proca}$  is an explicit modification of a fundamental theory  $S_{YM}$ , while  $S_{CF}$  aims at modelling the incomplete gauge fixing  $S_{FP}$ .