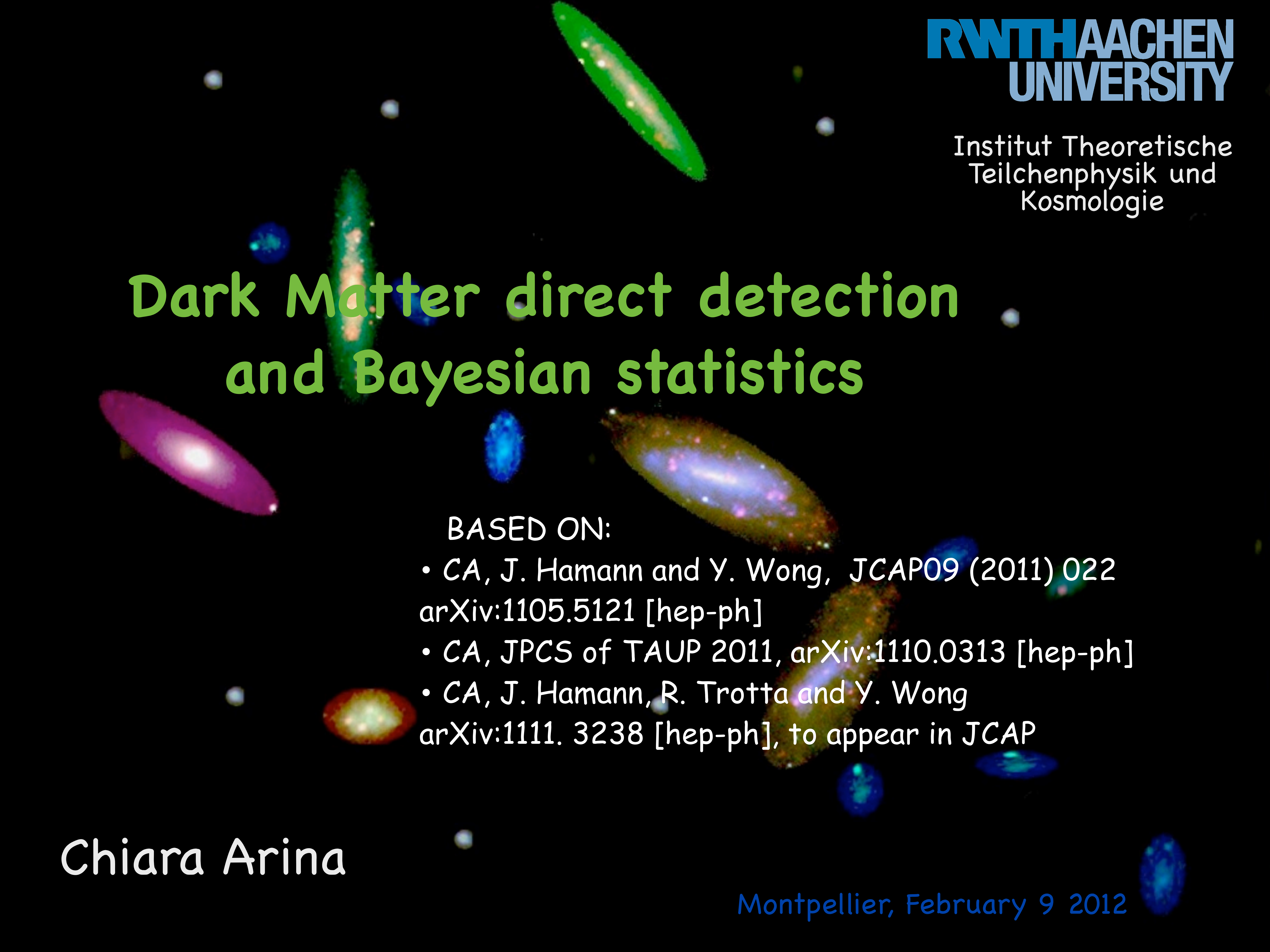


RWTH AACHEN UNIVERSITY

Institut Theoretische
Teilchenphysik und
Kosmologie

Dark Matter direct detection and Bayesian statistics

BASED ON:

- CA, J. Hamann and Y. Wong, JCAP09 (2011) 022 arXiv:1105.5121 [hep-ph]
- CA, JPCS of TAUP 2011, arXiv:1110.0313 [hep-ph]
- CA, J. Hamann, R. Trotta and Y. Wong arXiv:1111. 3238 [hep-ph], to appear in JCAP

Chiara Arina

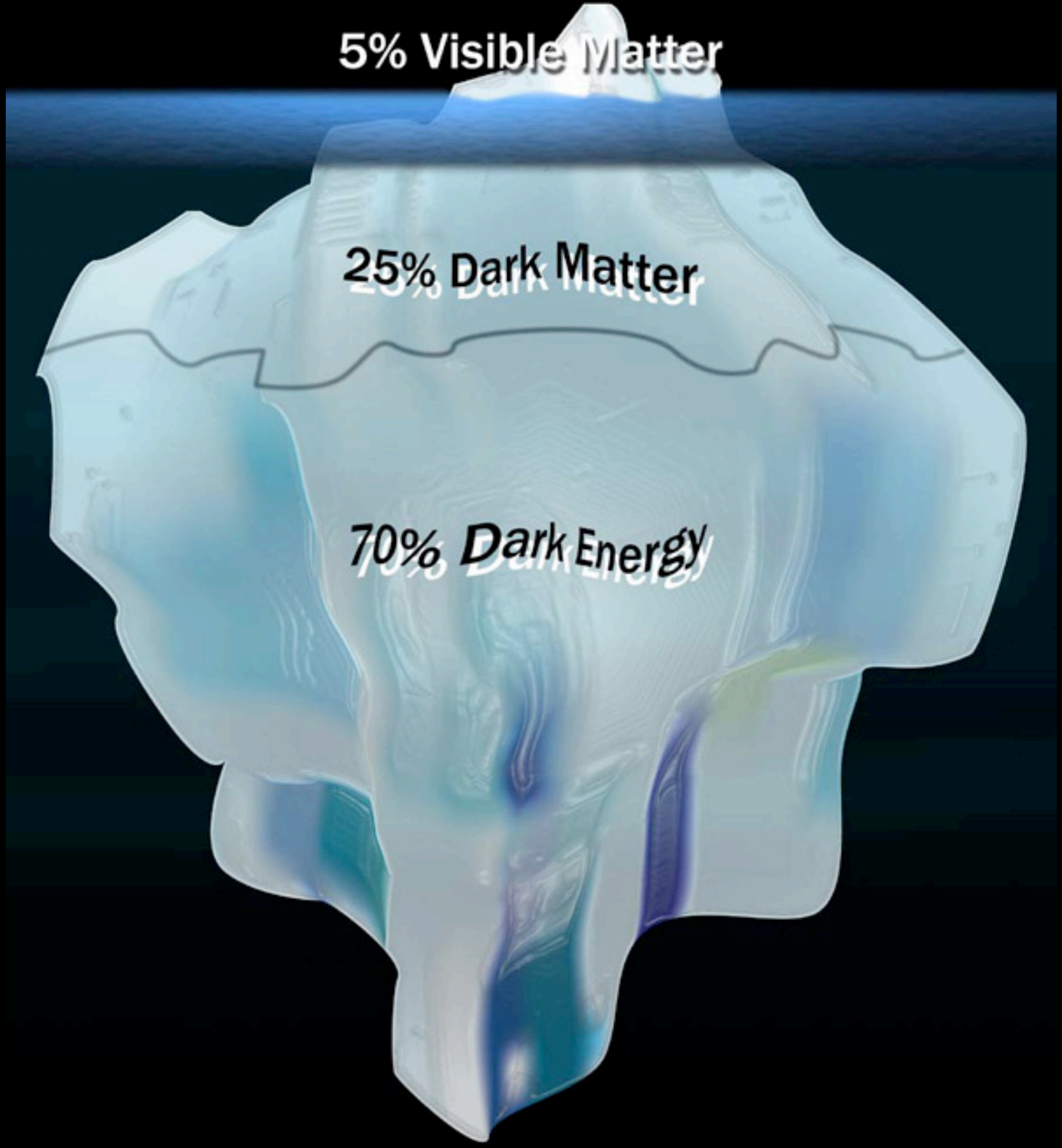
Montpellier, February 9 2012

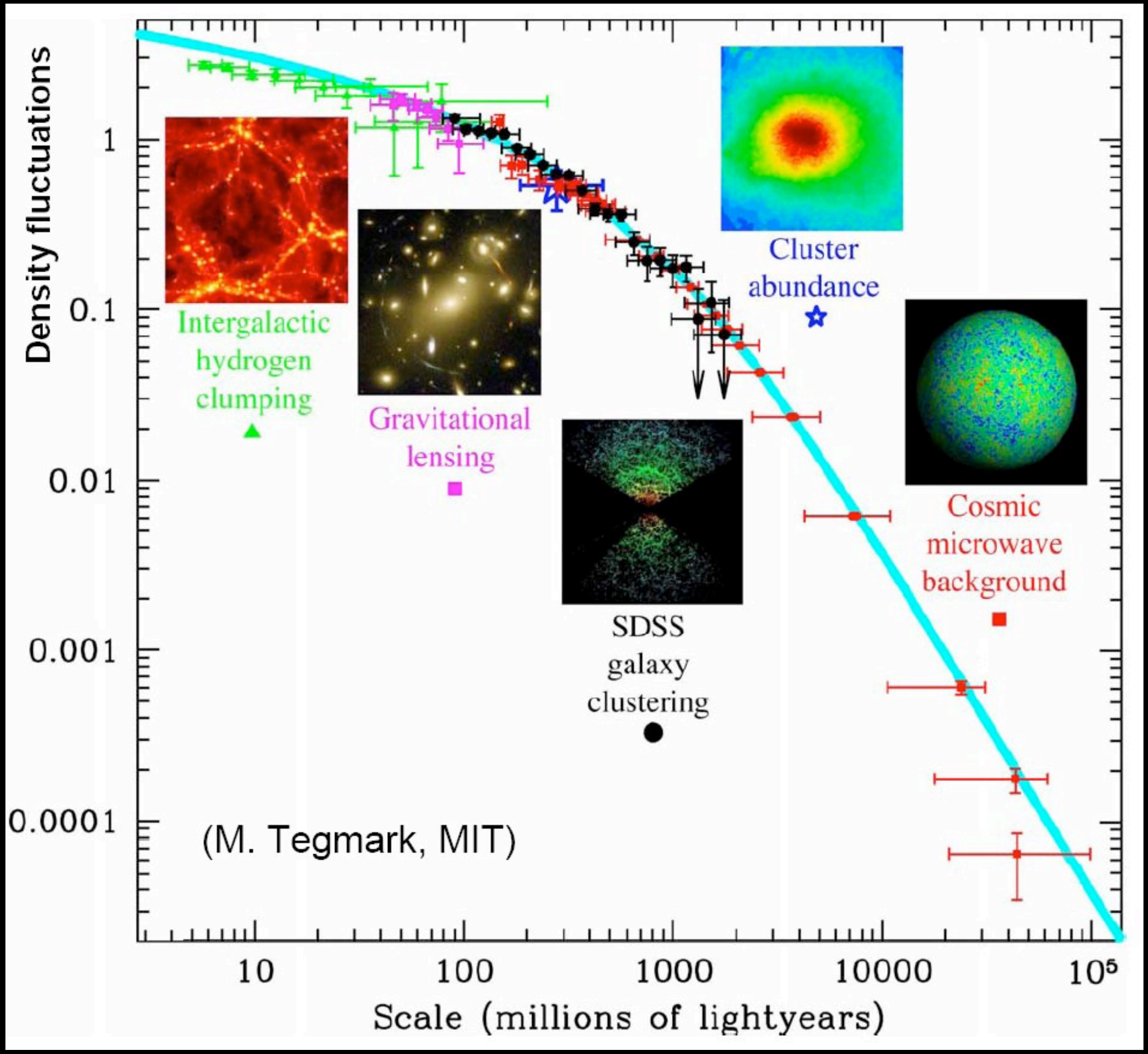
Standard Cosmological Model

Komatsu et al. '10, Larson et al. '10, Bennett et al. '10

CMB (WMAP) + BAO (clusters) + H0 (SNIa)

Gravitational hint of Dark Matter (DM) at all scales





+ Rotational curves of galaxies and clusters

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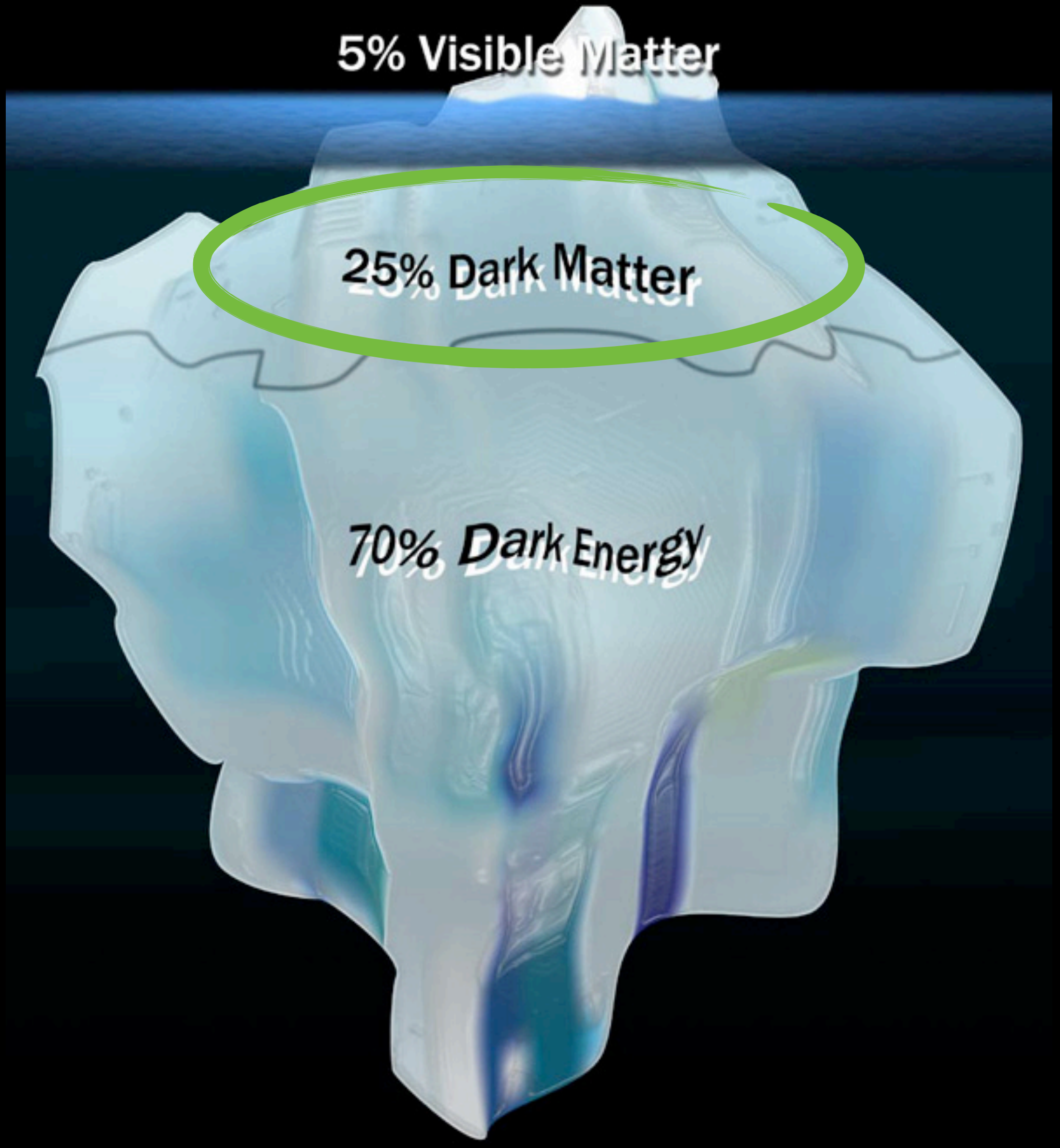
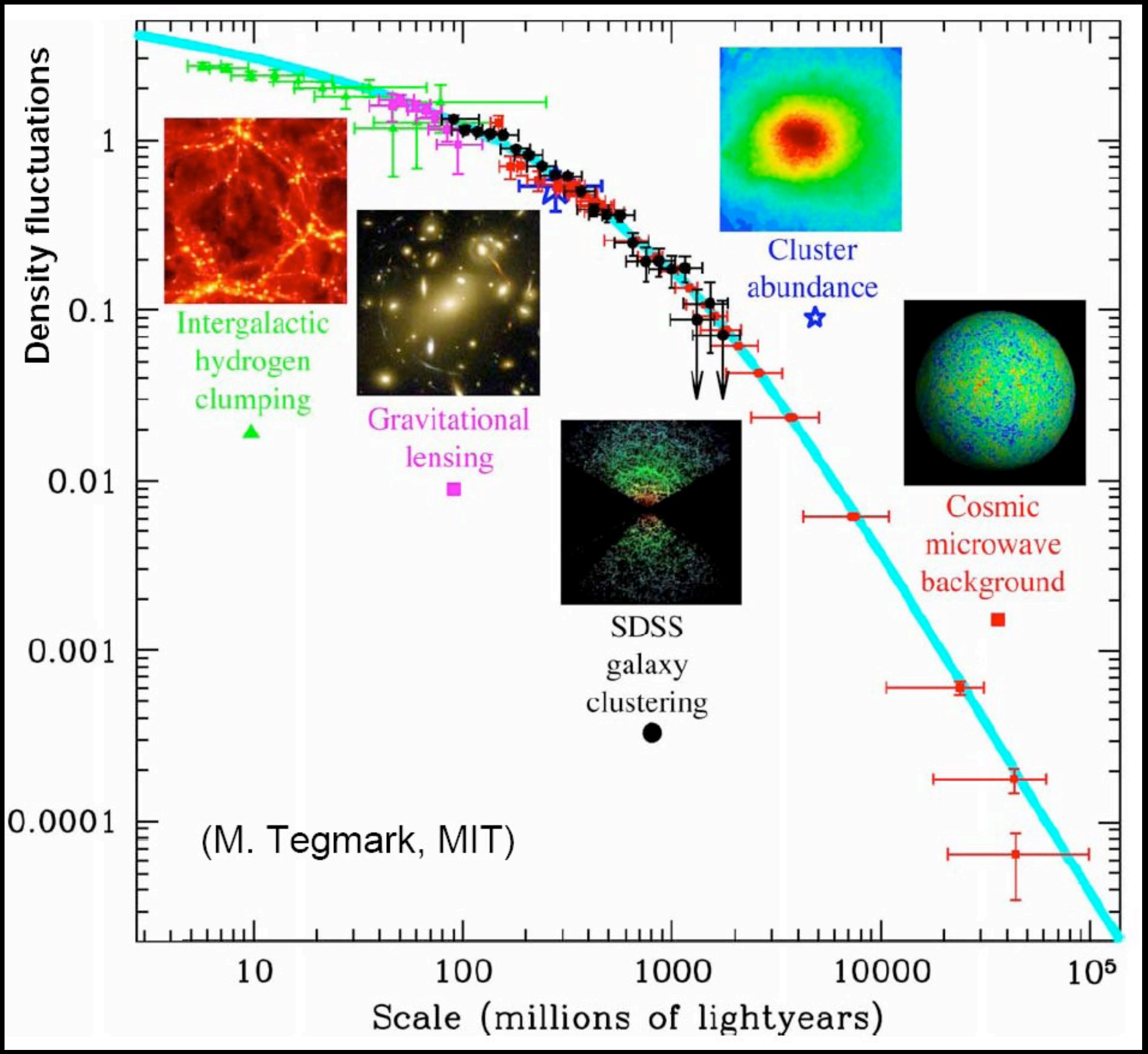
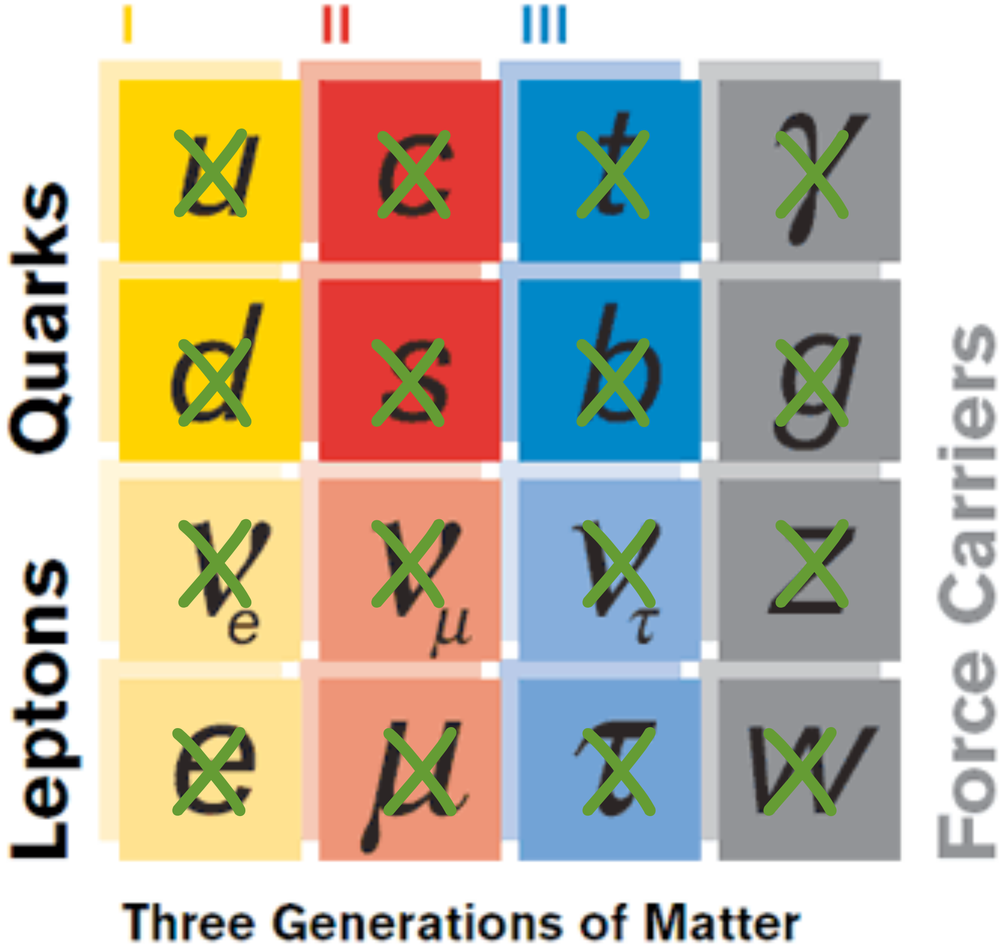




+ Rotational curves of galaxies and clusters

What do we know about Dark Matter?

- Neutral (and massive)
- Stable at least on cosmological scale
- Thermally (or non-thermally) produced: Ω_M = 0.227 +- 0.014
- Cluster to account for large scale structures and form halos



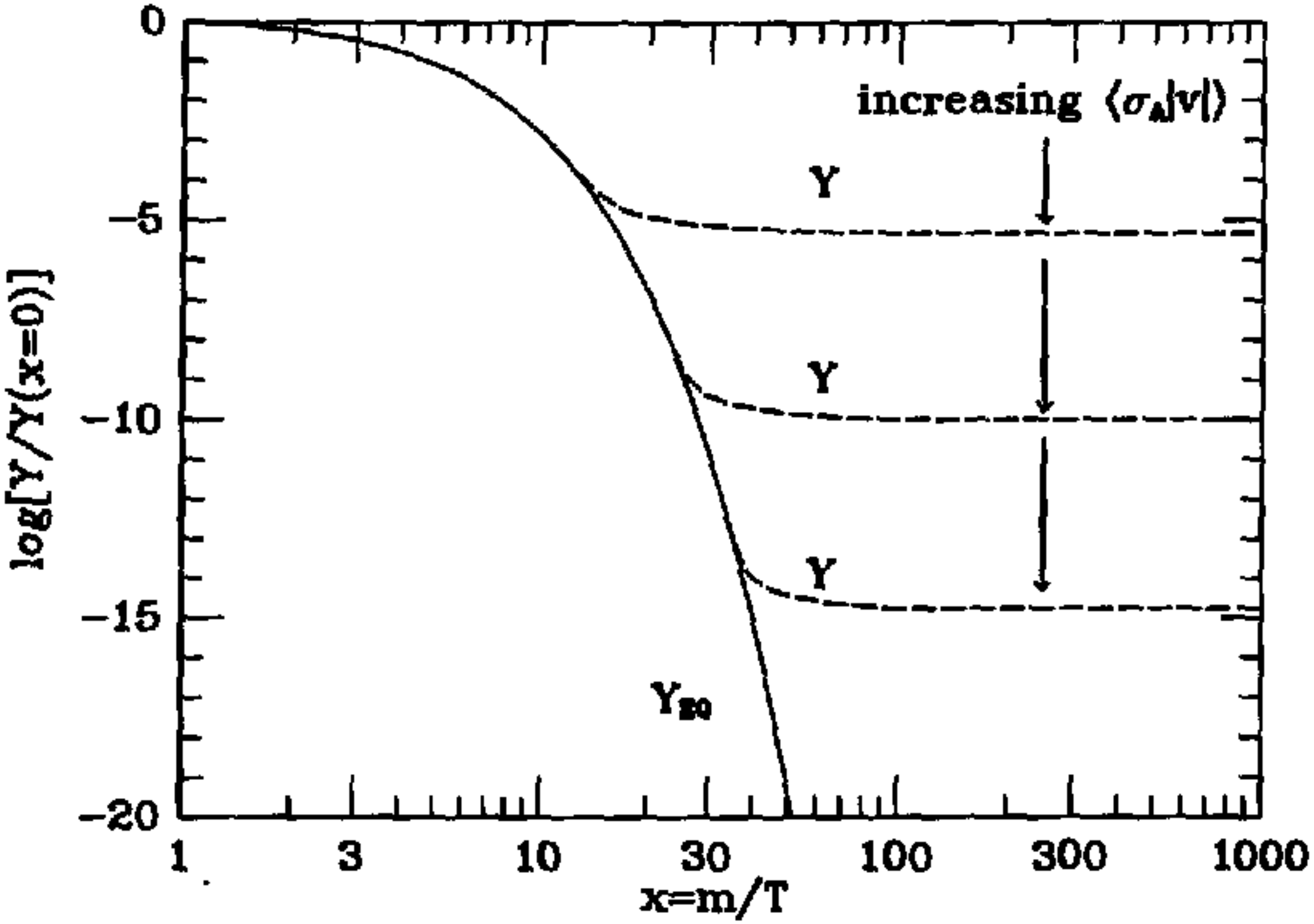
Non baryonic Dark Matter (DM)

New physics beyond the Standard Model (SM)

WIMPs: Weakly Interacting Massive Particles

Lee & Weinberg '77, Gunn et al. '78, Steigman et al. '78, Kolb & Turner '81, Ellis et al. '84, Scherrer & Turner '85, Griest & Seckel '91

$$\chi + \overline{\chi} \leftrightarrow \mathrm{SM} + \overline{\mathrm{SM}}$$



Freeze-out (chemical decoupling):

$$\Gamma = n < \sigma_A v > \sim \mathrm{H}$$

$$\Omega_{\rm DM}\mathrm{h}^2 \sim 0.3\left(\frac{10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}}{< \sigma_A v >}\right)$$

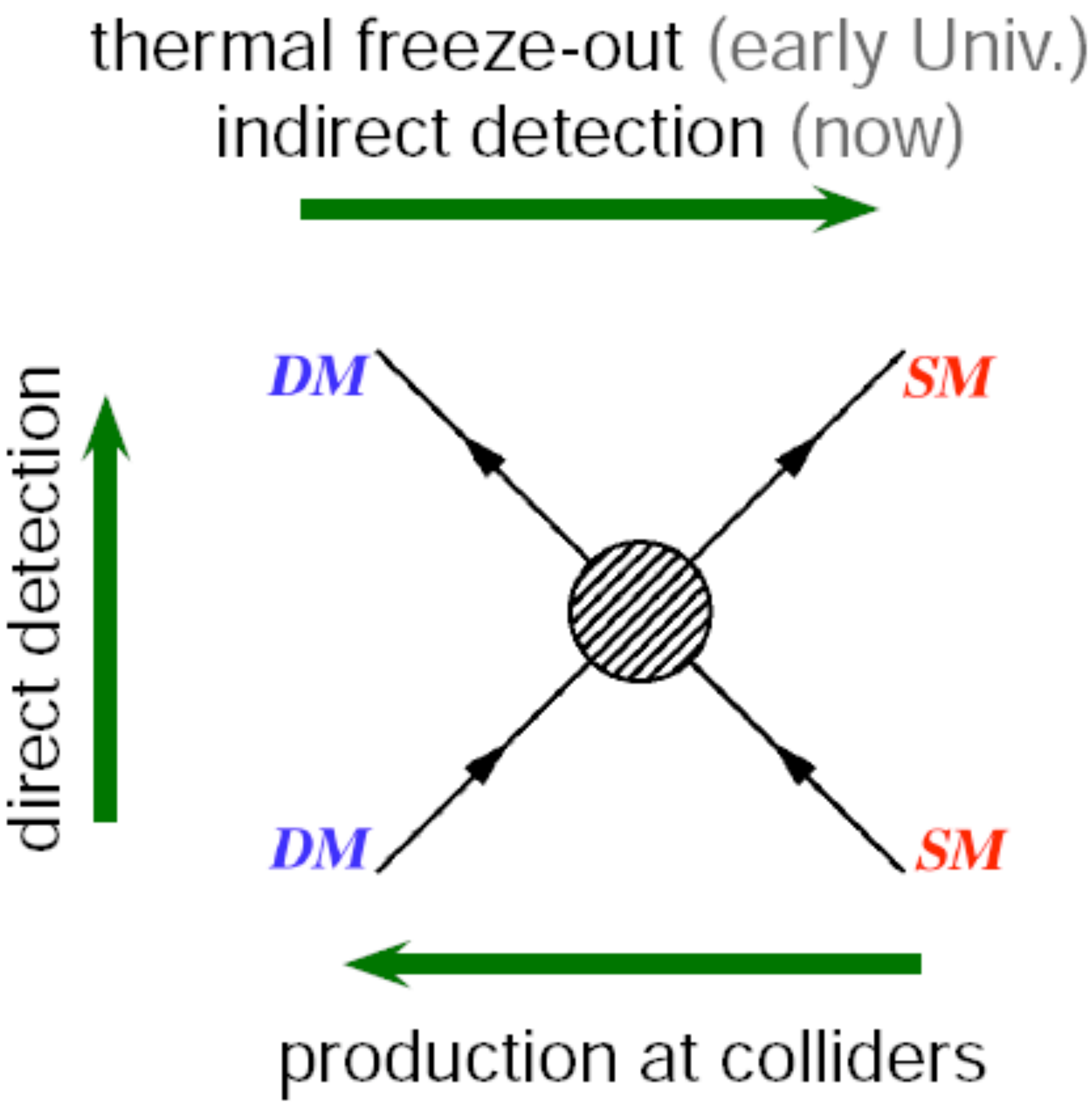
Example:

$$< \sigma_A v > \sim \frac{g^2}{m_\chi^2} \sim \frac{0.01^2}{(100\ \mathrm{GeV})^2} \sim 8 \times 10^{-25}\mathrm{cm}^3\mathrm{s}^{-1}$$

GeV ⟶ TeV scale DM candidates with weak scale interactions

WIMPs arise in SUSY theories, Hidden sectors, Kaluza-Klein models
(other DM candidates are axions, sterile neutrinos, ...)

GeV-TeV DM detection

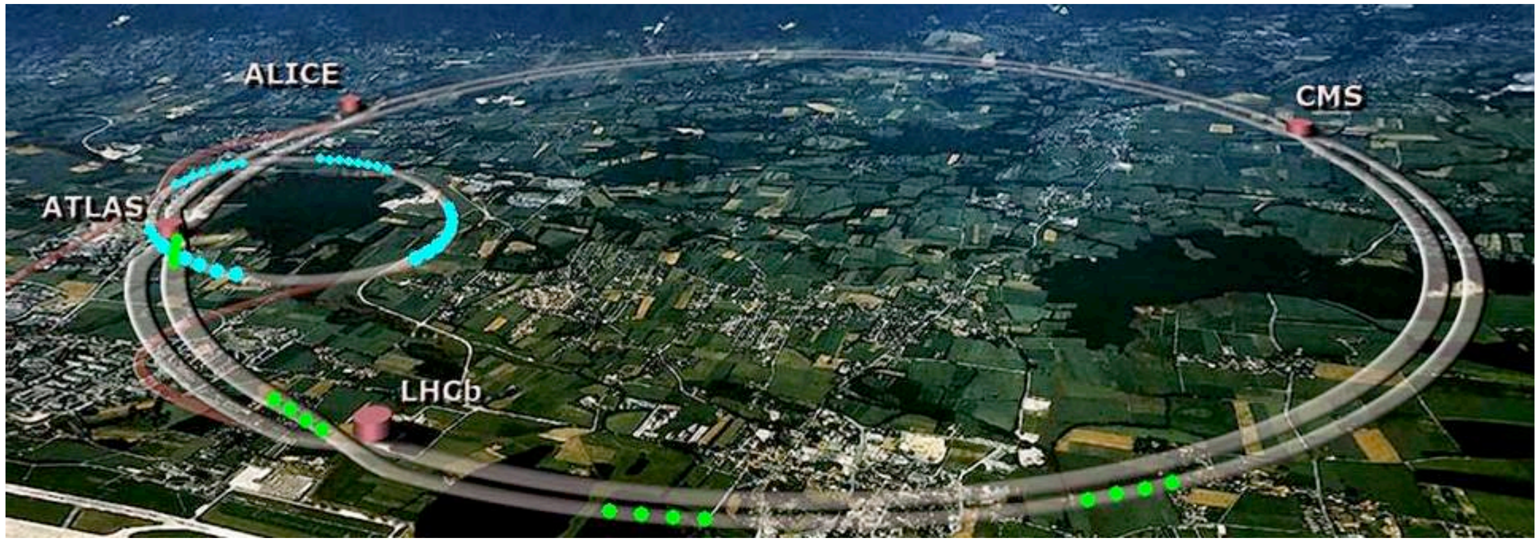


thermal freeze-out (early Univ.)

indirect detection (now)

direct detection

production at colliders



GeV-TeV DM detection

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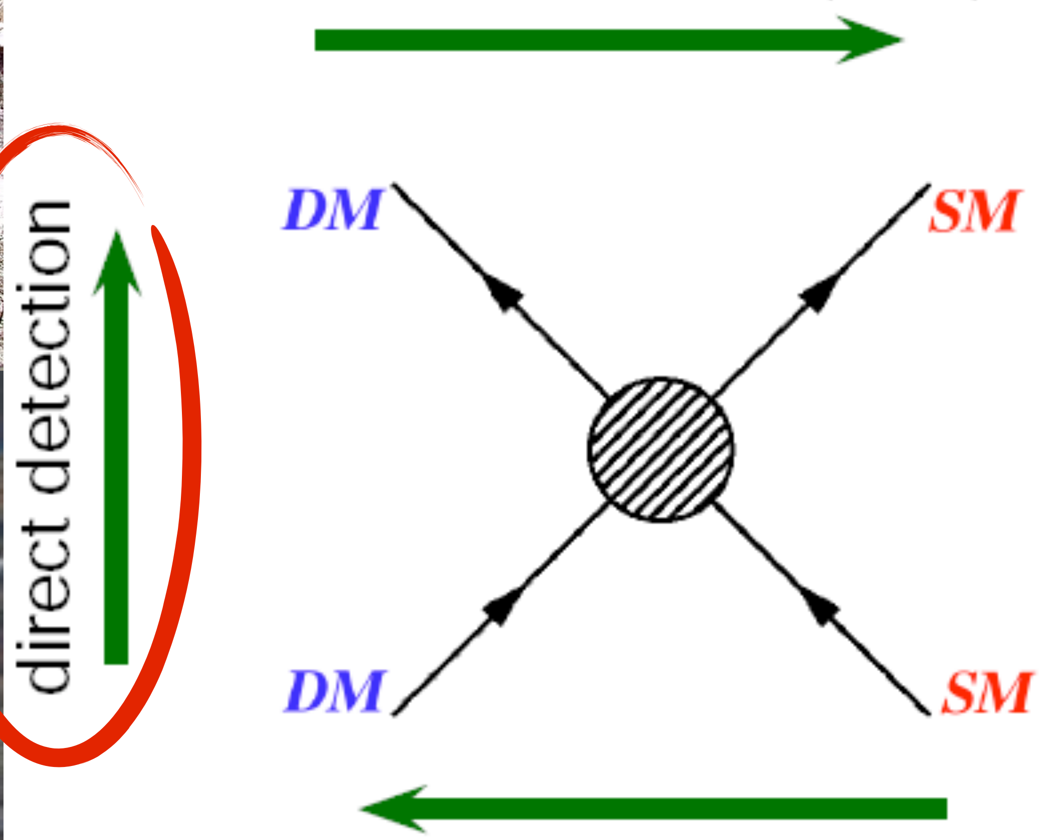

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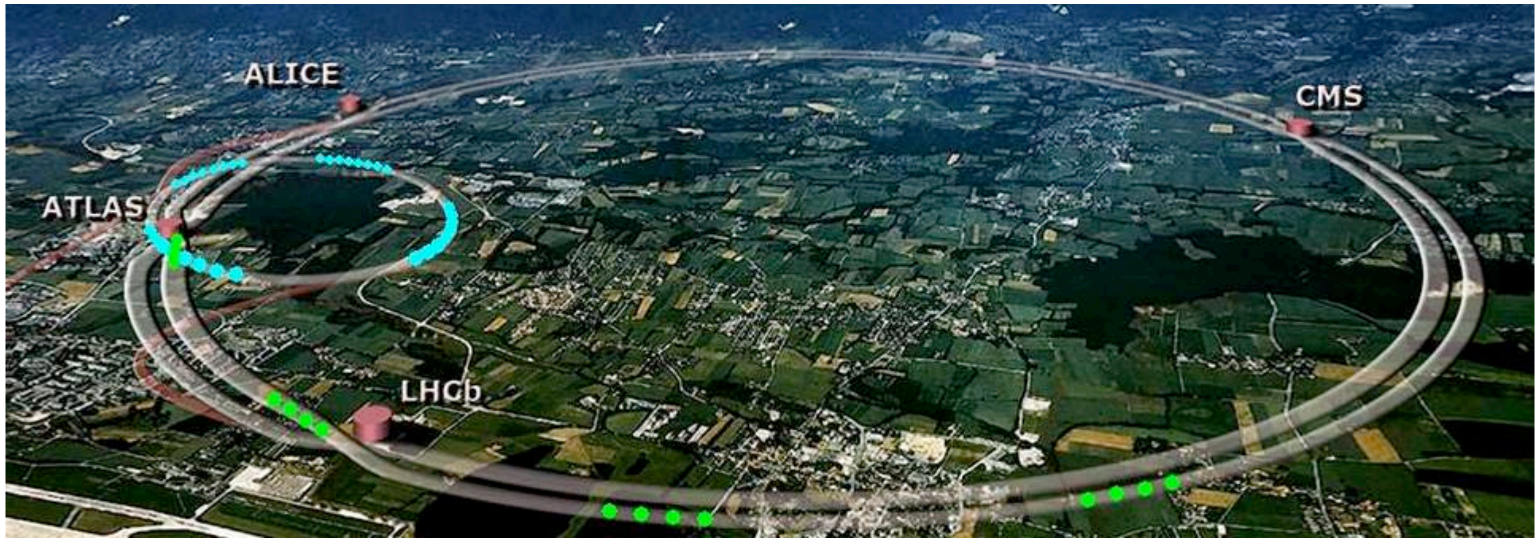
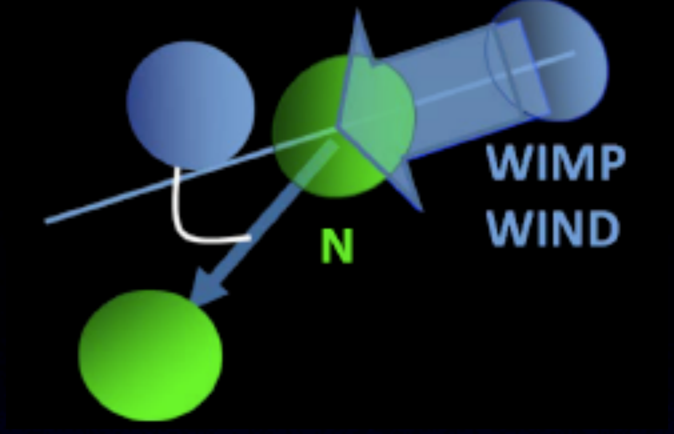


Outline

- Bayesian (brief remind of basic concepts) analysis of direct detection data motivated by
  - (a) tension between experiments
  - (b) experimental systematics
  - (c) astrophysical uncertainties
- Bayesian Evidence
- Results for model comparison
  - ☐ CoGeNT modulation
- Conclusions

WIMP Direct Detection (DD)

Goodman & Witten '85



$$\frac{\mathrm{dR}}{\mathrm{dE}} = \frac{\rho_\odot}{m_{\mathrm{DM}}} \frac{\mathrm{d}\sigma}{\mathrm{dE}} \int_{v' > v'_{\mathrm{min}}} \mathrm{d}^3\mathrm{v}' \, \frac{\mathrm{f}(\mathrm{v}'(\mathrm{t}))}{\mathrm{v}'}$$

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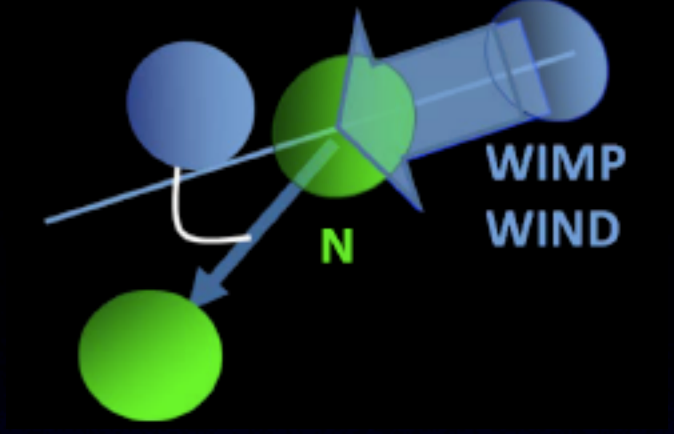

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$$\frac{\mathrm{d}\sigma}{\mathrm{dE}} = \frac{M_N \sigma_n^{\mathrm{SI}}}{2\mu_n^2} \frac{\Big(f_p Z + (A-Z) f_n\Big)^2}{f_n^2} \mathcal{F}^2(E)$$

- For equal coupling to n and p, A^2 dependence: light nuclei more sensitive to light WIMPs and viceversa
- spin-independent interaction (SI)

WIMP Direct Detection (DD)

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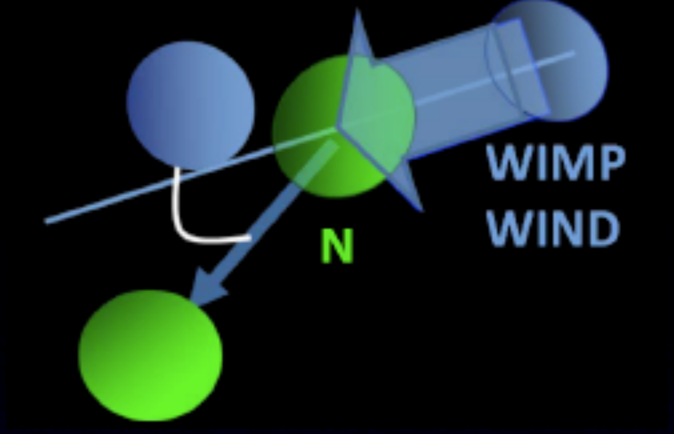

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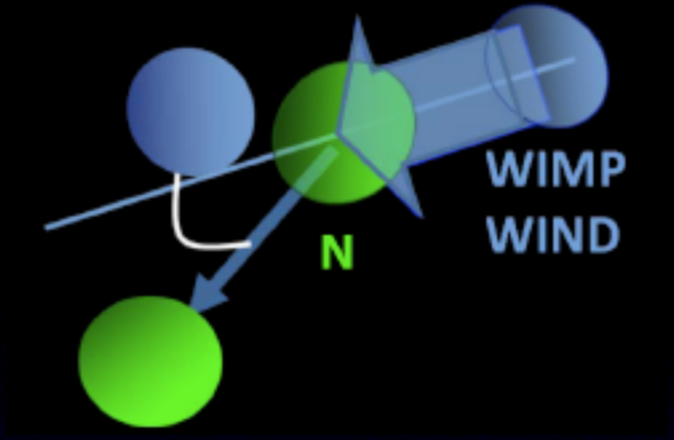

$$\frac{\mathrm{dR}}{\mathrm{dE}} = \frac{\boxed{\rho_\odot}}{m_{\mathrm{DM}}} \boxed{\frac{\mathrm{d}\sigma}{\mathrm{dE}}} \boxed{\int_{v'>v'_{\min}} \mathrm{d}^3\mathrm{v}' \, \frac{\mathrm{f}(\mathrm{v}'(\mathrm{t}))}{\mathrm{v}'}}$$

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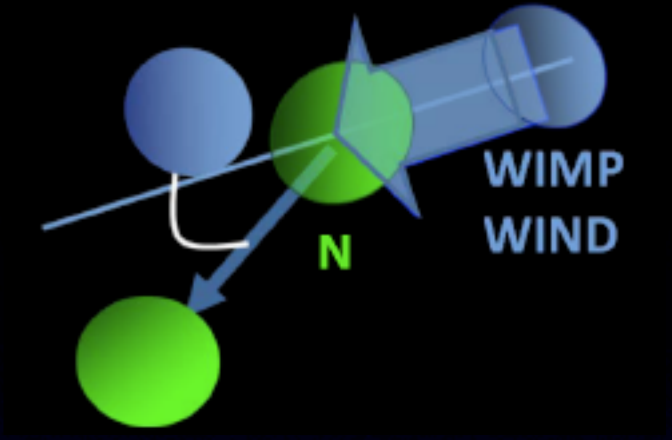
DM velocity distribution + astrophysical parameters at the Sun position

$$v'_{\min} = \sqrt{\frac{M_N E}{2\mu_N}}$$

WIMP Direct Detection (DD)

Goodman & Witten '85



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DM velocity distribution + astrophysical parameters at the Sun position

$$v'_{\mathrm{min}} = \sqrt{\frac{M_N E}{2\mu_N}}$$

Total rate = Integrate over energy times detector mass and exposure time

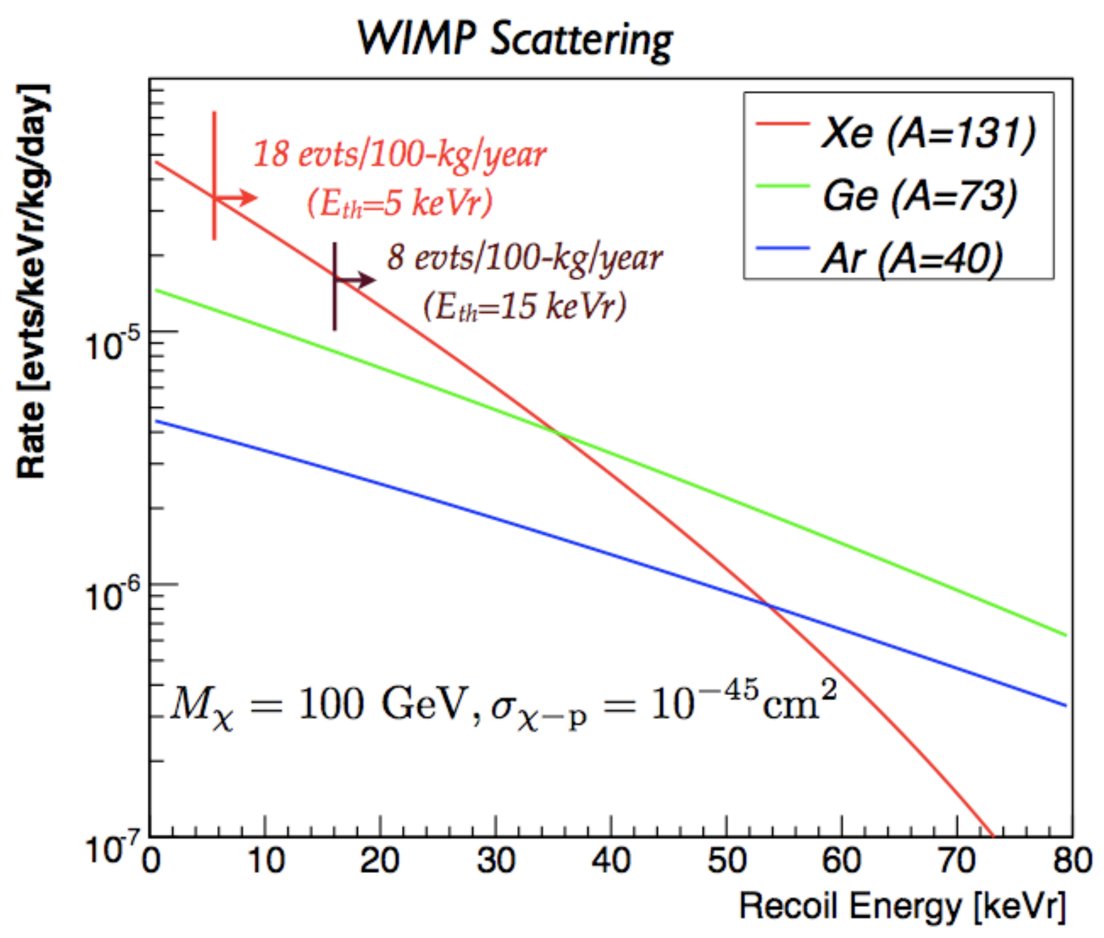
Experimental Issues

- Small recoil energy
  - lowest threshold possible

$$\langle E_R \rangle \sim \mathrm{keV} \left(\frac{m_N}{\mathrm{GeV}} \right) \left(\frac{m_{DM}}{m_{DM} + m_N} \right)^2$$

- Event rate very small
  - large detector mass and long exposure time

- Background discrimination -> SYSTEMATICS !!
  - misidentified electrons (surface events)
  - neutron in the recoil band
  - use of multiple detection techniques (ionization, heat, scintillation)
  - use of signature proper of the a WIMP



Annual Modulation

Signature of WIMP recoil in the detector

Drukier, Freese and Spergel '86,
Freese, Frieman and Gould '88

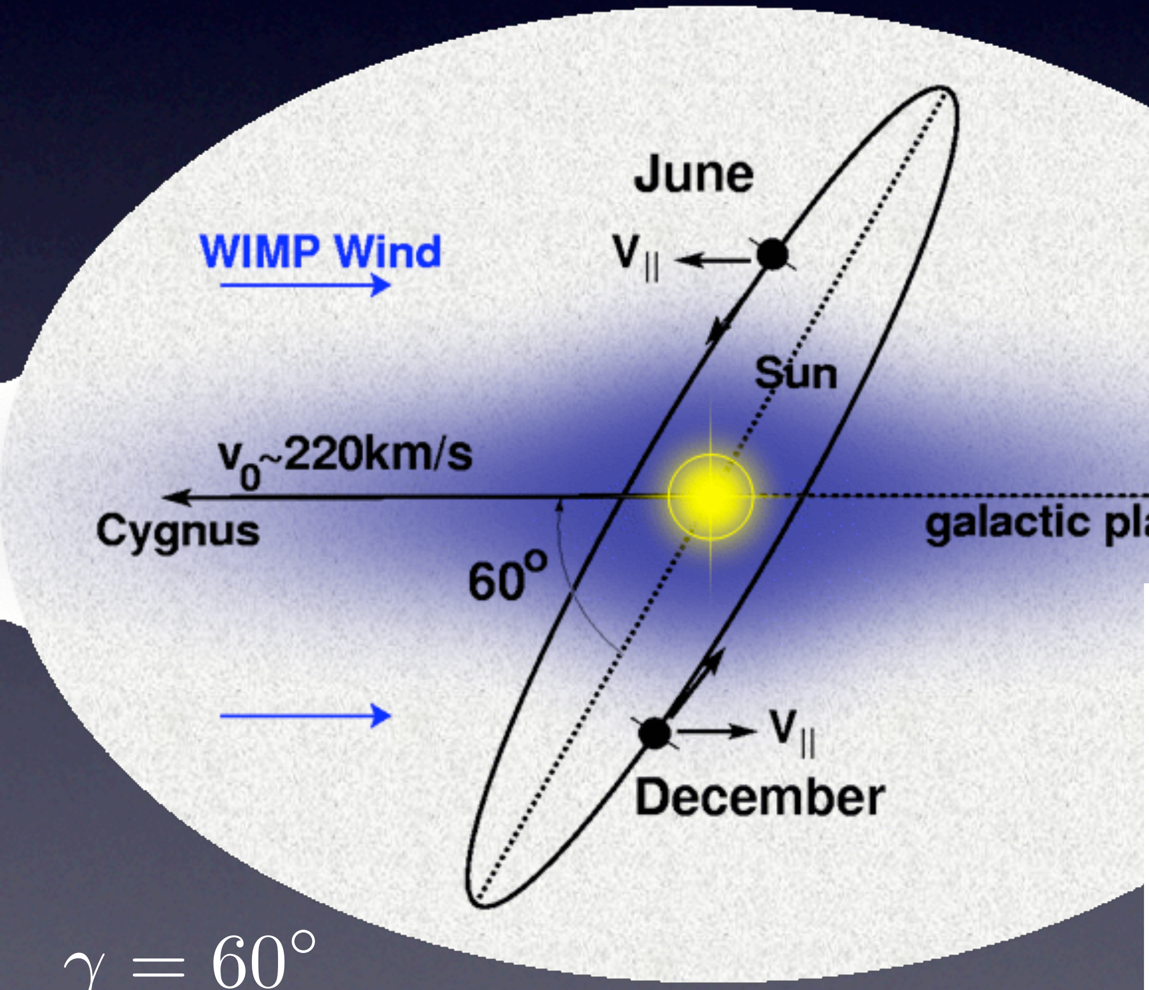
In the Earth's rest frame the DM velocity distribution acquires a time dependence, which follows a sinusoidal behavior

Projecting along the galactic plane:

$$\eta(E,t) = \int_{v'>v_{\min}} \mathrm{d}^3\mathrm{v}' \, \frac{\mathrm{f}(\mathrm{v}'(\mathrm{t}))}{\mathrm{v}'}$$

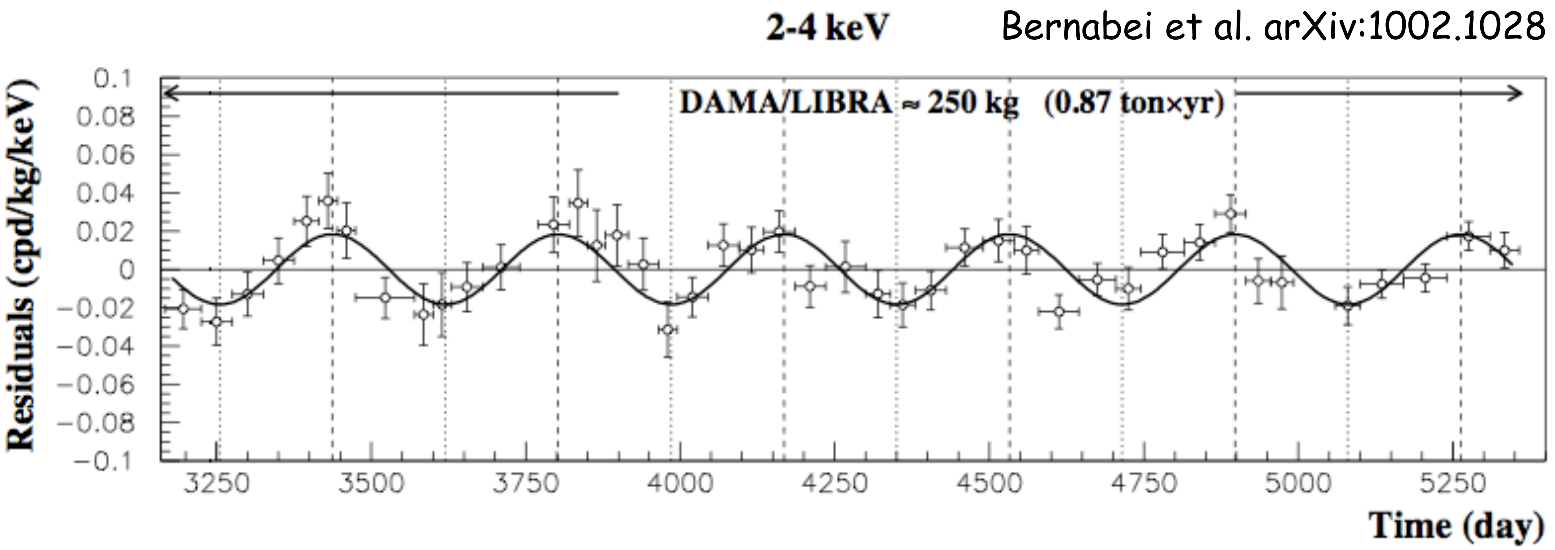
$$v^2 = |\vec{v'} + \vec{v}_\oplus|^2$$

$$v_\oplus = |\vec{v}_\odot + \vec{v''}_{\oplus,\mathrm{rot}}|$$
$$= v_\odot + v''_{\oplus,\mathrm{rot}} \cos\gamma \cos[2\pi(t - t_0)/T]$$



$\gamma = 60^\circ$

effect of O(10%)



Bernabei et al. arXiv:1002.1028

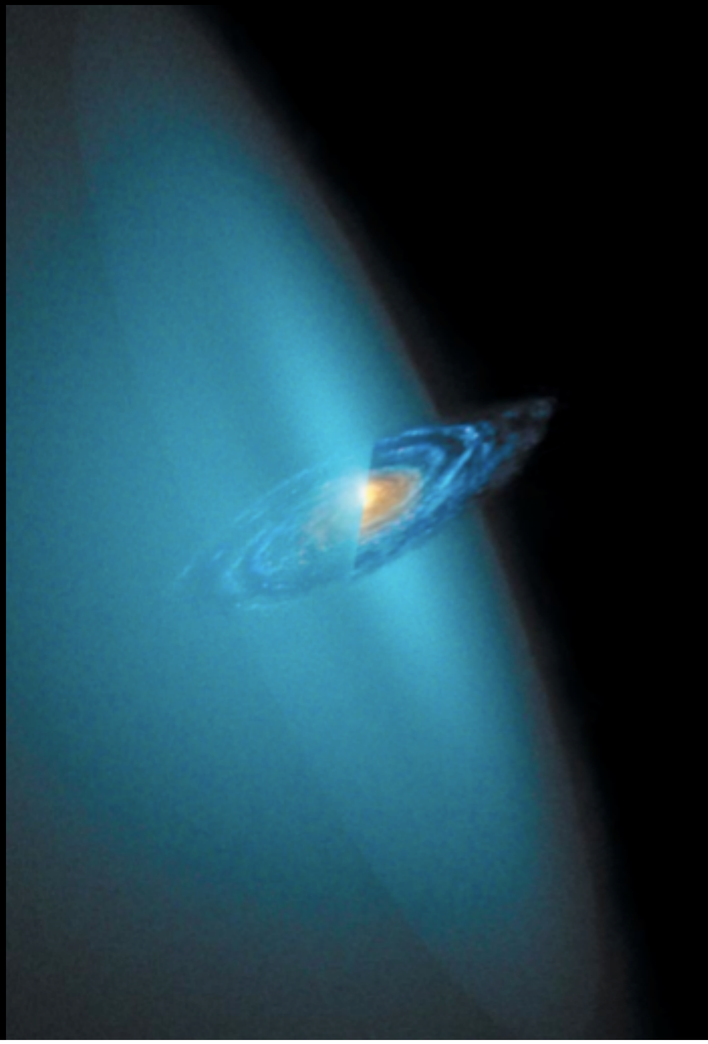
Theoretical Issues

- WIMP-nucleon cross-section can span several order of magnitude: model dependent quantity ⟶ theoretical model parameter together with the WIMP mass

- DM velocity distribution

  ☐ depends on the solar neighborhood quantities and properties

  ☐ approximated with Standard Model Halo (SMH), that is a spherically symmetric and isotropic Maxwellian distribution



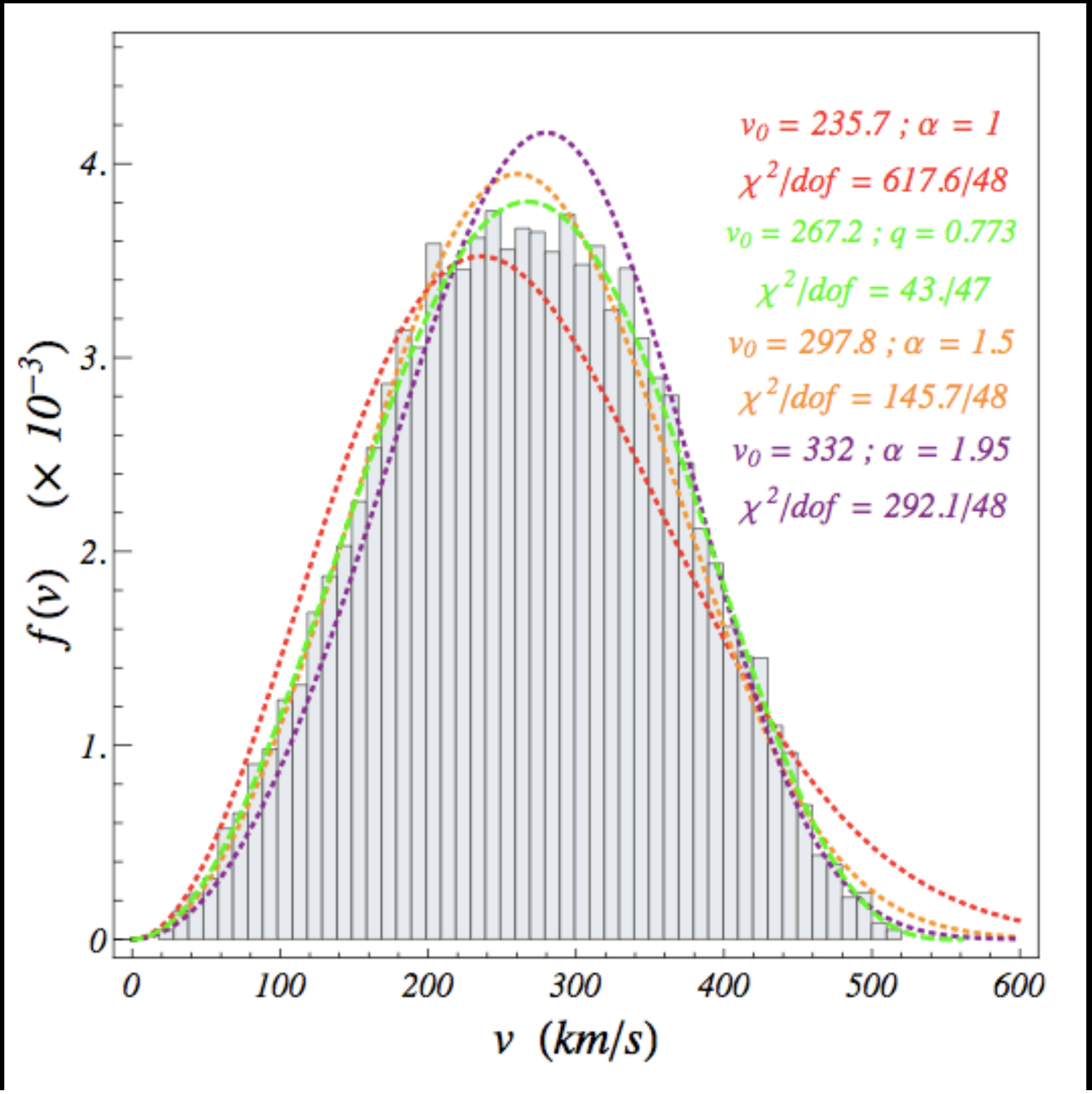
SMH disfavoured by N-body simulations

⟶

Velocity distribution in a shell 7<R<9 kpc

Milky way like galaxy simulated with RAMSES: DM + baryons

Ling et al. '09



Bayesian Inference framework

X data

$$\theta = \{\theta_1, \ldots, \theta_n, \psi_a, \ldots, \psi_z\}$$

θ_i theoretical model parameters

ψ_k nuisance parameters = astrophysics and systematics

$$\mathcal{P}(\theta|X)\mathrm{d}\theta \propto \mathcal{L}(X|\theta) \cdot \pi(\theta)\mathrm{d}\theta$$

Posterior probability function (PDF)

Likelihood (proper of each EXP)

Prior

Bayesian Inference framework

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$\mathcal{P}(\theta|X)\mathrm{d}\theta \downarrow$ Posterior probability function (PDF)

$\mathcal{L}(X|\theta) \downarrow$ Likelihood (proper of each EXP)

$\pi(\theta)\mathrm{d}\theta \downarrow$ Prior

Observable	Prior
WIMP mass (θ_1)	$\log(m_{\rm DM}/{\rm GeV}):\ 0 \to 3$
SI cross-section (θ_2)	$\log(\sigma_n^{\rm SI}/{\rm cm}^2):\ -44(-46) \to -38$

$$\pi_{\log}(\log\theta)\ \mathrm{d}\log\theta = \begin{cases} \mathrm{d}\log\theta, & \text{if } \theta_{\rm min} \leq \theta \leq \theta_{\rm max}, \\ 0, & \text{otherwise,} \end{cases}$$

Common prior choices that do not favour any parameter region

$$\pi_{\rm flat}(\theta)\mathrm{d}\theta \propto \begin{cases} \mathrm{d}\theta, & \text{if } \theta_{\rm min} \leq \theta \leq \theta_{\rm max}, \\ 0, & \text{otherwise,} \end{cases}$$

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Posterior sampled via MCMC techniques (Markov-Chain Monte Carlo) given the likelihood and the prior and marginalized over nuisance parameters

$$\mathcal{P}_{\mathrm{mar}}(\theta_1, ..., \theta_n|X) \propto \int \mathrm{d}\psi_1 ... \mathrm{d}\psi_m\ \mathcal{P}(\theta_1, ..., \theta_n, \psi_1 ..., \psi_m|X)$$

Bayesian Inference framework

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Profile Likelihood -> comparison with frequentist approach, prior independent

$$\mathcal{L}_{\mathrm{prof}}(X|\theta_1, ..., \theta_n) \propto \max_{\psi_1 ... \psi_m} \mathcal{L}(X|\theta_1, ..., \theta_n, \psi_1 ..., \psi_m) \qquad \Delta\chi^2_{\mathrm{eff}}(m_{\mathrm{DM}}, \sigma_n^{\mathrm{SI}}) \equiv -2\ln \mathcal{L}_{\mathrm{prof}}(m_{\mathrm{DM}}, \sigma_n^{\mathrm{SI}})$$

Construction of DM velocity distribution

$$\int_{v'>v'_{\rm min}} {\rm d}^3v' \, \frac{f(\vec{v}'(t))}{v'} \longrightarrow \begin{array}{l} f(\vec{v}'(t)) \equiv F(\vec{v}, \vec{R}_\odot)/\rho_\odot \\ \rho_\odot \equiv \rho_{\rm DM}(R_\odot) \end{array}$$

DD depends on the distribution function (DF) at the sun position arising from the WIMPs phase-space distribution $F(\vec{r}, \vec{v}) \; d^3r \; d^3v$.

$$\rho_{\rm DM}(\vec{r}) = \int {\rm d}^3v \; F(\vec{v}, \vec{r})$$

- DF obtained inverting the equation above
- Symmetries assumed: density profile spherically symmetric and f(v) isotropic -> DF only function of the energy

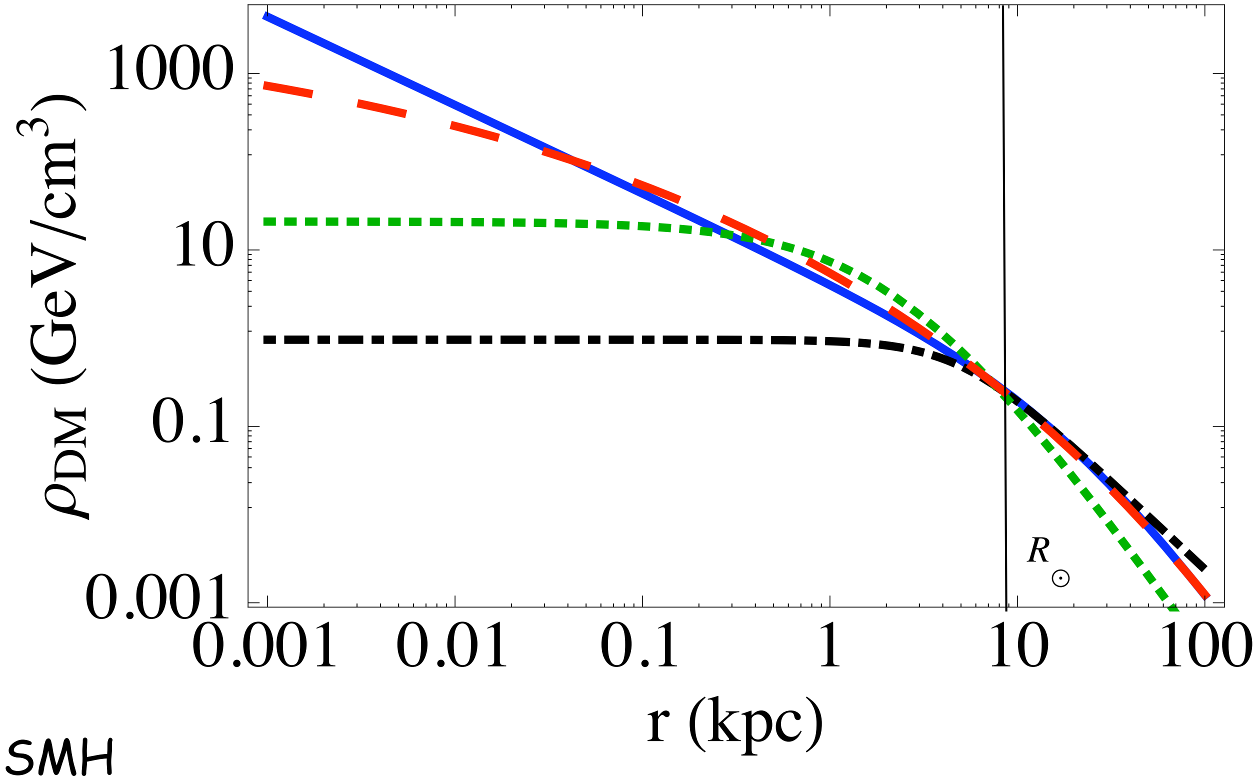
$$F(\varepsilon) = \frac{1}{\sqrt{8}\pi^2} \left[\int_0^\varepsilon \frac{{\rm d}^2\rho_{\rm DM}}{{\rm d}\Psi^2} \frac{{\rm d}\Psi}{\sqrt{\varepsilon - \Psi}} + \frac{1}{\sqrt{\varepsilon}} \left(\frac{{\rm d}\rho_{\rm DM}}{{\rm d}\Psi} \right) \Big|_{\Psi=0} \right]$$

- f(v) is a function of the gravitational potential (including baryon contribution)
- f(v) is a function of the DM density profile

Construction of DM velocity distribution

Spherically symmetric DM density profiles $\rho_{\rm DM} = \rho_{\rm DM}(c_{\rm vir}, M_{\rm vir})$:

- NFW
- Einasto
- Cored Isothermal
- Burkert



They mostly differ near the galactic center, at the sun position they give similar behavior for f(v)
In what follow only shown comparison between NFW and SMH

Likelihood for astrophysical observables (nuisance parameters for ALL EXP)

$$\ln \mathcal{L}_{\rm Astro} = -\frac{(v_0 - \bar{v}_0^{\rm obs})^2}{2\sigma^2_{v_0}} - \frac{(v_{\rm esc} - \bar{v}_{\rm esc}^{\rm obs})^2}{2\sigma^2_{v_{\rm esc}}} - \frac{(\rho_\odot - \bar{\rho}_\odot^{\rm obs})^2}{2\sigma^2_{\rho_\odot}} - \frac{(M_{\rm vir} - \bar{M}_{\rm vir}^{\rm obs})^2}{2\sigma^2_{M_{\rm vir}}}$$

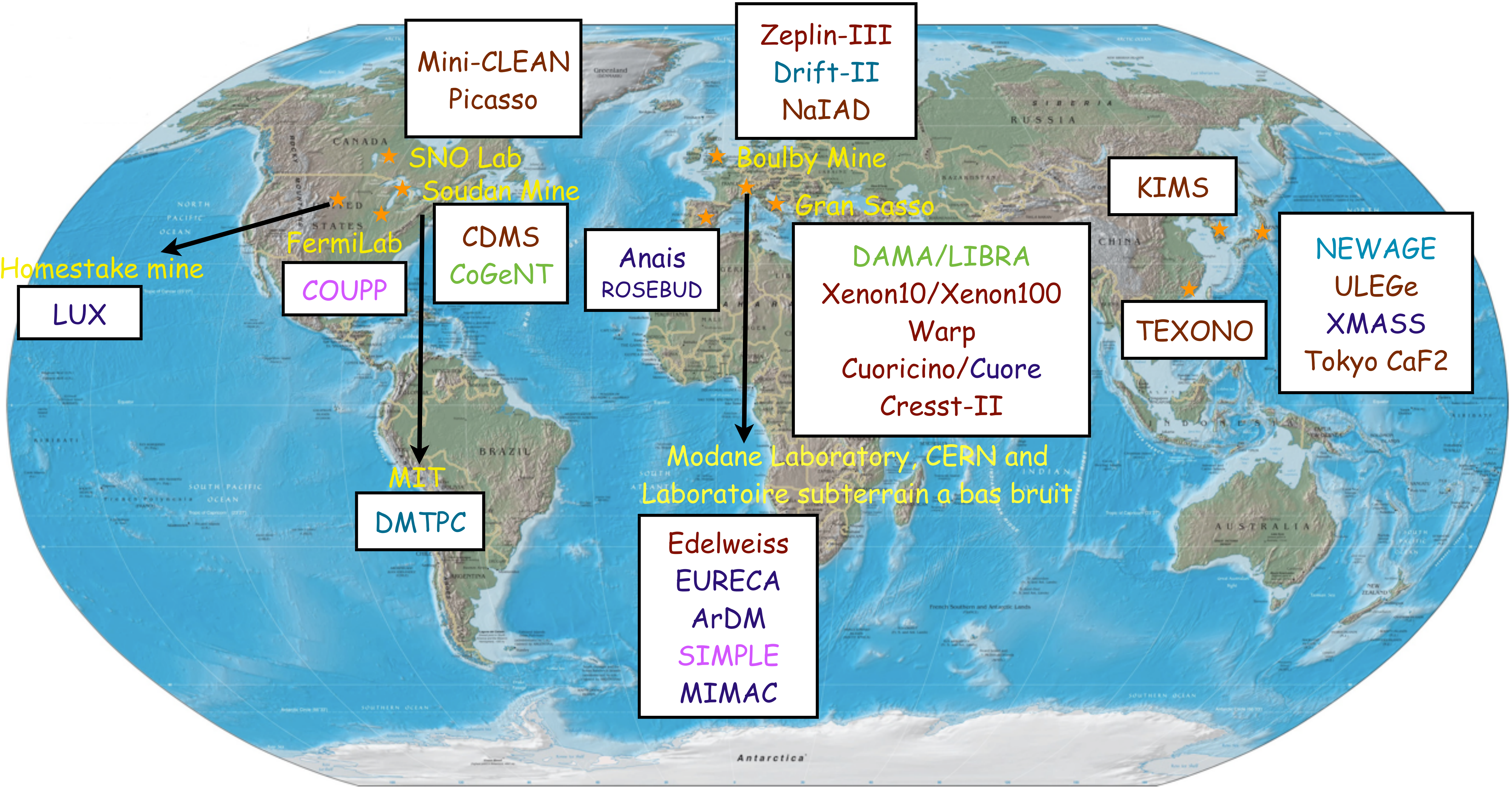
Observable/Parameter	Constraint/Prior
Local standard of rest	$v_0^{\rm obs} = 230 \pm 24.4$ km s^{-1}
Escape velocity	$v_{\rm esc}^{\rm obs} = 544 \pm 39$ km s^{-1}
Local DM density	$\rho_\odot^{\rm obs} = 0.4 \pm 0.2$ GeV cm^{-3}
Virial mass	$M_{\rm vir}^{\rm obs} = 2.7 \pm 0.3 \times 10^{12} M_\odot$
Concentration parameter (NFW, Einasto)	$c_{\rm vir}$: $5 \rightarrow 20$
Concentration parameter (ISO, Burkert)	$c_{\rm vir}$: $50 \rightarrow 200$

$$v_{\rm esc} = \sqrt{2\Psi}\Big|_{r=R_\odot}$$

$$v_0 \equiv \sqrt{-r\frac{{\rm d}\Psi}{{\rm d}r}}\Bigg|_{r=R_\odot}$$

$$\rho_\odot \equiv \rho_{\rm DM}(R_\odot)$$

Direct Detection Experiment Map

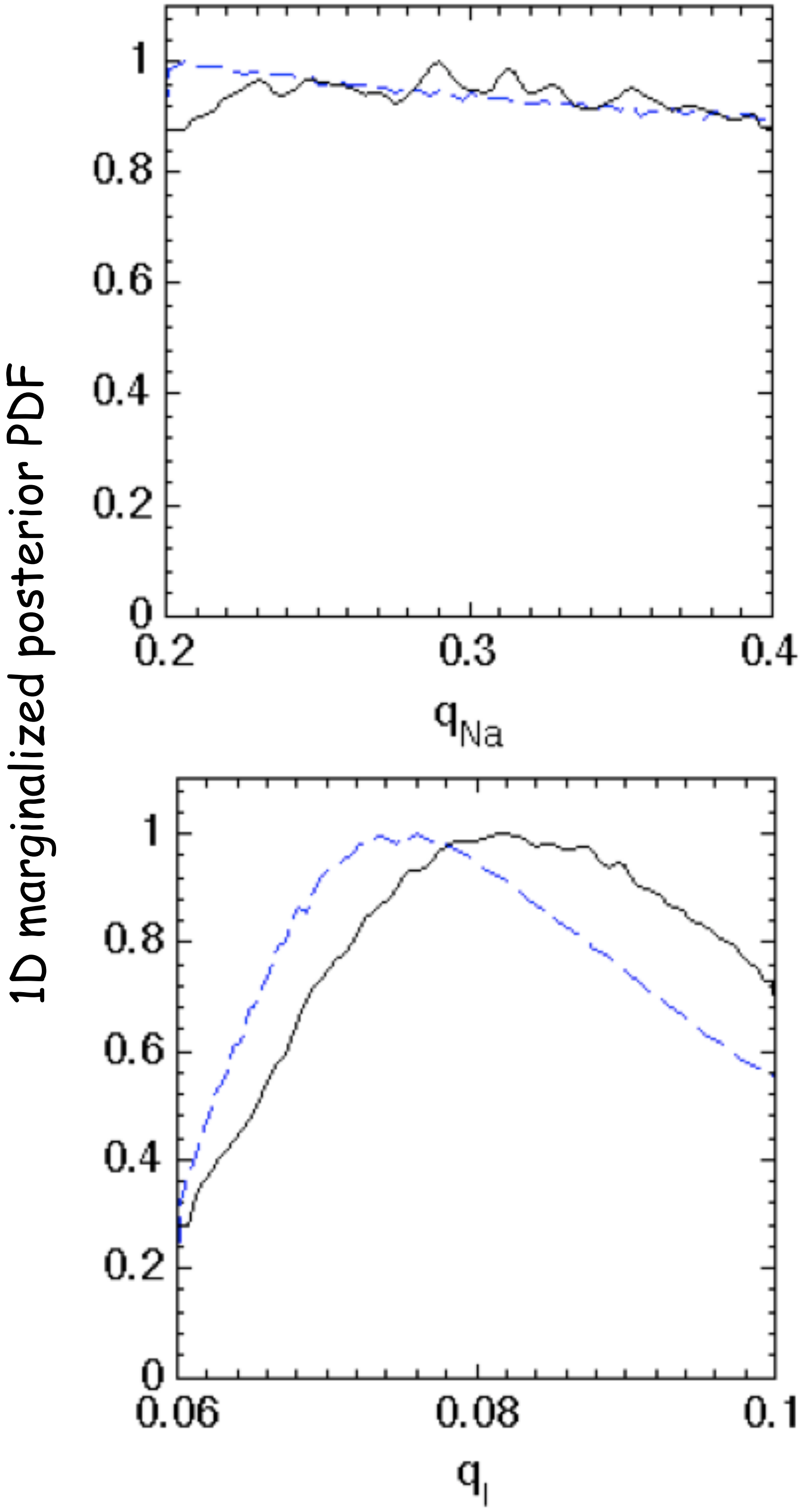


- background rejection technique
- directional signature
- annual modulation signature
- bubble chamber
- planned or under construction (prototypes)

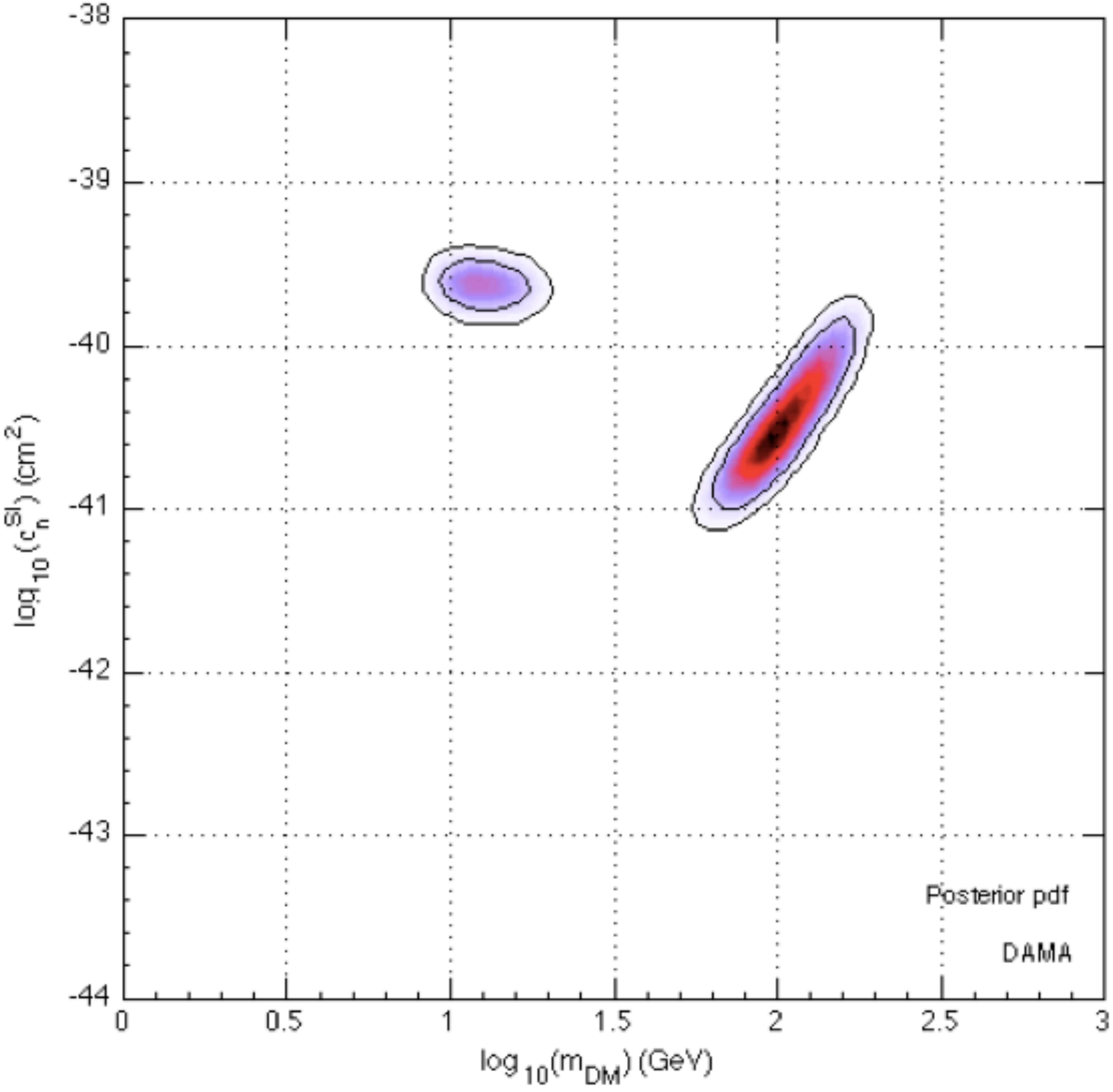
Inference: results for DAMA/LIBRA and SMH

Data given by modulated rate as a function of the energy (13 annual cycles, 1.17 ton x yr) : gaussian likelihood

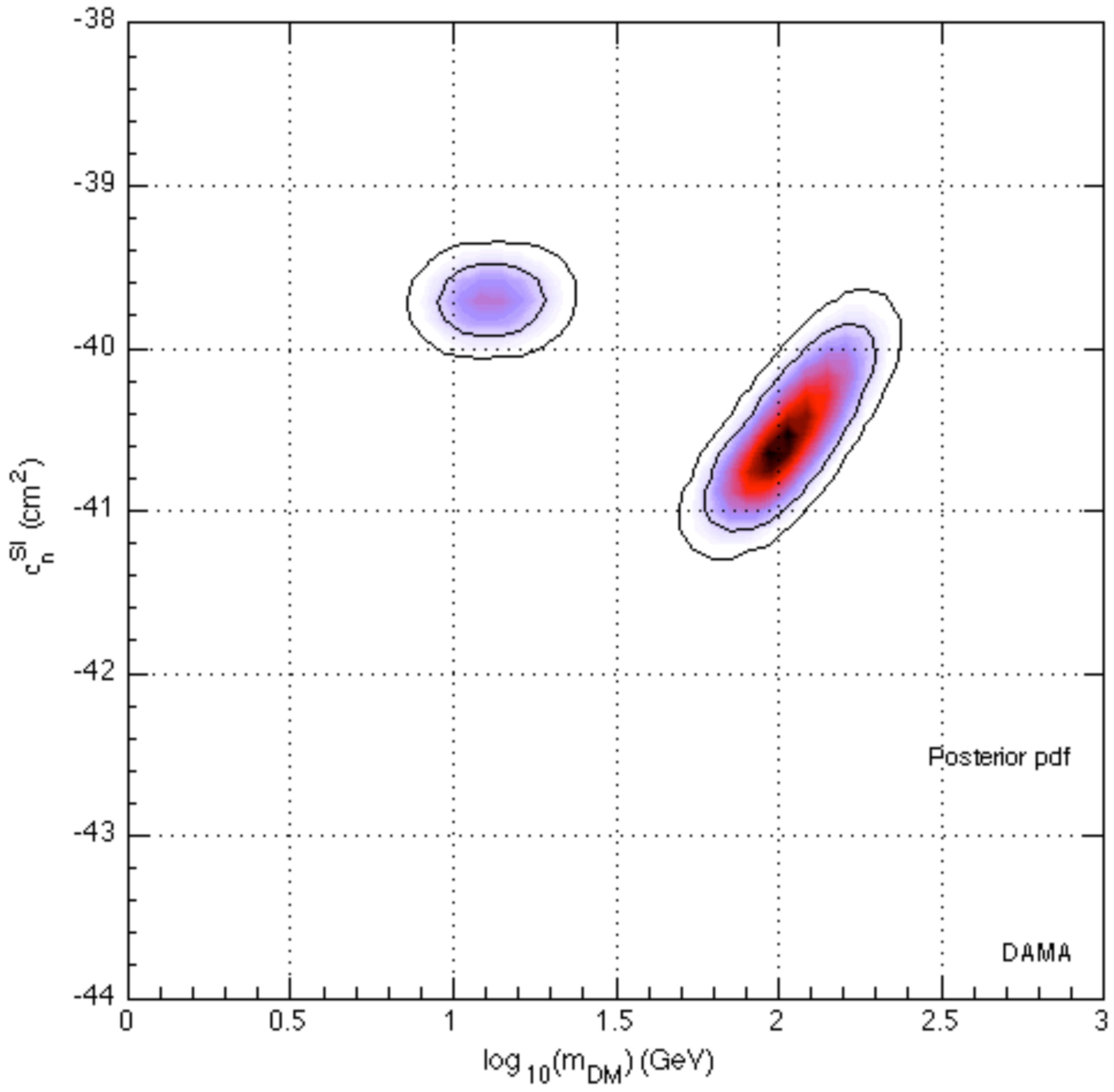
Scintillator made by Na and I: quenching factors are nuisance parameters $\mathcal{E} = qE$

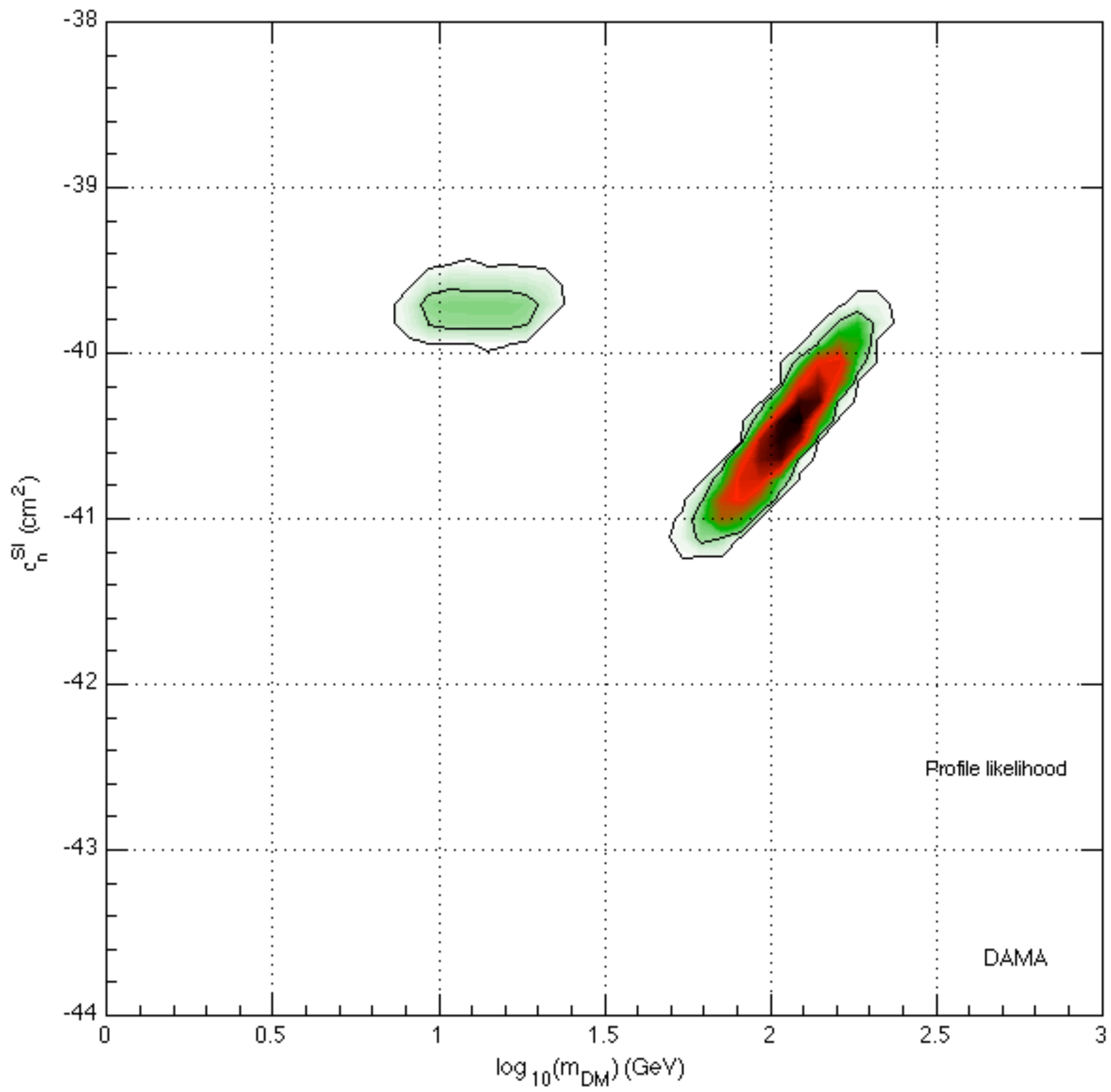


2D marginal credible regions at 90 and 99%



Varying astrophysics results for DAMA/LIBRA inference, NFW DM profile





- 2D posterior pdf matches with profile likelihood for constraining data
- 1D marginalized posterior PDF for the quenching factors as in the SMH case
- 2D regions at 90, 99% are larger than SMH case because of volume effects due to the integration over all possible velocities and density values of the halo at the Sun position
- very similar behavior for Einasto, Burkert and cored isothermal profile

Preferred values for astrophysics:

$$v_0^{\rm obs} = 220^{+40}_{-20}\ {\rm km\ s^{-1}}$$
$$v_{\rm esc}^{\rm obs} = 558^{+19}_{-16}\ {\rm km\ s^{-1}}$$
$$\rho_\odot^{\rm obs} = 0.38^{-0.09}_{+0.15}\ {\rm GeV\ cm^{-3}}$$

CoGeNT 2011

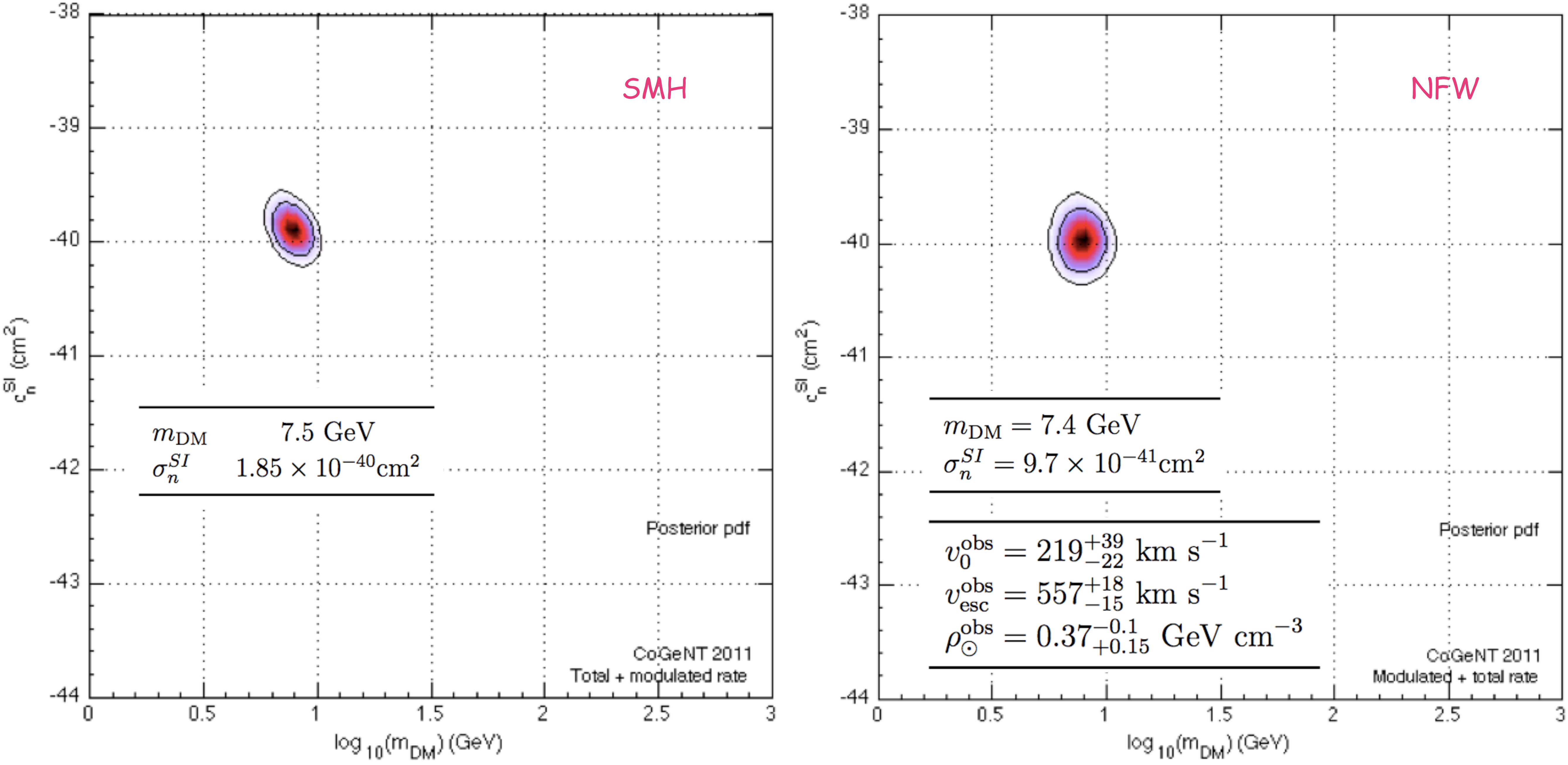
(Aalseth et al. arXiv:1106.0650
data courtesy of CoGeNT coll.)

Ge detector, 146 kg days
Very low threshold:
0.4 keVee = 2.7 keV

Gaussian likelihood $\ln \mathcal{L}_{\mathrm{CoGeNT}} = \ln \mathcal{L}_{\mathrm{TR}} + \ln \mathcal{L}_{\mathrm{MR}}$

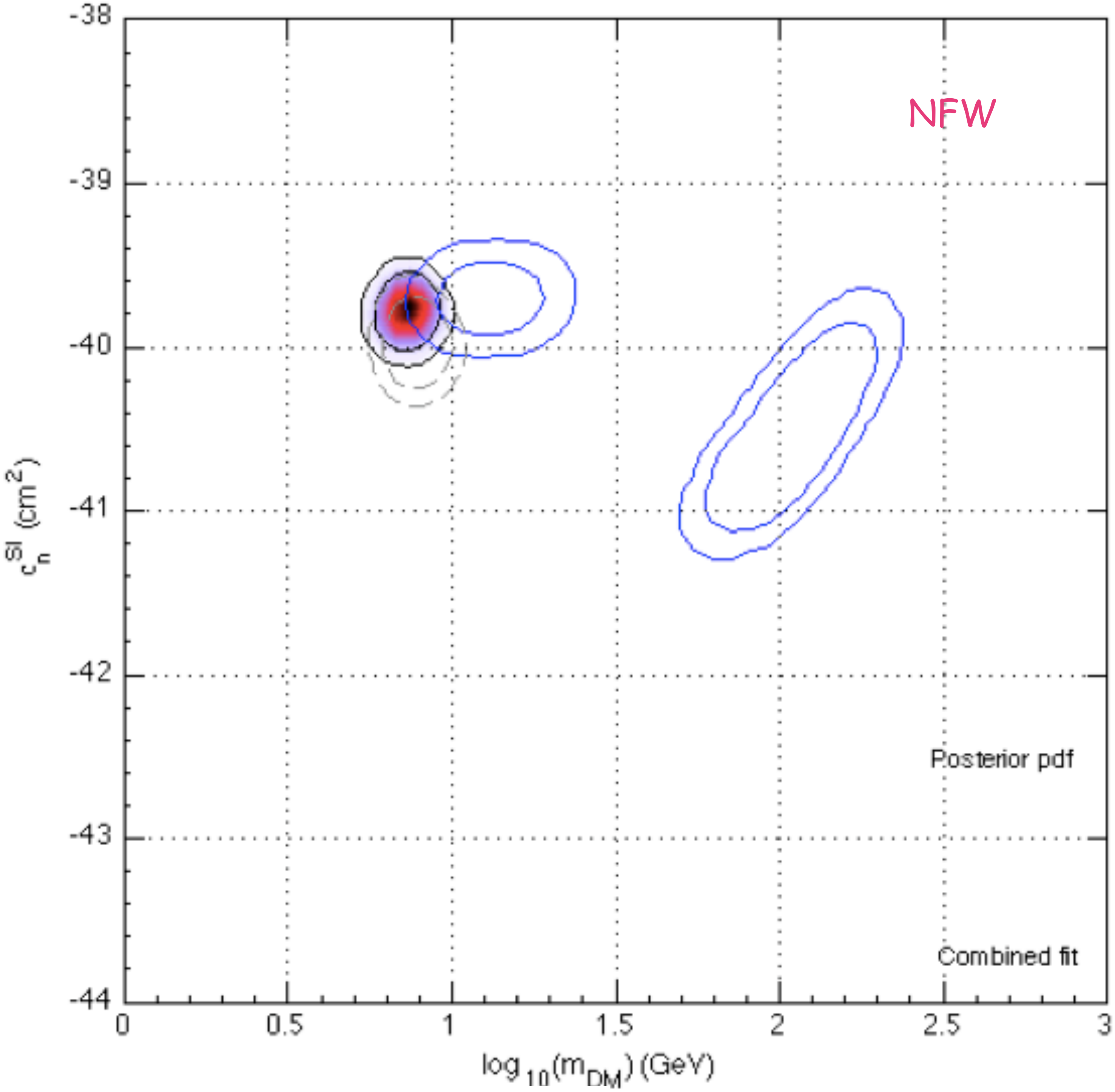
- Background
  1. does not modulate, included only for the total rate
  2. constant + exponential background (mimic surface events)
  3. 3 nuisance parameters
- Radioactive peaks subtracted

2D marginal credible regions at 90 and 99%

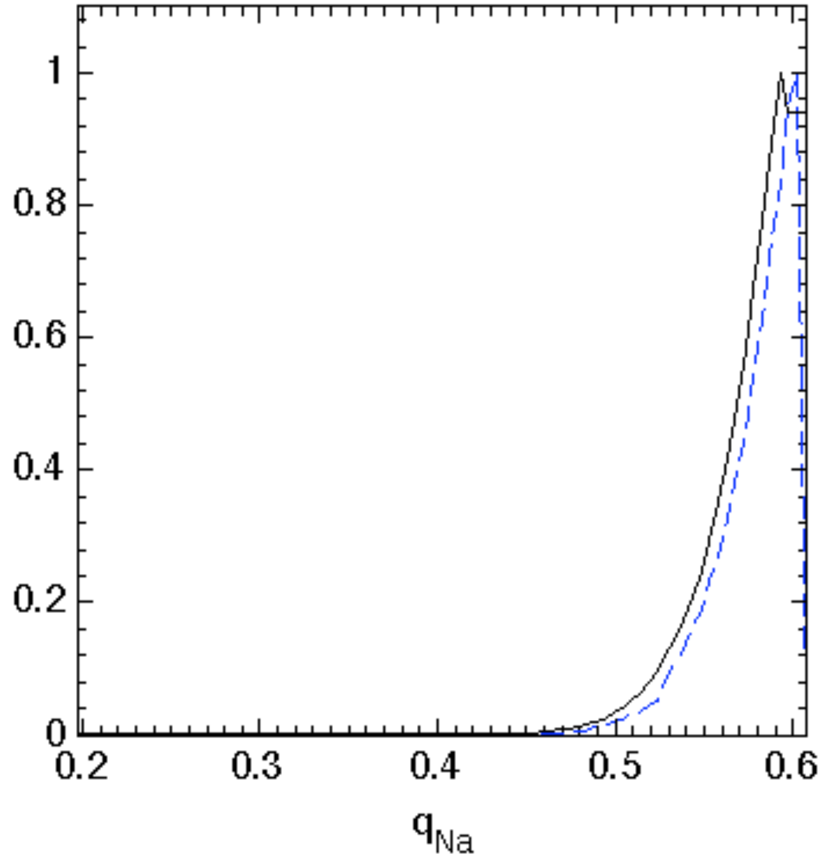


DAMA and CoGeNT, combined fit

2D marginal credible regions at 90 and 99%



- quenching factor prefers now the value 0.57 (same behavior also for SMH)

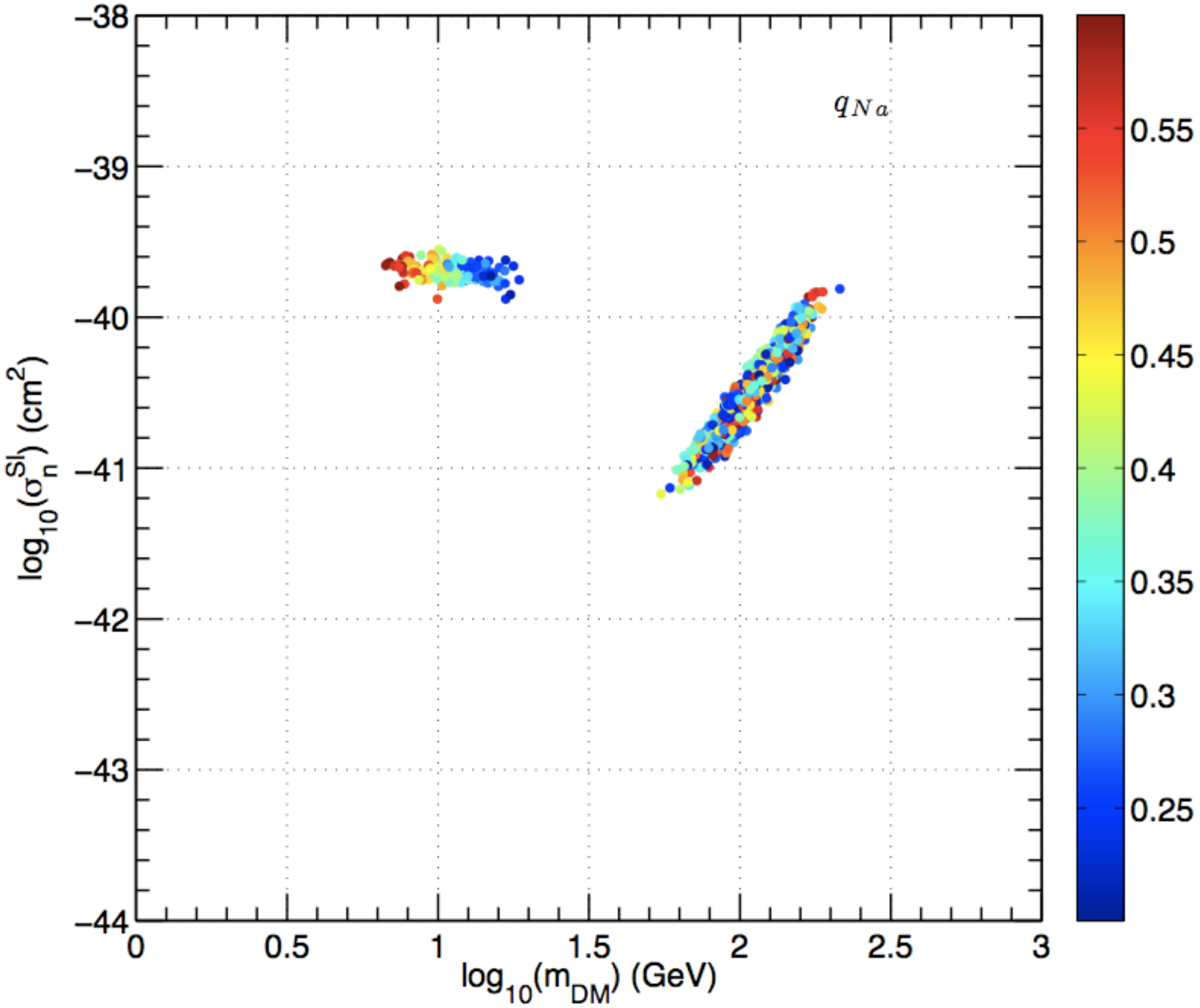


$$v_0 = 214^{+32}_{-21}\ (\mathrm{km\ s}^{-1})$$
$$v_{\mathrm{esc}} = 556^{+14}_{-15}\ (\mathrm{km\ s}^{-1})$$
$$\rho_\odot = 0.35^{+0.14}_{-0.09}\ (\mathrm{GeV\ cm}^{-3})$$

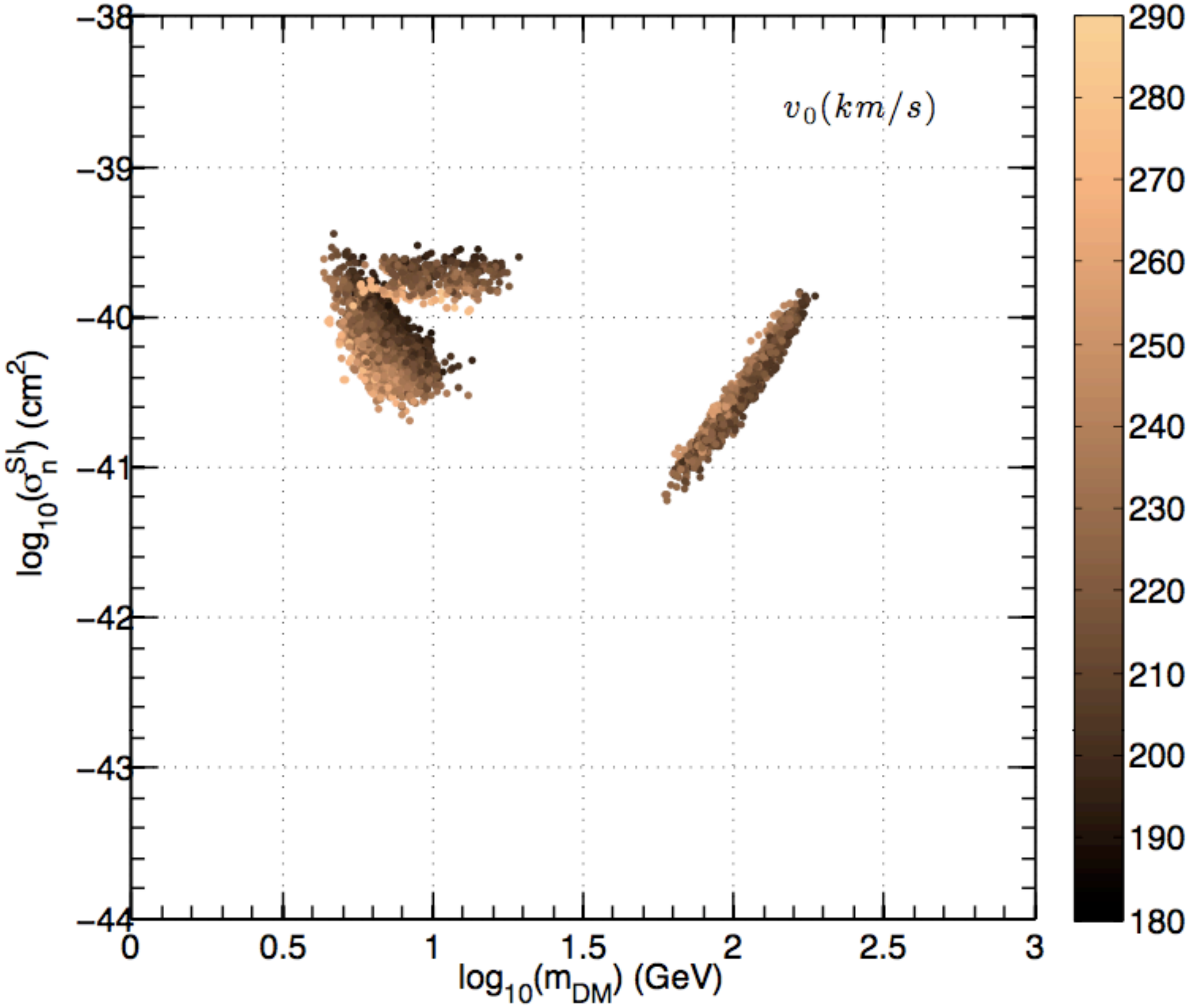
$$m_{\mathrm{DM}} = 7.\ (\mathrm{GeV})$$
$$\sigma_n^{SI} = 1.53 \times 10^{-40}\ (\mathrm{cm}^2)$$

- combined fit prefers small values of the local standard at rest, the escape velocity and density

DAMA and CoGeNT, combined fit



- the larger qNa the smaller the WIMP mass
- low mass region is independent on qI



- similar behavior for the DM density at the sun position
- less sensitive to the escape velocity value

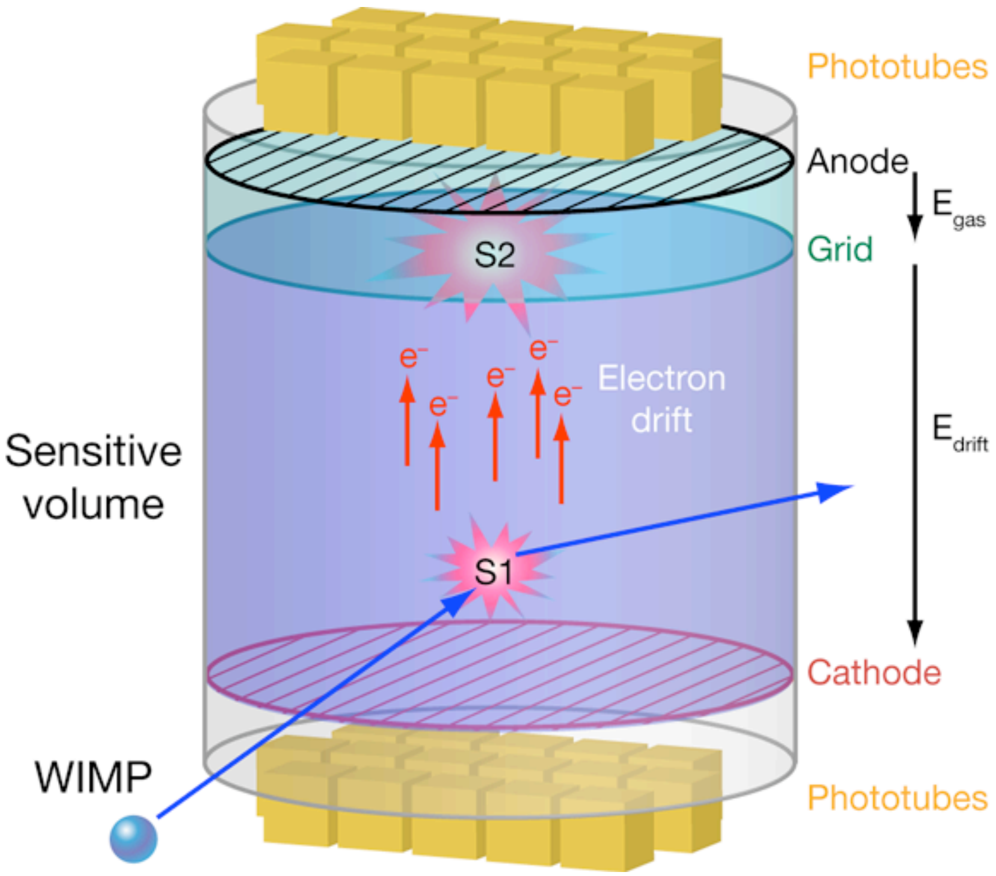
What about the compatibility with current exclusion bounds?

Xenon100

- S = 3 (seen events), likelihood follows a Poisson distribution
- B = 1.8 +- 0.6, numerical marginalization
- considered Poisson fluctuations below threshold
- energy range from 4 PE (5-8 keV) -> 30 PE
- total exposure 1481 kg days

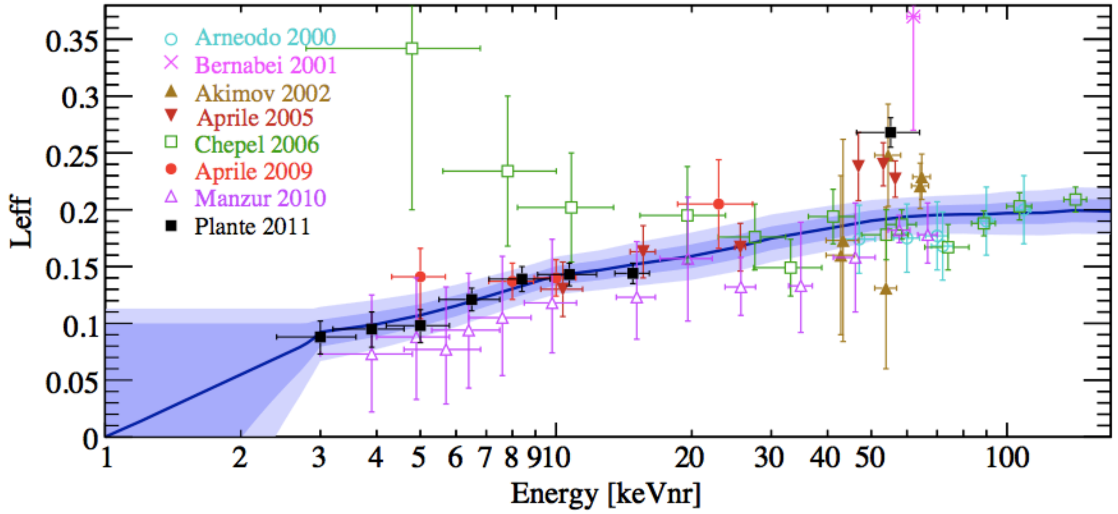
$$\ln \mathcal{L}_{\mathrm{Xenon}} = \ln \mathcal{L}_{\mathrm{Events}} + \ln \mathcal{L}_{\mathrm{L_{eff}}}$$

$$\ln \mathcal{L}_{\mathrm{Events}} = -S - B + 3 + \sum_{i=1}^{3} \ln \left(\frac{\mathrm{d}R}{\mathrm{d}S_1}\bigg|_i + \frac{B}{\bar{B}} \frac{\mathrm{d}N_B}{\mathrm{d}S_1}\bigg|_i \right) + C_{\mathrm{norm}}$$



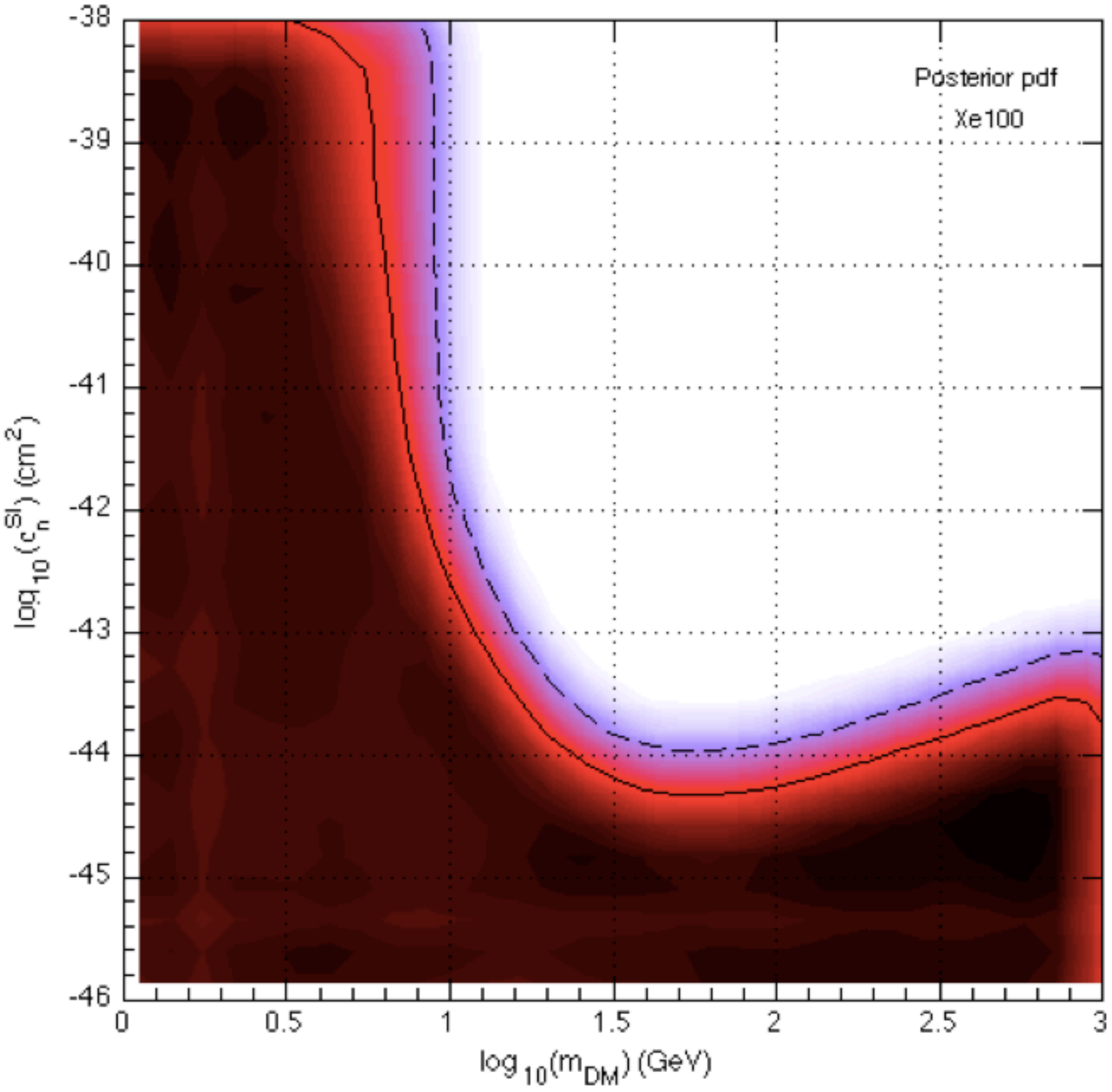
$$S_1(E) = \mathrm{L_{eff}}(E)\ \mathrm{L}_y\ E\ \frac{S_{\mathrm{nr}}}{S_{\mathrm{ee}}}$$

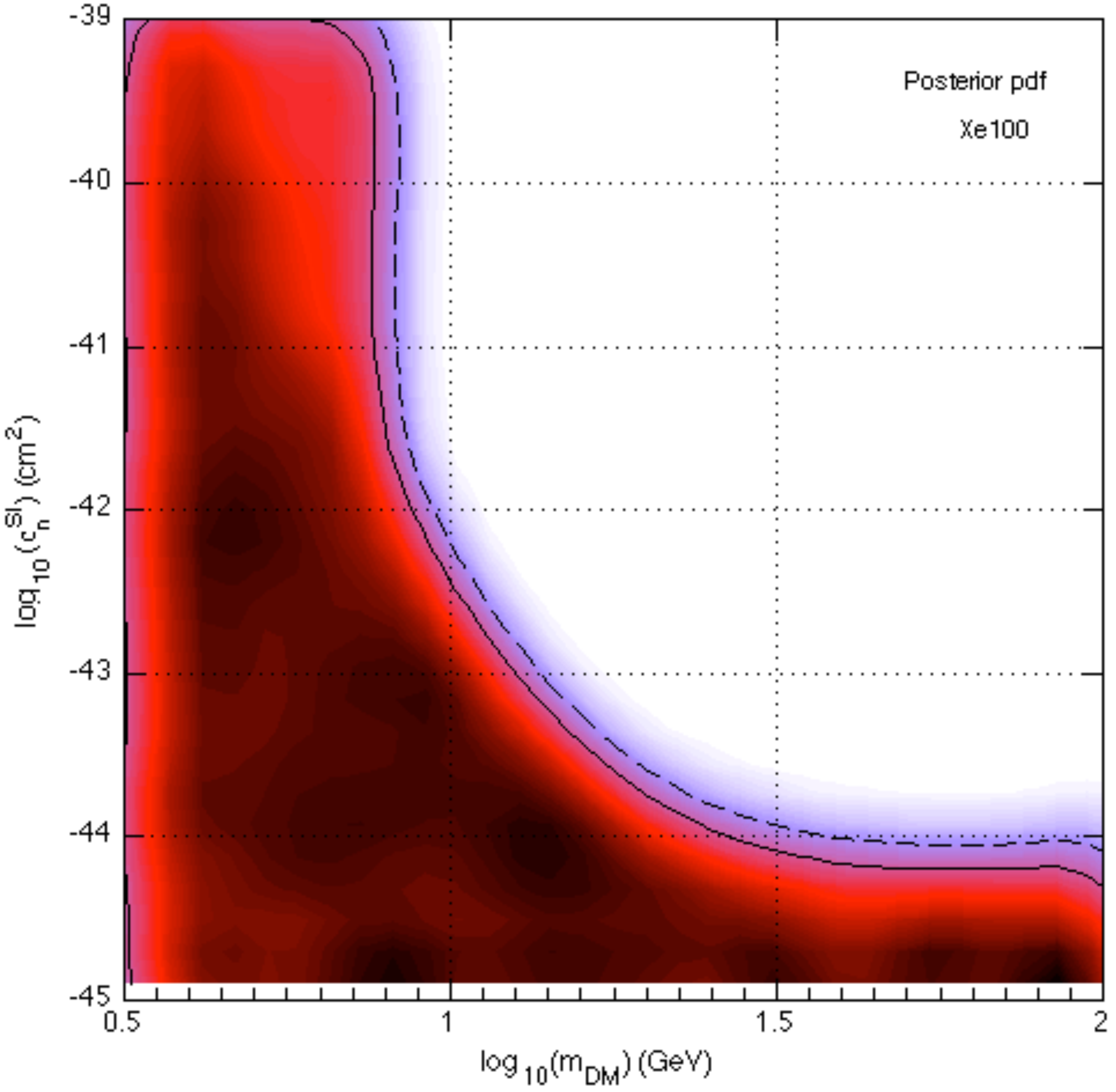
Aprile et al. arXiv:1104.2549



- Scintillation efficiency is a systematic of the experimental set-up
- treated as nuisance parameter with truncated gaussian prior and marginalized over

Unconstraining data: prior dependence





2D marginal credible regions at 90% + $90_S\%$.

$$\Delta\chi^2_{\rm eff} \leq 2.7$$

$$\mathcal{P}_{\rm mar}(m_{\rm DM}, \sigma_n^{\rm SI}|X) = \mathcal{P}_{\rm mar}(S_x|X)$$

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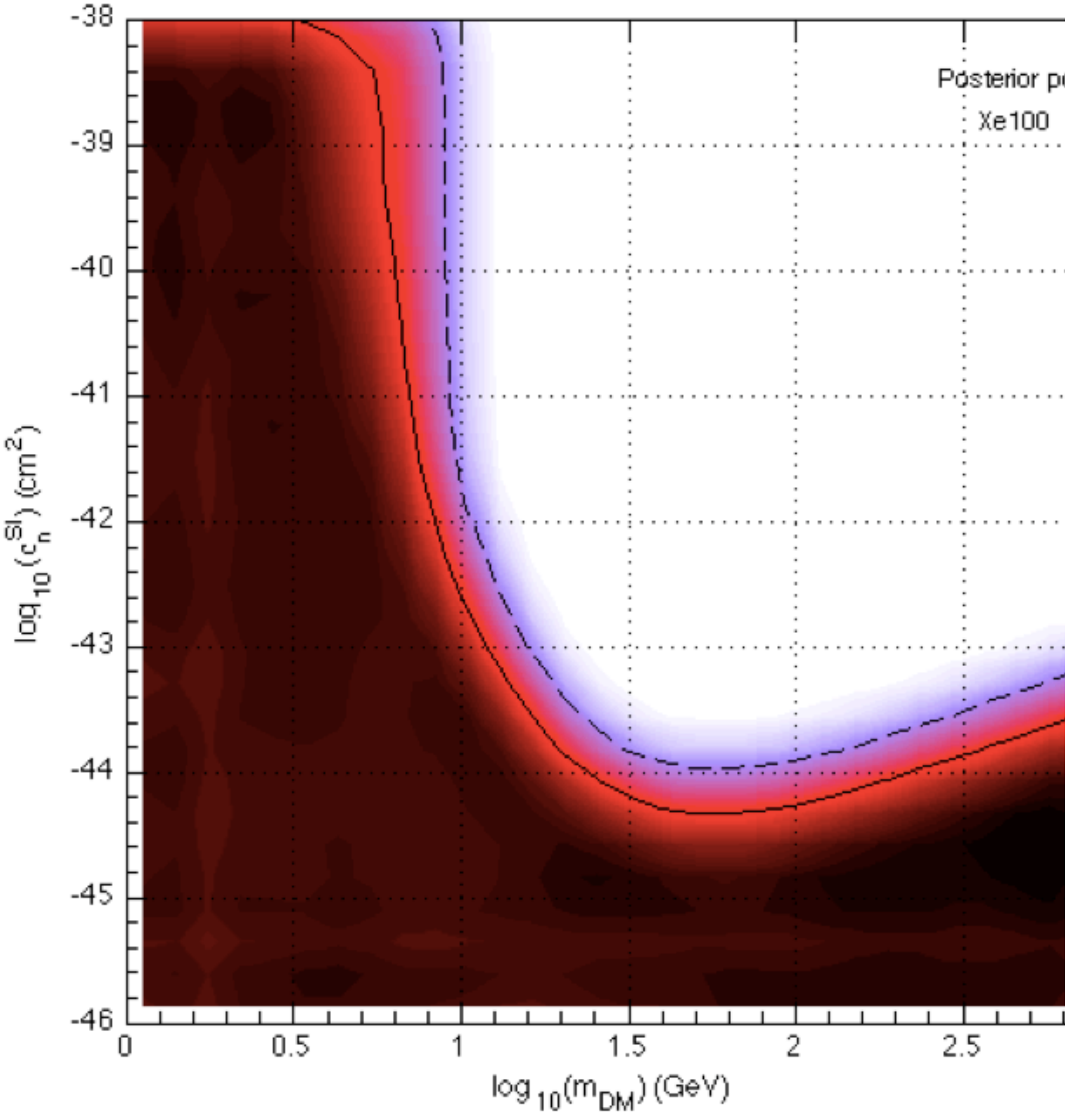

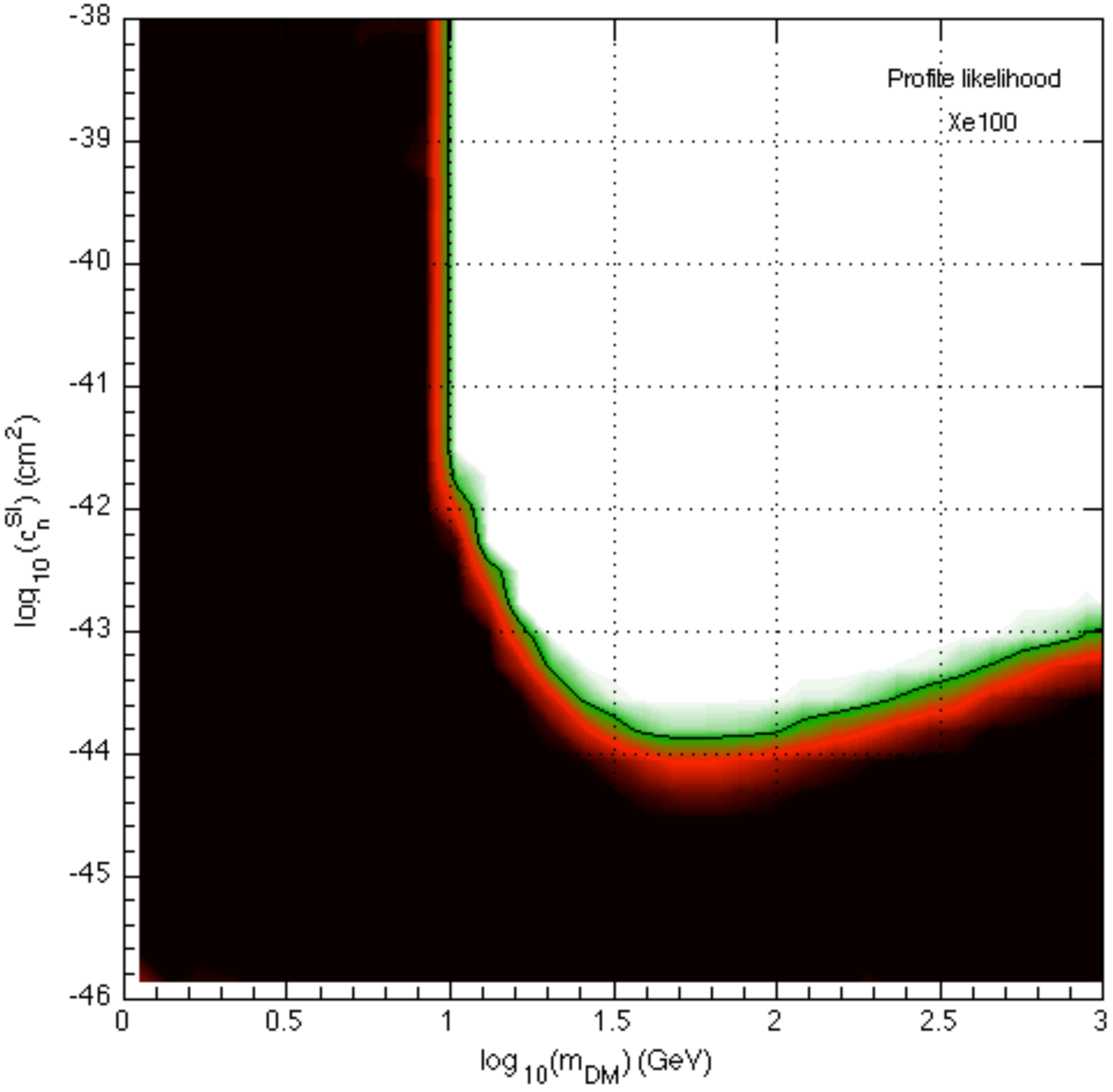

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CDSM Ge and Si

Akerib et al. '05, Ahmed et al '09

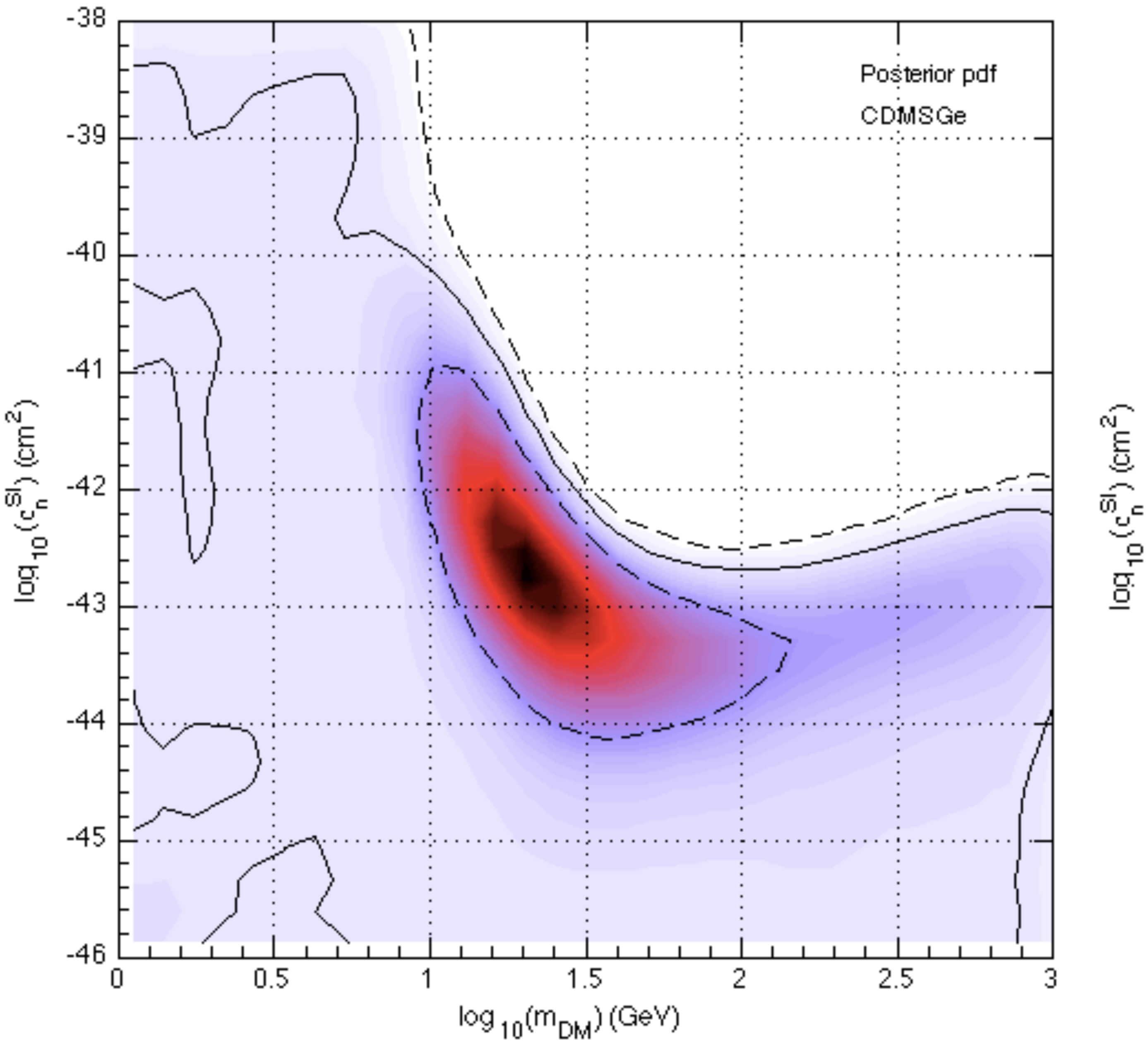
No nuisance parameters, background accounted for by analytical marginalization

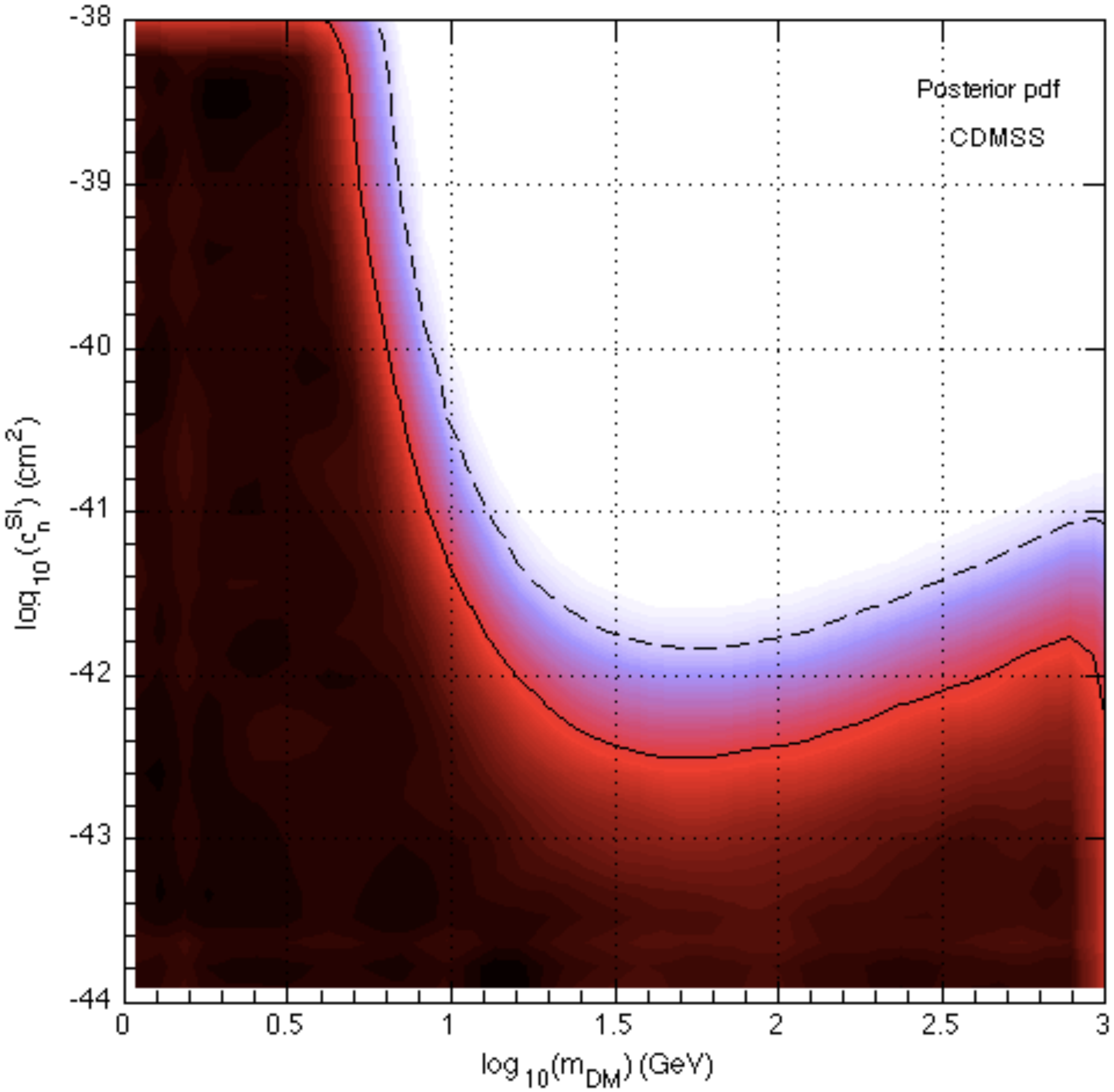
- N = 2, B= 1.38 +- 0.38
- exposure of 1063.2 kg days (all runs combined)
- energy range from 10 -> 100 keV

$$\ln \mathcal{L}_{\rm CDMSGe} = -S - B + 2 + \sum_{i=1,2} \ln\left(\frac{\rm dR}{\rm dE_i} + \frac{B}{\bar{B}}\frac{dN_B}{dE_i}\right) + C_{\rm norm}$$

- N = 2, B= 4.4 +- 0.6
- exposure of 65.8 kg days
- energy range from 5 -> 100 keV

$$\ln \mathcal{L}_{\rm CDMSSi}(2|S,B) = -S - B + 2 + 2\ln\left(\frac{S+B}{2}\right)$$





Low energy analyses

- CDMS Ge (Ahmed et al. arXiv: 1011.2482)

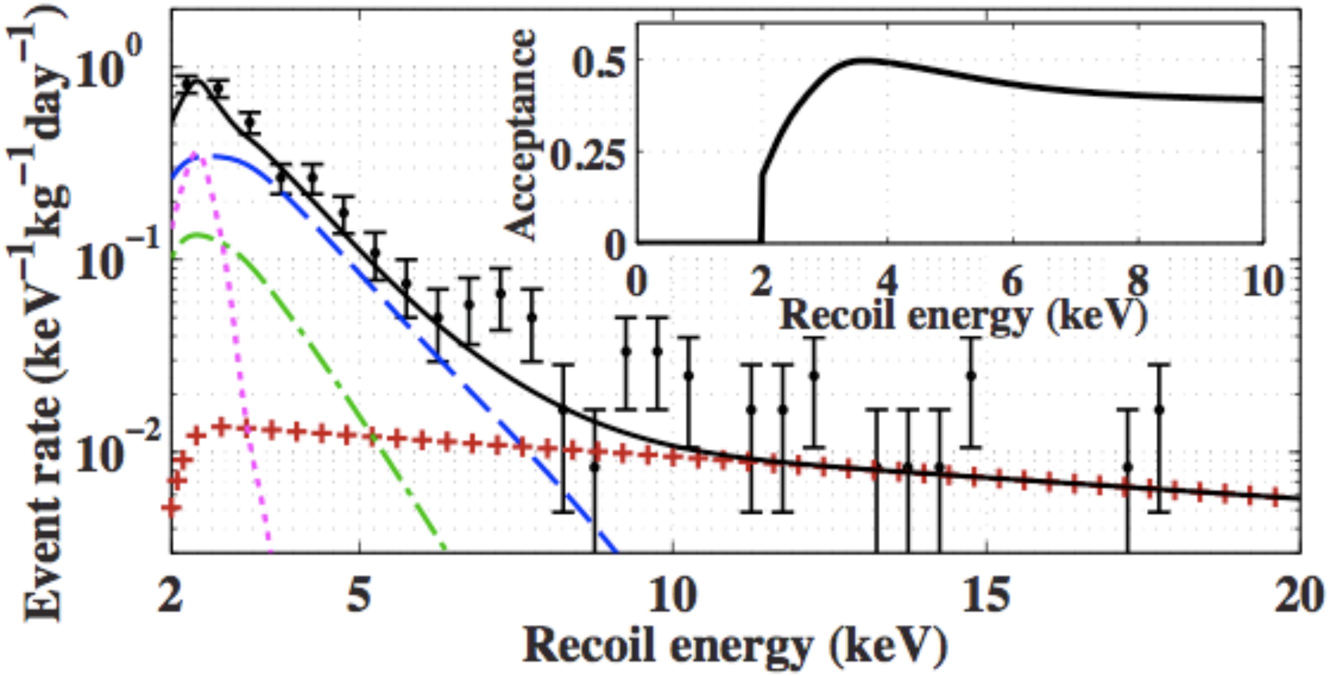
  (A) threshold lowered down from 10 to 2 KeV

  (B) lower threshold -> lower ability in discriminating background events, because ionization signal missing

  (C) 427 events in 214 kg days

  (D) calibration of recoil energy extrapolated as well

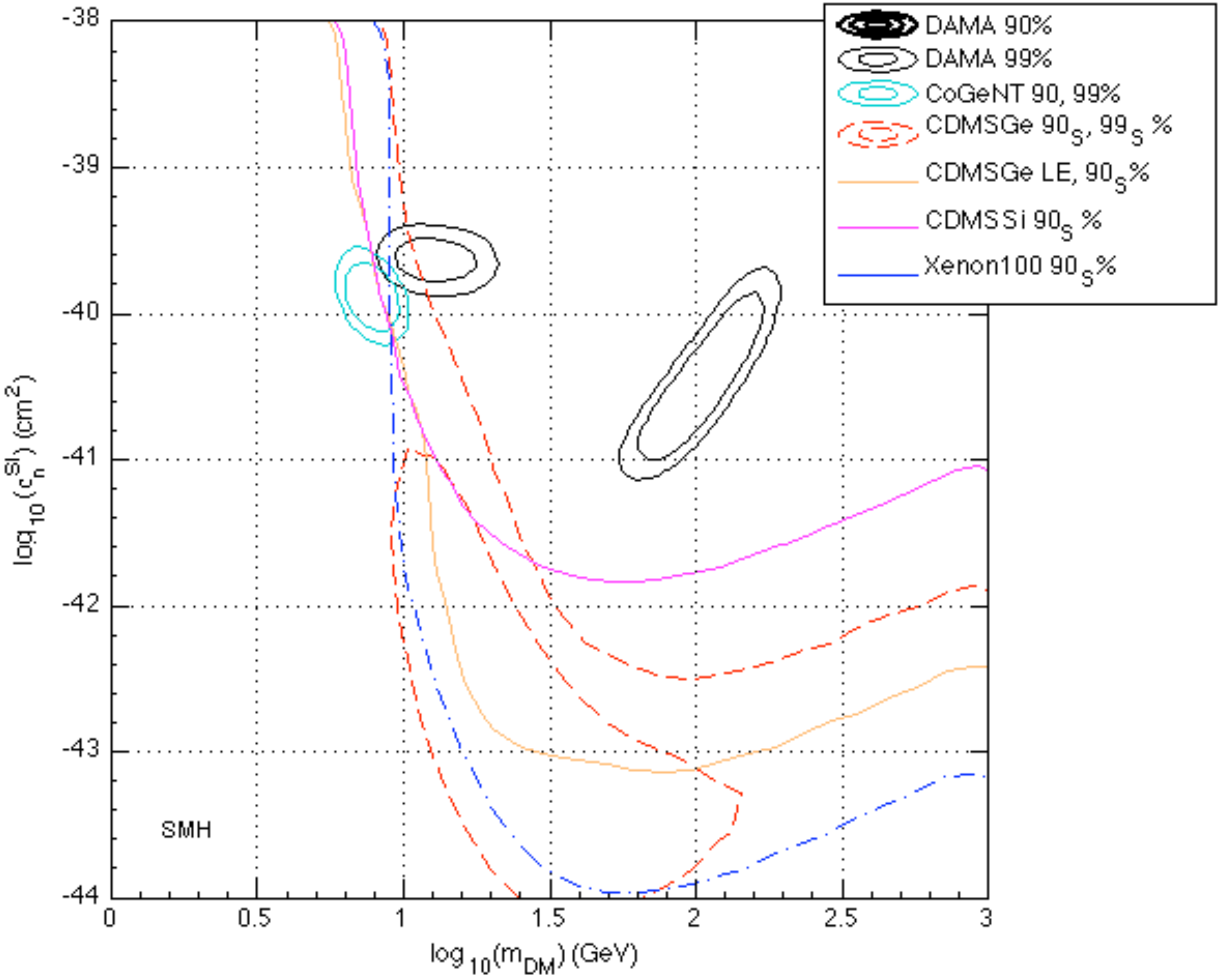
  (E) background as nuisance parameter



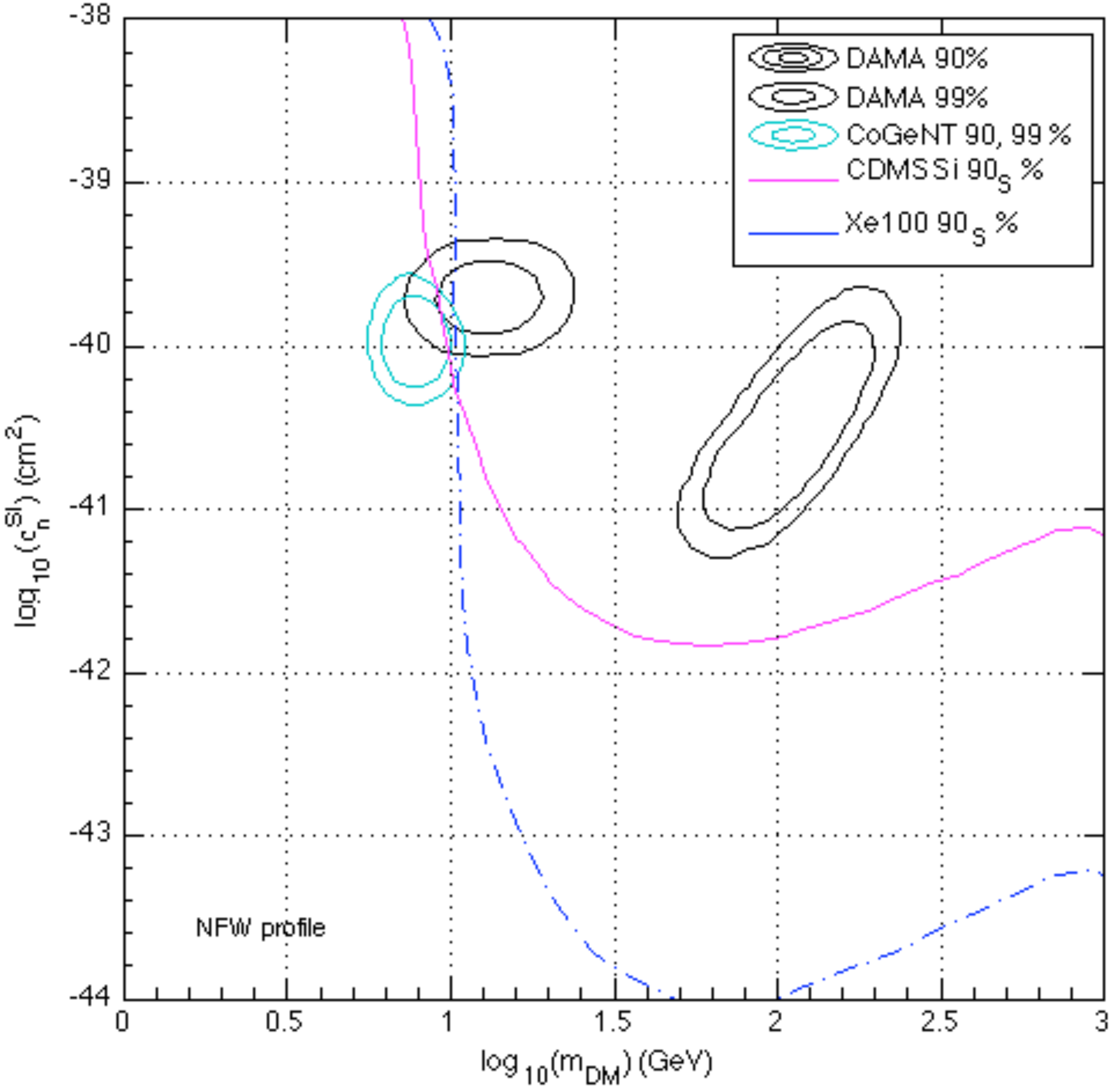
NOT CONSIDERED:

- Xenon10 -> S2 only based analysis, lowered threshold at 1 KeV but the background can not be modelled (Angle et al. arXiv:1104.3088)
- Combined Ge + Si -> unknown low energy background as well (Akerib et al. arXiv: 1010.4290)

2D region for SMH, all experiments



2D credible regions for NFW density profile case



- Einasto, Burkert and ISO density profiles give very similar results

Preferred values for the astrophysical observables

	v_0 (km s^{-1})	$v_{\rm esc}$ (km s^{-1})	$\rho_\odot$ (GeV cm^{-3})
Cored Isothermal			
DAMA	210^{+26}_{-16}	628^{+22}_{-17}	$0.31^{+0.05}_{-0.03}$
CoGeNT	209^{+14}_{-21}	628 ± 18	0.31 ± 0.04
CDMSGe	208^{+22}_{-16}	628^{+23}_{-21}	0.31 ± 0.05
CDMSSi	210^{+29}_{-16}	628 ± 21	$0.31^{+0.05}_{-0.04}$
Xenon100	211^{+26}_{-19}	629 ± 21	0.31 ± 0.04
NFW			
DAMA	220^{+40}_{-20}	558^{+19}_{-16}	$0.37^{+0.15}_{-0.09}$
CoGeNT	219^{+38}_{-18}	559 ± 17	$0.37^{+0.20}_{-0.08}$
CDMSGe	218^{+41}_{-18}	559 ± 18	$0.37^{+0.16}_{-0.08}$
CDMSSi	218^{+44}_{-19}	560^{+19}_{-18}	$0.36^{+0.18}_{-0.09}$
Xenon100	219^{+43}_{-20}	559 ± 18	$0.37^{+0.16}_{-0.08}$
Einasto			
DAMA	221^{+39}_{-19}	560^{+13}_{-18}	$0.36^{+0.14}_{-0.08}$
CoGeNT	222^{+42}_{-19}	562^{+11}_{-21}	$0.36^{+0.15}_{-0.08}$
CDMSGe	221^{+44}_{-19}	561^{+11}_{-22}	$0.36^{+0.15}_{-0.08}$
CDMSSi	221^{+44}_{-19}	561^{+11}_{-22}	$0.36^{+0.15}_{-0.08}$
Xenon100	221^{+44}_{-19}	562^{+11}_{-22}	$0.36^{+0.15}_{-0.08}$
Burkert			
DAMA	214^{+36}_{-21}	548^{+29}_{-16}	$0.44^{+0.16}_{-0.12}$
CoGeNT	216^{+35}_{-22}	550 ± 20	$0.44^{+0.16}_{-0.12}$
CDMSGe	215^{+35}_{-23}	549 ± 19	$0.44^{+0.18}_{-0.12}$
CDMSSi	215^{+35}_{-23}	550 ± 22	$0.44^{+0.18}_{-0.13}$
Xenon100	216^{+35}_{-23}	550 ± 21	$0.44^{+0.16}_{-0.13}$

Bayesian Model comparison

$$\mathcal{P}(\theta \mid X) = \pi(\theta)\, \frac{\mathcal{L}(X|\theta)}{\mathcal{Z}(X)}$$

$$\mathcal{Z} = \int \mathcal{L}(X|\theta)\pi(\theta) d^D\theta$$

Bayesian evidence

1. model averaged likelihood
2. contains notion of Occam's razor principle
3. used for model comparison

Posterior pdf for a model:

$$\mathcal{P}(\mathcal{M}|X) \propto \mathcal{Z}\ \pi(\mathcal{M})$$

$$\pi(\mathcal{M}_0) = \pi(\mathcal{M}_1)$$

(non committal prior)

$$\frac{\mathcal{P}(\mathcal{M}_0|X)}{\mathcal{P}(\mathcal{M}_1|X)} = \boxed{B_{01}} \frac{\pi(\mathcal{M}_0)}{\pi(\mathcal{M}_1)}$$

Empirical Jeffreys' scale

$\ln B_{10}$	Odds $\mathcal{M}_1 : \mathcal{M}_0$	Strength of evidence
< -5.0	$< 1 : 150$	Strong evidence for $\mathcal{M}_0$
$-5.0 \to -2.5$	$1 : 150 \to 1 : 12$	Moderate evidence for $\mathcal{M}_0$
$-2.5 \to -1.0$	$1 : 12 \to 1 : 3$	Weak evidence for $\mathcal{M}_0$
$-1.0 \to 1.0$	$1 : 3 \to 3 : 1$	Inconclusive
$1.0 \to 2.5$	$3 : 1 \to 12 : 1$	Weak evidence against $\mathcal{M}_0$
$2.5 \to 5.0$	$12 : 1 \to 150 : 1$	Moderate evidence against $\mathcal{M}_0$
> 5.0	$> 150 : 1$	Strong evidence against $\mathcal{M}_0$

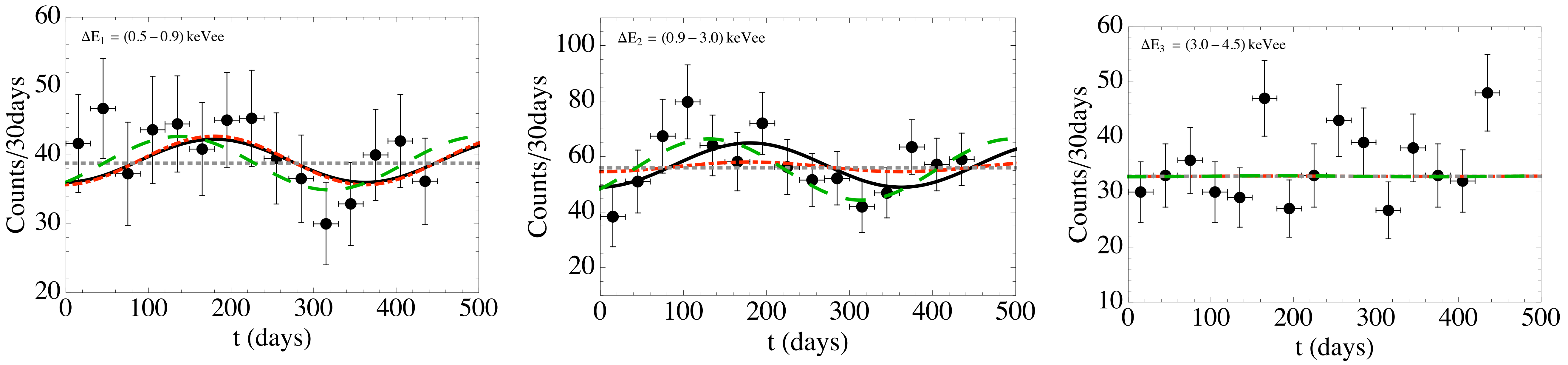
Bayes factor: ratio of model's evidences

Is there an evidence for DM modulation in CoGeNT data?

Comparison between 5 phenomenological models that describe a sinusoidal modulation:

$$R_i(t) = U^i_{\rm m} \left(1 + S^i_{\rm m} \cos[2\pi(t - t_{\rm max} - 28)/T]\right)$$

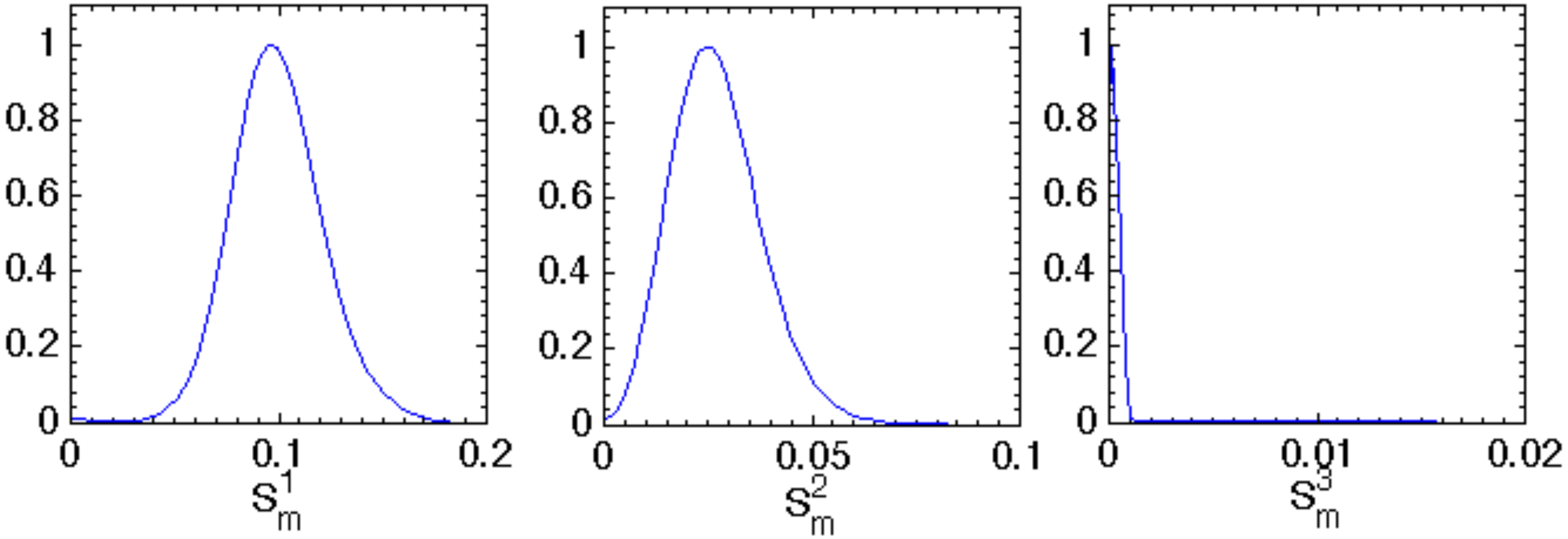
Model	Description	Fractional modulation $S^i_{\rm m}$	Phase $t_{\rm max}$ (days)	Period T (days)	Extra params
0	No modulation	0	—	—	$\nu = 0, 0$
1a	Pheno-DM	$S^{1,2}_{\rm m} = 0 \rightarrow 0.2$ $S^3_{\rm m} = 0$	152	365	$\nu = 1, 2$
1b	Consistent DM	Gaussian, clipped at 0 $(S^i_{\rm m} \geq 0)$ $S^1_{\rm m} = 0.098 \pm 0.021$ $S^2_{\rm m} = 0.026 \pm 0.011$ $S^3_{\rm m} = (0.37 \pm 36) \times 10^{-4}$	152	365	$\nu = 1, 3$
2a	Non-DM, annual	$0 \rightarrow 1$	$0 \rightarrow 365$	365	$\nu = 2, 4$
2b	Non-DM, free period	$0 \rightarrow 1$	$0 \rightarrow 365$	$1 \rightarrow 365$	$\nu = 3, 5$

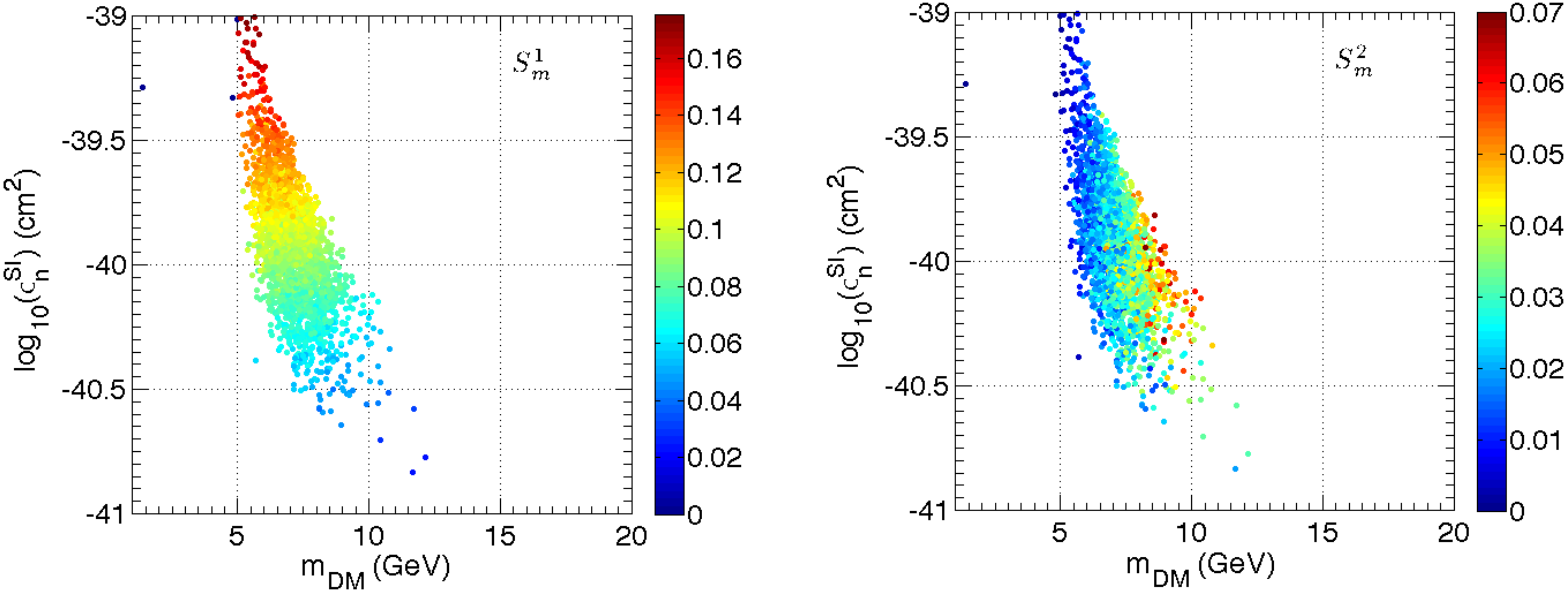


Model 1b: consistent DM

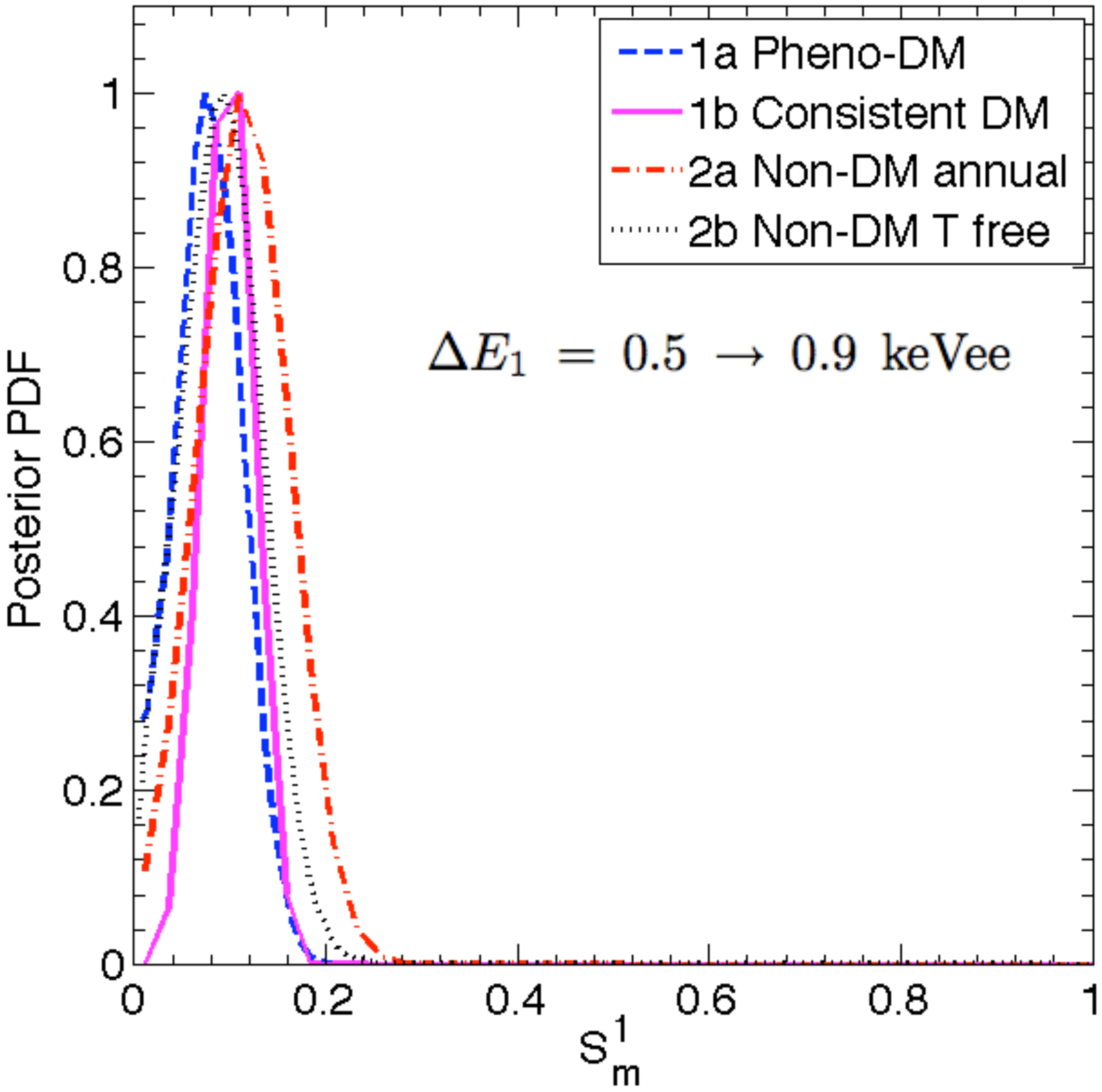
Priors on the fractional modulated amplitude predicted from configurations of DM mass and sigma that account for the CoGeNT total rate $R(t) = S(t) + B$

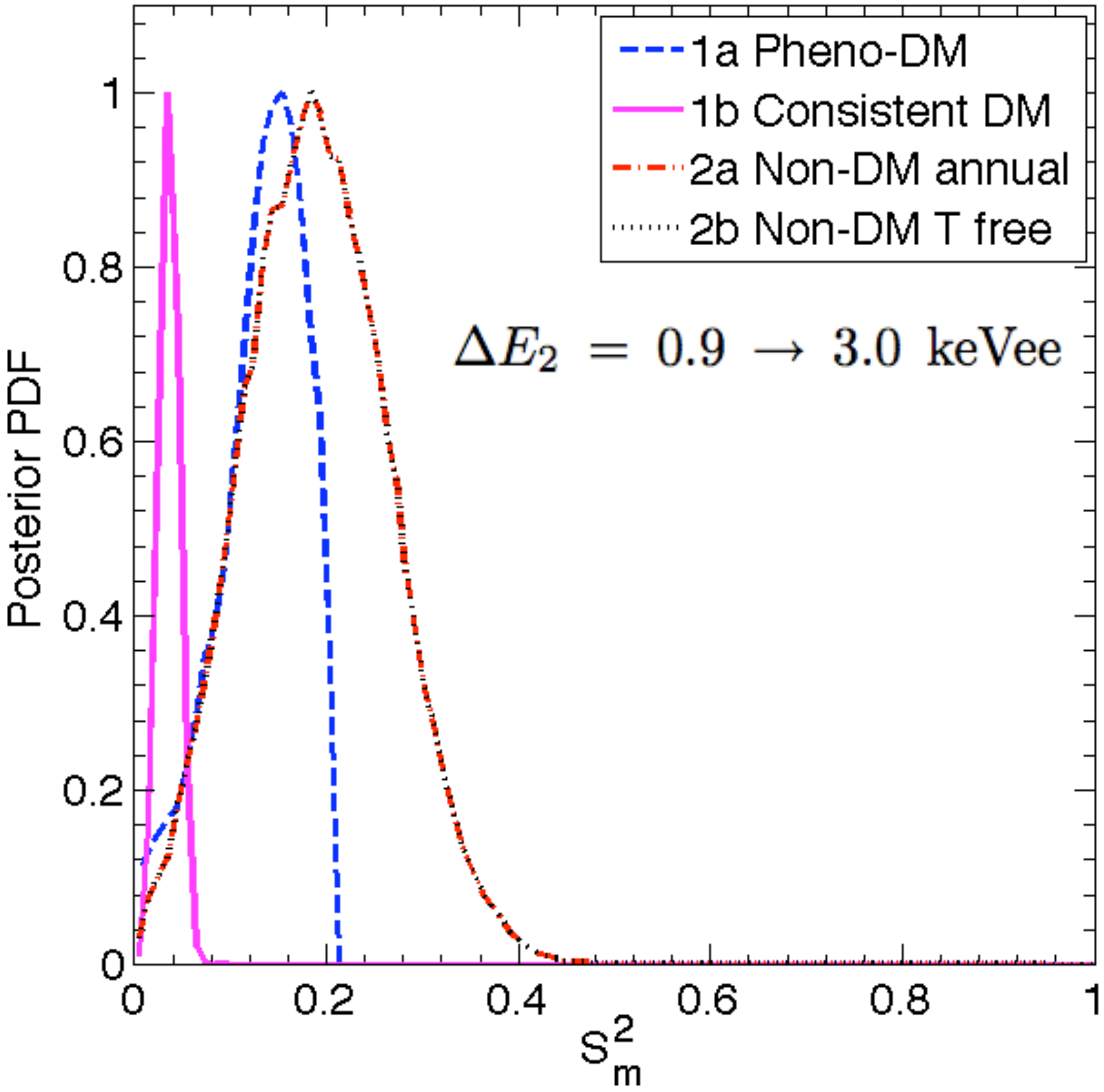
$$S_{\rm m} = \frac{R(t_{\rm max}) - R(t_{\rm min})}{R(t_{\rm max}) + R(t_{\rm min})}$$





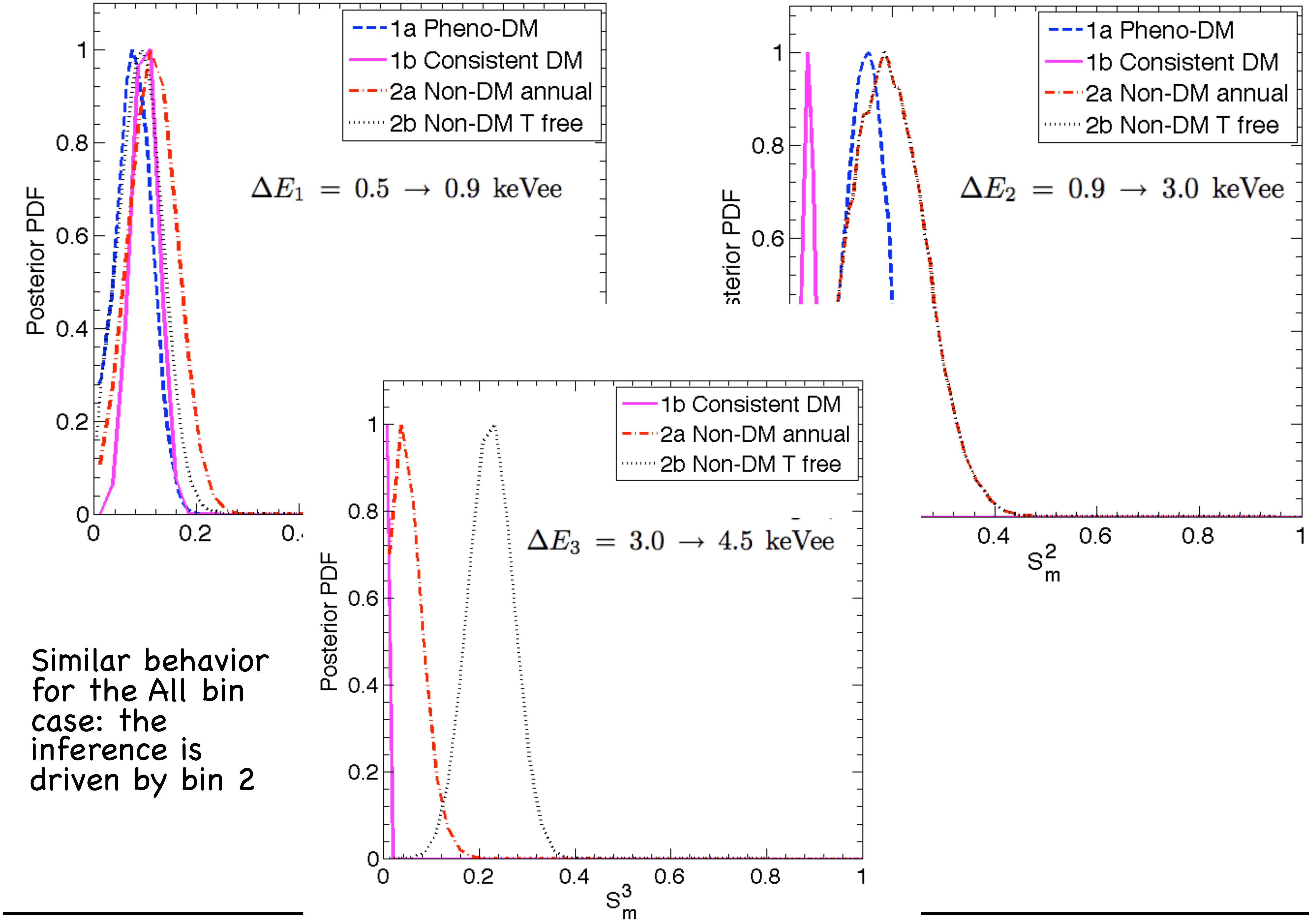
Parameter inference: amplitude of modulation





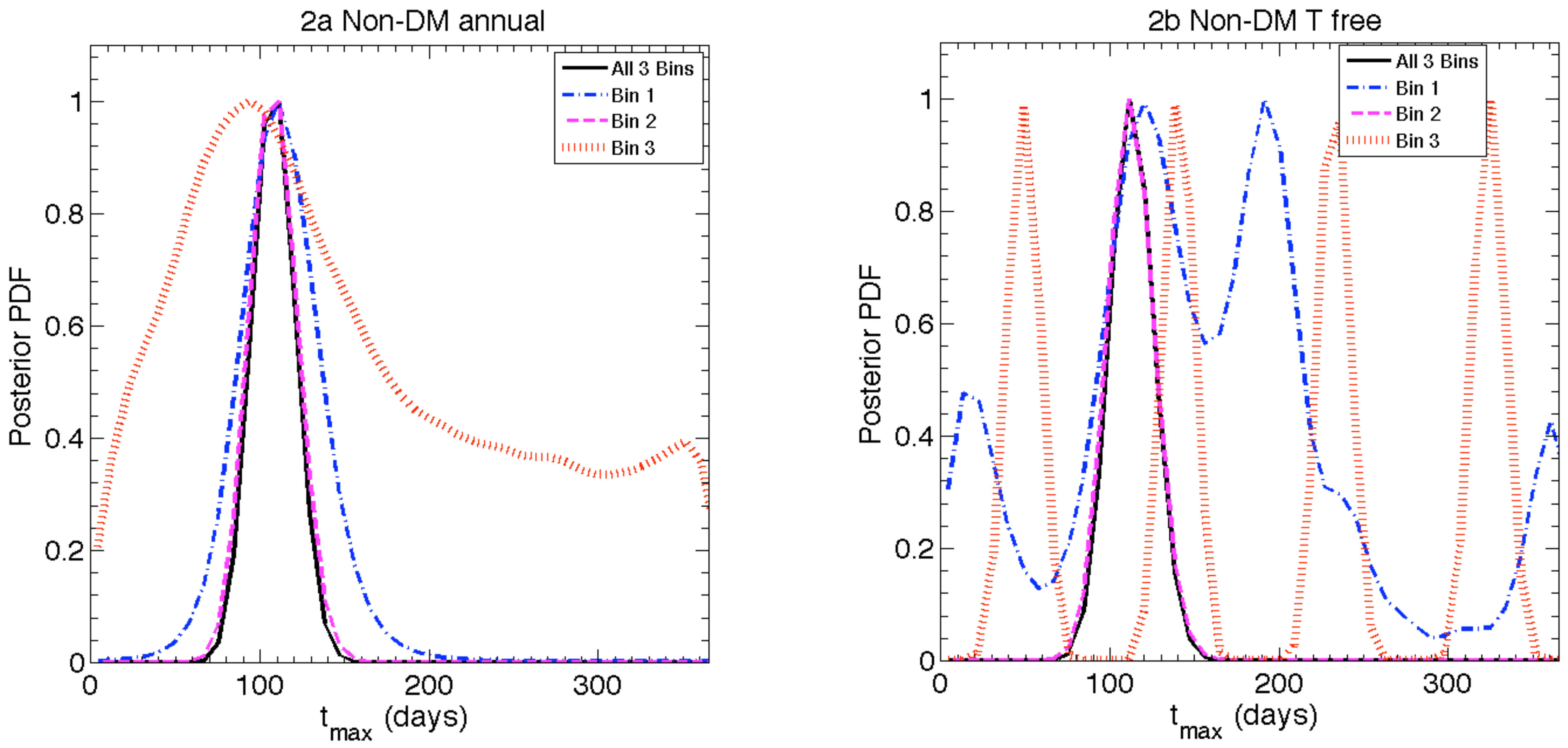
Similar behavior for the All bin case: the inference is driven by bin 2

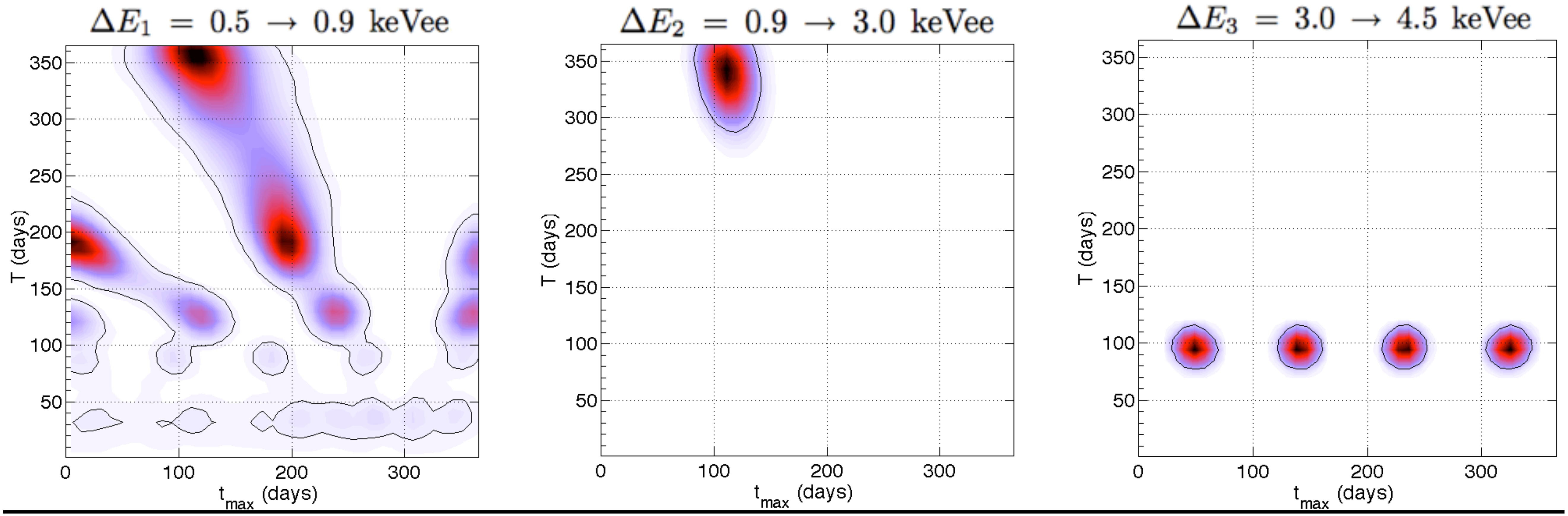
Parameter inference: amplitude of modulation



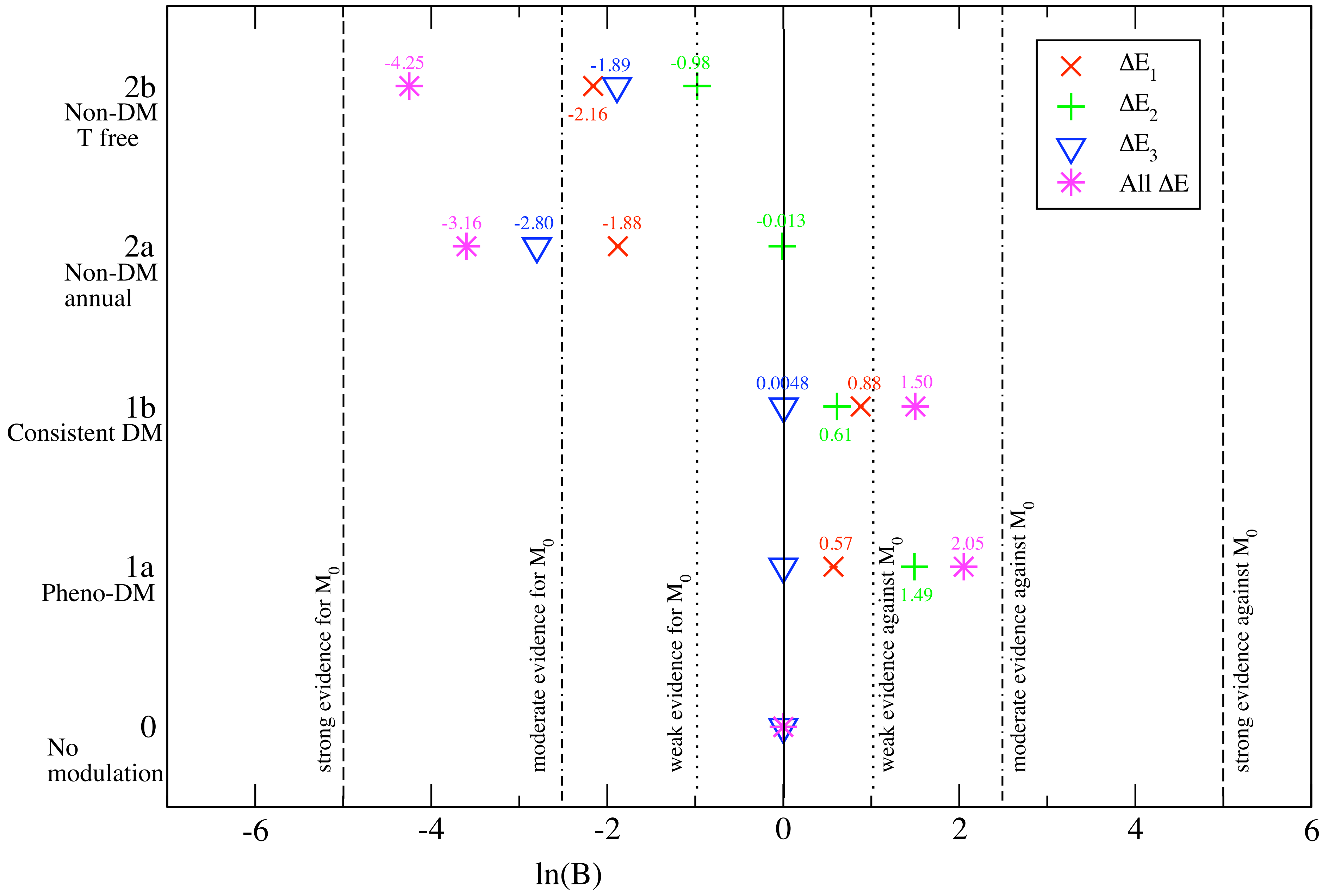
Similar behavior for the All bin case: the inference is driven by bin 2

Parameter inference: phase and period (models 2a and 2b)





Bayes factor: results for model comparison



	$\mathcal{M}_i : \mathcal{M}_0$			
Model $\mathcal{M}_i$	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2 : 1	4 : 1	1 : 1	8 : 1
1b	2 : 1	2 : 1	1 : 1	5 : 1
2a	1 : 7	1 : 1	1 : 16	1 : 37
2b	1 : 9	1 : 3	1 : 6	1 : 70

$\mathcal{M}_i : \mathcal{M}_j$	Bin 1	Bin 2	Bin 3	All 3 bins
1a:2a	12 : 1	5 : 1	16 : 1	183 : 1
1a:2b	15 : 1	12 : 1	7 : 1	545 : 1
1b:2a	16 : 1	2 : 1	17 : 1	107 : 1
1b:2b	21 : 1	5 : 1	7 : 1	314 : 1

Classical p-values

$$\wp \equiv \int_{t_{\rm obs}}^{\infty} p(t|H_0)$$

probability of obtaining more extreme data than observed assuming the null hypothesis is correct and NOT probability for hypothesis

$$\Delta\chi^2_{\rm eff} \equiv -2\ln\left[\frac{\mathcal{L}(\vartheta^\star, \hat{\psi})}{\mathcal{L}(\hat{\hat{\vartheta}}, \hat{\hat{\psi}})}\right]$$

test statistics for nested models if

1. additional dof distributed as a gaussian
2. unbounded likelihood
3. all additional dof identifiable under the null

	$\Delta\chi^2_{\rm eff}$ relative to model 0			
Model	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2.04	4.23	–	6.26
	$\wp = 0.08$	$\wp = 0.02$	–	$\wp = 0.02$
	$(\nu = 1)$	$(\nu = 1)$		$(\nu = 2)$
1b	1.94	1.88	0.020	3.84
	$\wp = 0.08$	$\wp = 0.09$	$\wp = 0.4$	$\wp = 0.1$
	$(\nu = 1)$	$(\nu = 1)$	$(\nu = 1)$	$(\nu = 3)$
2a	3.61	8.36	0.025	10.63
2b	3.70	8.87	10.88	10.86

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Chernoff’s theorem

$$\wp = \sum_{i=0}^{N} 2^{-\nu}\binom{\nu}{i} p(\chi^2_i > \Delta\chi^2_{\rm eff})$$

	$\Delta\chi^2_{\rm eff}$ relative to model 0			
Model	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2.04 $\wp = 0.08$ $(\nu = 1)$	4.23 $\wp = 0.02$ $(\nu = 1)$	– –	6.26 $\wp = 0.02$ $(\nu = 2)$
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1b	1.94 $\wp = 0.08$ $(\nu = 1)$	1.88 $\wp = 0.09$ $(\nu = 1)$	0.020 $\wp = 0.4$ $(\nu = 1)$	3.84 $\wp = 0.1$ $(\nu = 3)$ 1.6σ
2a	3.61	8.36	0.025	10.63
2b	3.70	8.87	10.88	10.86

Classical p-values

$$\wp \equiv \int_{t_{\rm obs}}^{\infty} p(t|H_0)$$

probability of obtaining more extreme data than observed assuming the null hypothesis is correct and NOT probability for hypothesis

$$\Delta\chi^2_{\rm eff} \equiv -2\ln\left[\frac{\mathcal{L}(\vartheta^\star,\hat{\psi})}{\mathcal{L}(\hat{\hat{\vartheta}},\hat{\hat{\psi}})}\right]$$

test statistics for nested models if

1. additional dof distributed as a gaussian
2. ~~2.~~ unbounded likelihood
3. all additional dof identifiable under the null

Chernoff's theorem

$$\wp = \sum_{i=0}^{N} 2^{-\nu}\binom{\nu}{i} p(\chi_i^2 > \Delta\chi^2_{\rm eff})$$

	$\Delta\chi^2_{\rm eff}$ relative to model 0			
Model	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2.04 $\wp = 0.08$ $(\nu = 1)$	4.23 $\wp = 0.02$ $(\nu = 1)$	– –	6.26 $\wp = 0.02$ $(\nu = 2)$ 2.3σ
1b	1.94 $\wp = 0.08$ $(\nu = 1)$	1.88 $\wp = 0.09$ $(\nu = 1)$	0.020 $\wp = 0.4$ $(\nu = 1)$	3.84 $\wp = 0.1$ $(\nu = 3)$ 1.6σ
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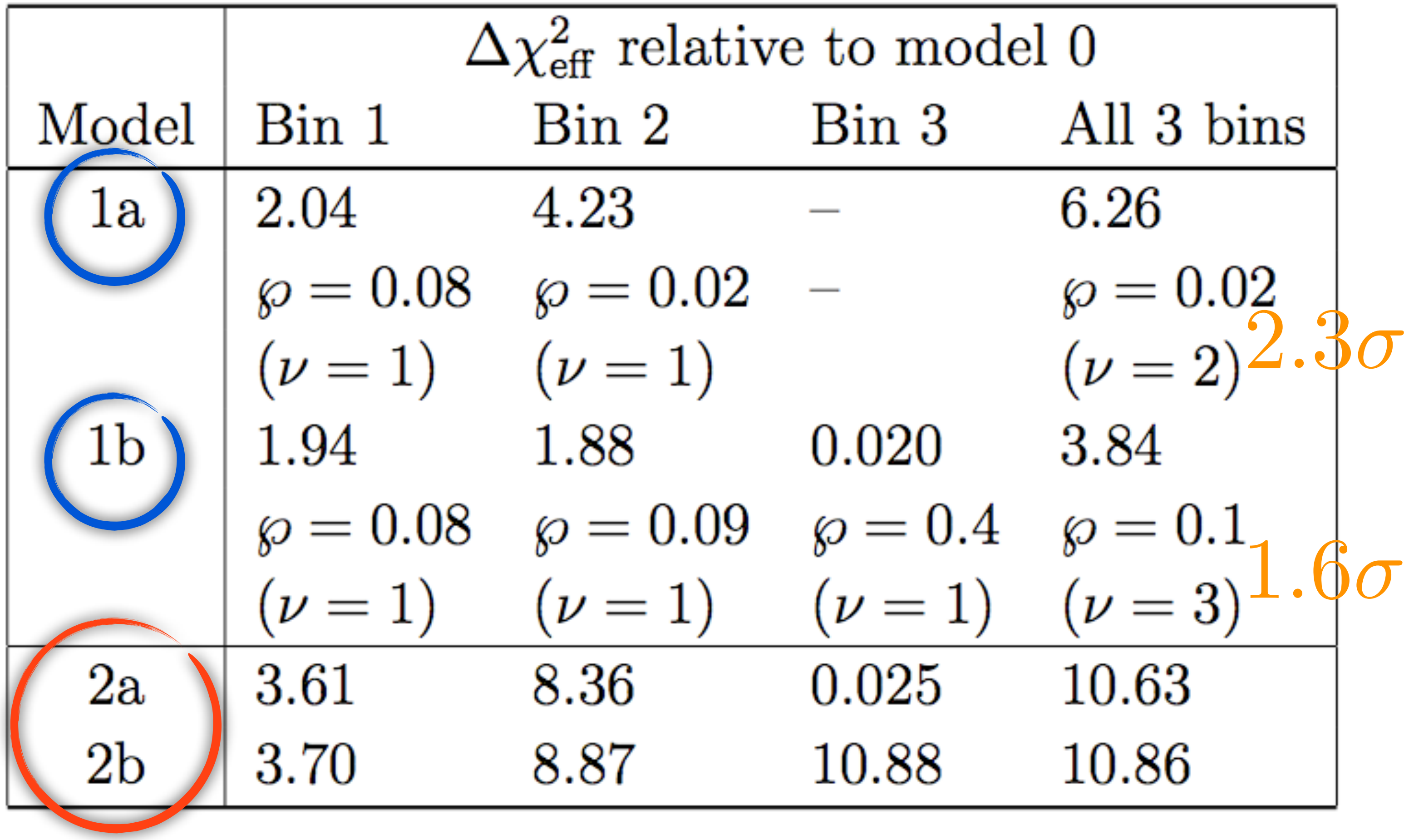

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2.3σ

1.6σ

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2.3σ

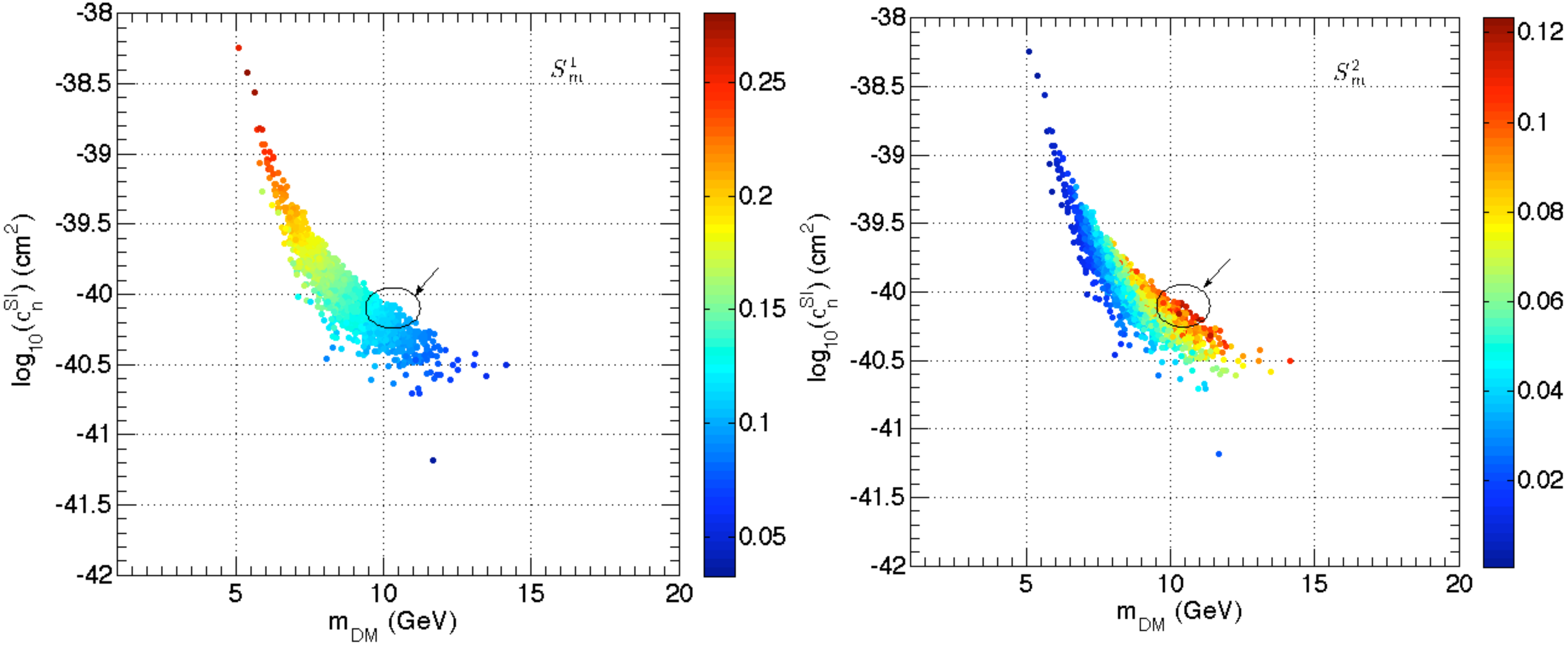
1.6σ

Rely on Monte Carlo simulation for mapping the t statistic into p-values

Locally anisotropic DM velocity distribution

Ellipsoidal, triaxial DM halo model gives rise to a triaxial gaussian velocity distribution:

$$f(\vec{v'}(t)) = \frac{1}{(2\pi)^{3/2}\sigma_R\sigma_\phi\sigma_z} \exp\left[-\frac{v'^2_R}{2\sigma^2_R} - \frac{(v'_\phi + v_\oplus)^2}{2\sigma^2_\phi} - \frac{v'^2_z}{2\sigma^2_z}\right]$$



Alleviate the tension between modulated amplitude and total rate in bin 2

Summary

- DD experiments and Bayesian inference
    - ☐ marginalization over experimental systematics
    - ☐ considered velocity distributions arising from motivated DM halo densities and marginalized over astrophysical uncertainties
    - ☐ Tension between exclusion experiments and 'hints' of detection is alleviated
    - ☐ Combined fit of DAMA and CoGeNT selects a large quenching factor for DAMA, same WIMP mass region as selected by recent 'hints' of CRESST-II (Angloher et al. arXiv:1109.0702)
    - ☐ Combined fit can constrain astrophysical parameters
- Model comparison and CoGeNT modulated rate
    - ☐ weak evidence for DM annual modulation in all the energy range
    - ☐ "other physics" models strongly disfavoured because of additional parameters not supported by the data
    - ☐ CoGeNT total rate predicts too little modulation in the second bin, tension alleviated by assuming anisotropic velocity distribution

Thanks for your attention!

Back up slides

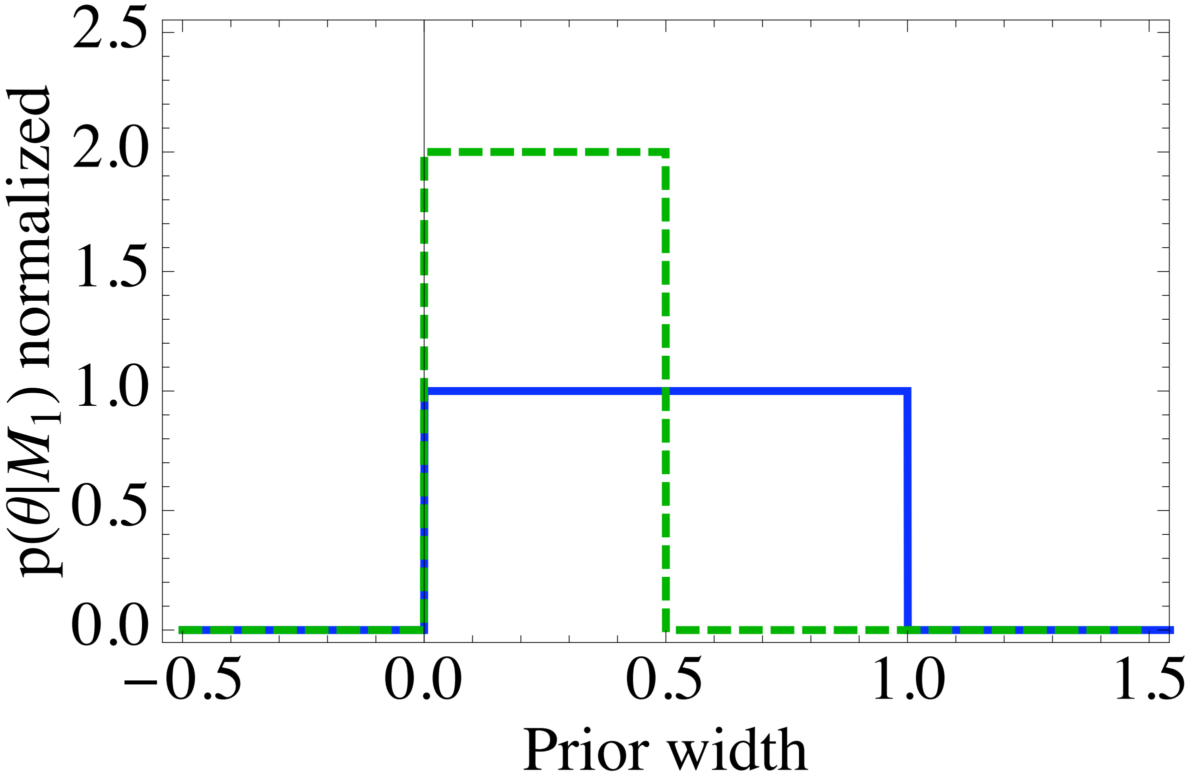
Sensitivity analysis

For nested models with parameter priors separable the Savage Dickey density ratio (SDDR) gives an analytical estimate of the effect on lnB changing the width of the prior

marginal normalized prior density computed at fixed value of ϑ

$$B_{10} = \frac{p(\vartheta^\star|\mathcal{M}_1)}{p(\vartheta^\star|d, \mathcal{M}_1)}$$

marginal posterior pdfs, computed at fixed value of the parameters



EXAMPLE

$$S^i_{\rm m} = 0 \rightarrow 0.5$$

$$\ln 2^3 \simeq 2.1$$

$$\ln B_{2a} = -1.06$$

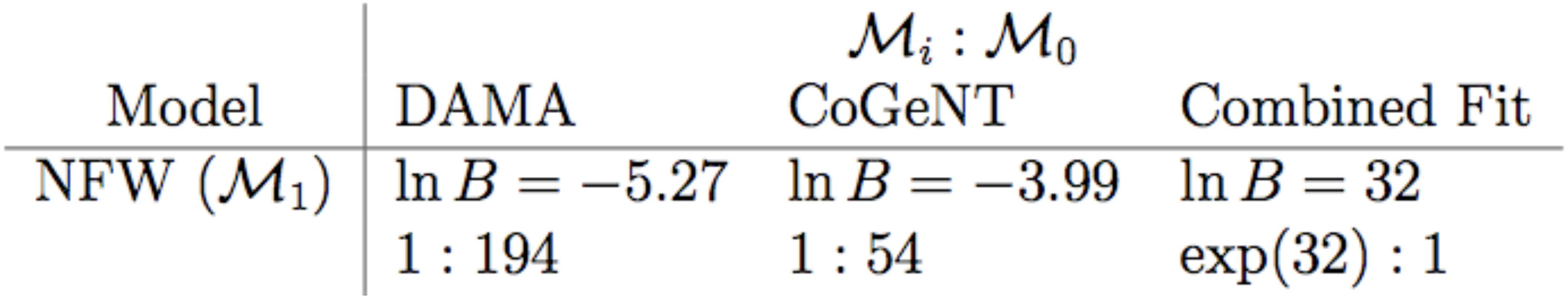
- lnB of 1a:2a is now 3.11 instead of 5.21, still moderate evidence
- Results are robust from a Bayesian point of view!

DM Astrophysical distributions, what can be said using DD?

$\mathcal{M}_0$ SMH velocity distribution with fixed astrophysical quantities

$\mathcal{M}_i$ motivated f(v) with 5 free parameters

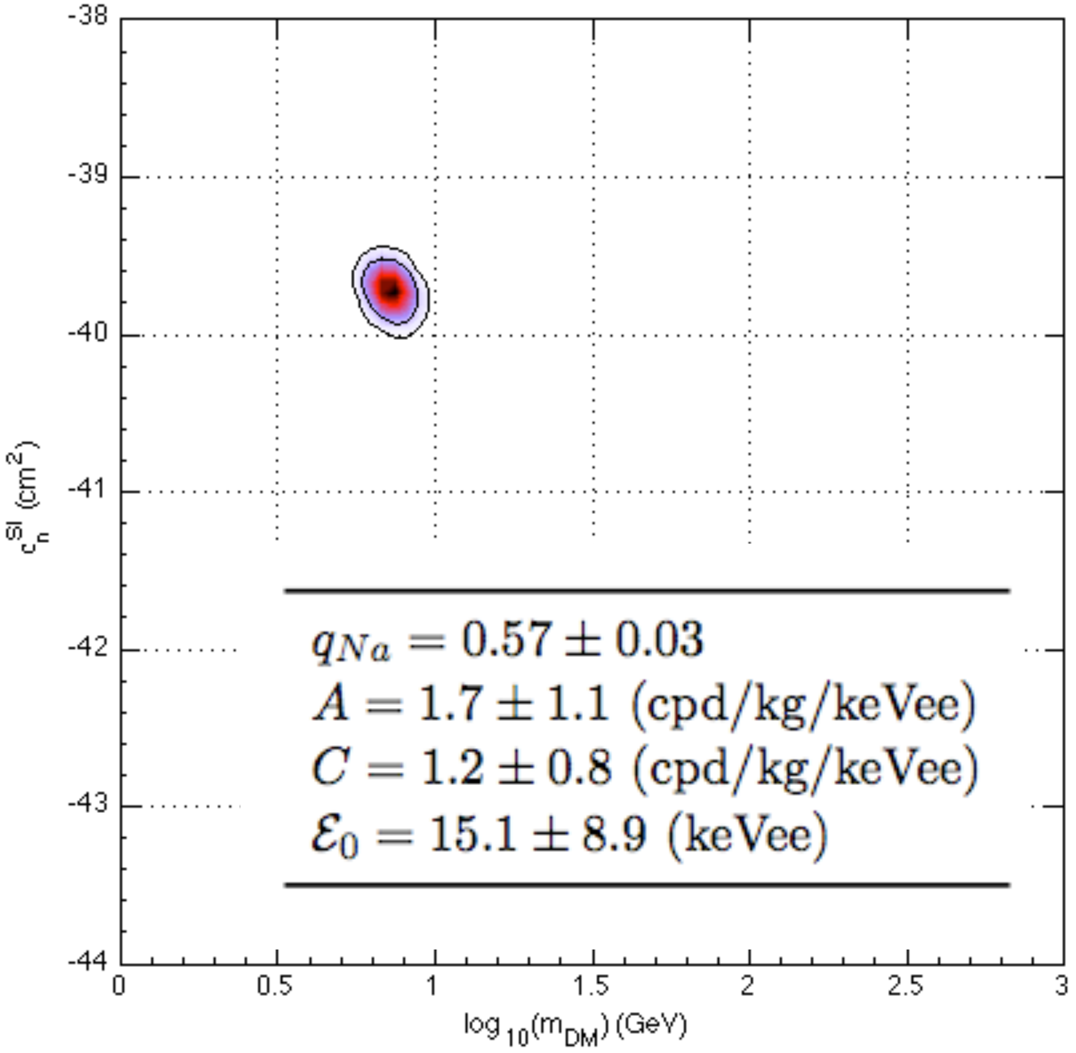
$v_0 \quad v_{\rm esc} \quad \rho_\odot \quad M_{\rm vir} \quad c_{\rm vir}$

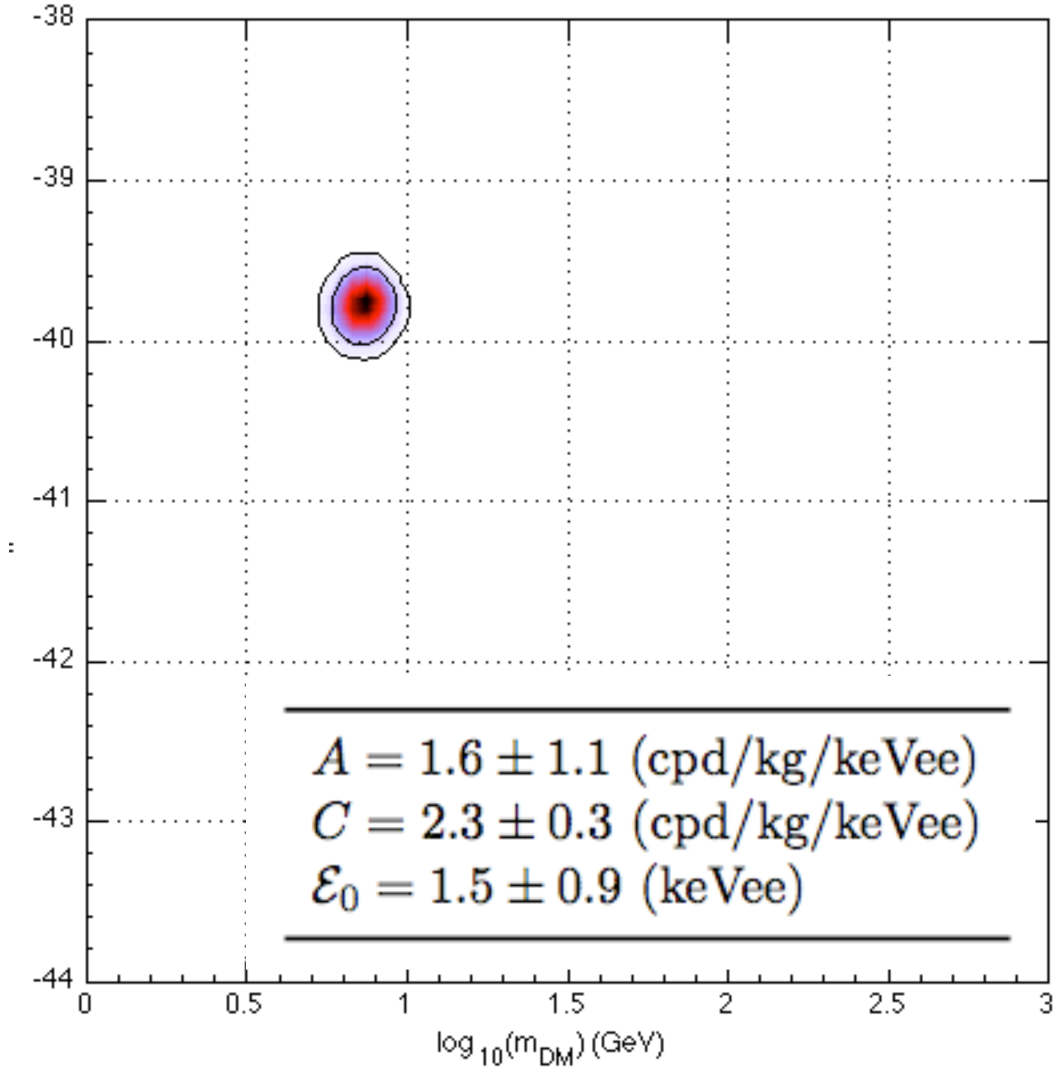


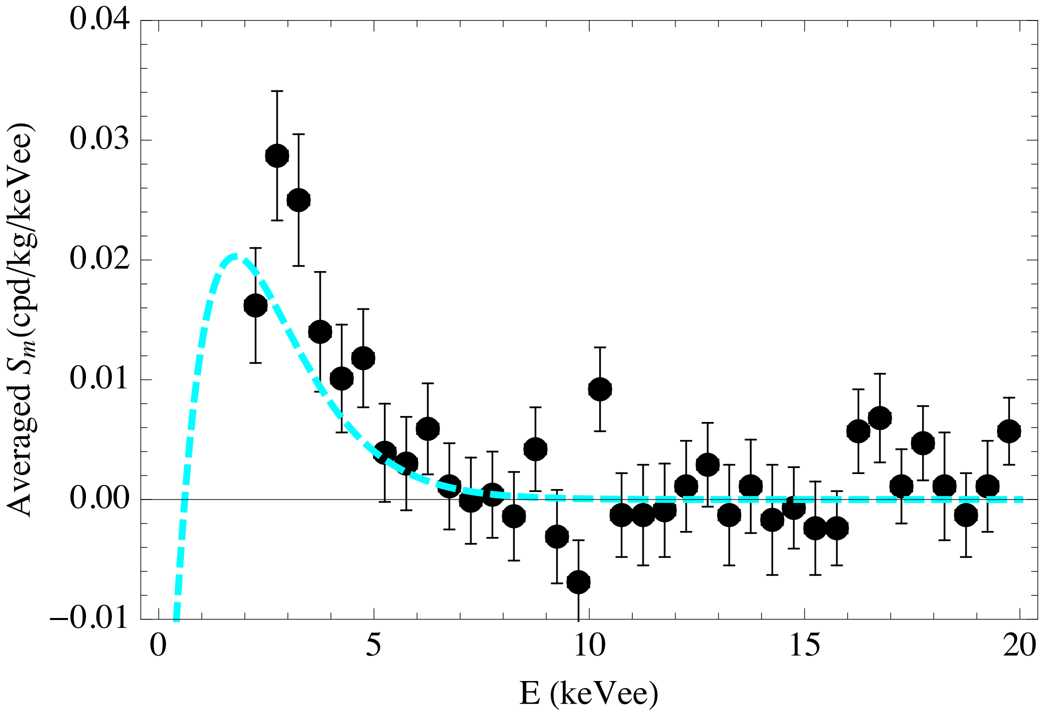
	$\mathcal{M}_i : \mathcal{M}_0$		
Model	DAMA	CoGeNT	Combined Fit
NFW ($\mathcal{M}_1$)	$\ln B = -5.27$	$\ln B = -3.99$	$\ln B = 32$
	$1 : 194$	$1 : 54$	$\exp(32) : 1$

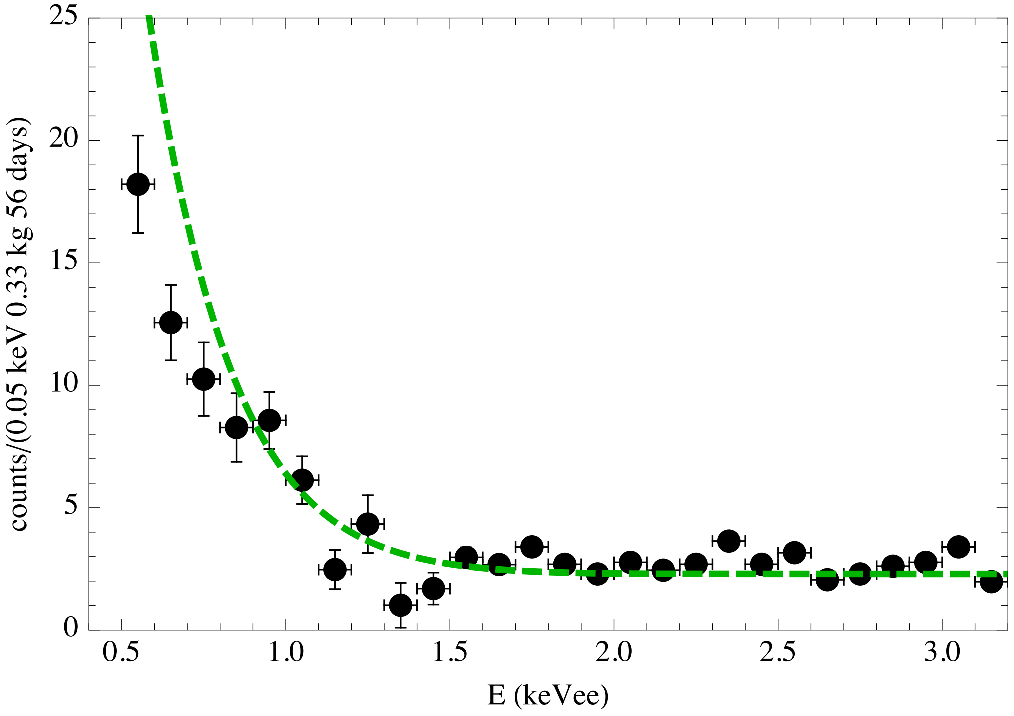
- Single experiment fit: moderate to strong evidence against inclusion of astrophysics
- A single direct detection experiment can not constrain astrophysical DM models
- Combined fit: very strong evidence for inclusion of astrophysics
- Combined experiments need astrophysical parameters for compatibility

More on combined fit of DAMA and CoGeNT

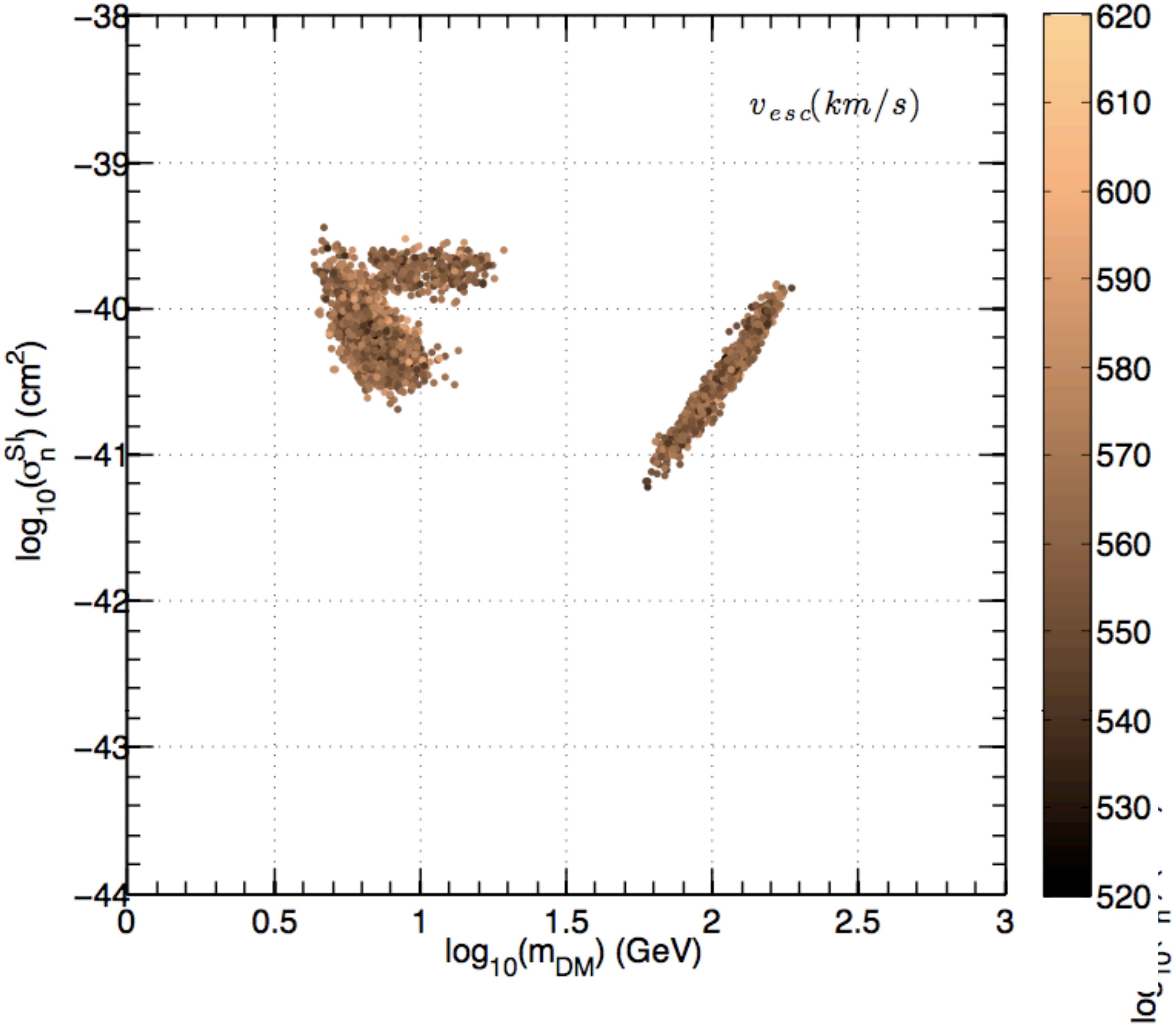


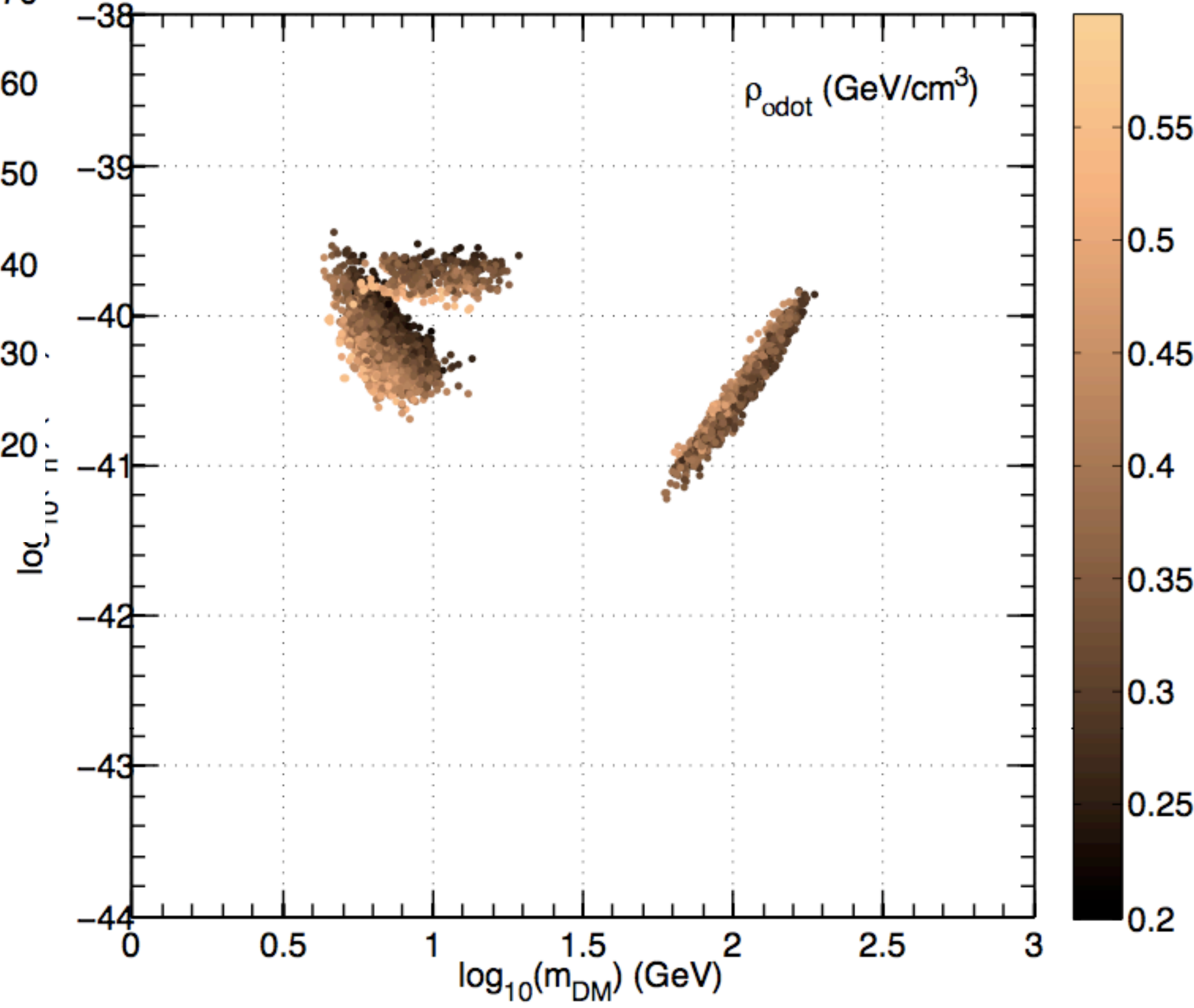






DAMA and CoGeNT, combined fit





Velocity distribution from DM density profile

Assuming equilibrium between gravitational force and pressure:

$$F(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\left[\int_0^\varepsilon \frac{\mathrm{d}^2\rho_{\mathrm{DM}}}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\varepsilon-\Psi}} + \frac{1}{\sqrt{\varepsilon}}\left(\frac{\mathrm{d}\rho_{\mathrm{DM}}}{\mathrm{d}\Psi}\right)\Big|_{\Psi=0}\right]$$

Eddigton formula for spherically symmetric DM density profiles that lead to isotropic f(v)

Poisson equation for the gravitational potential including contribution from the bulge and disk:

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\Psi}{\mathrm{d}r} = -4\pi G[\rho_{\mathrm{DM}} + \rho_{\mathrm{disk}} + \rho_{\mathrm{bulge}}]$$

$$\rho_{\mathrm{DM}}(r) = \rho_s\left(\frac{r}{r_s}\right)^{-1}\left(1+\left(\frac{r}{r_s}\right)\right)^{-2}$$ NFW

$$\rho_{\mathrm{disk}}(r) = \frac{M_{\mathrm{disk}}}{4\pi r_{\mathrm{disk}}^2}\frac{e^{-r/r_{\mathrm{disk}}}}{r}$$

$$\rho_{\mathrm{bulge}}(r) = M_{\mathrm{bulge}}\delta_D^{(3)}(\vec{r})$$

The velocity distribution is translated to the reference frame of the Earth:

$$\int_{v'>v'_{\mathrm{min}}} \mathrm{d}^3v'\,\frac{f(\vec{v'}(t))}{v'} \rightarrow 2\pi\rho_\odot^{-1}\int_{v'>v'_{\mathrm{min}}}\mathrm{d}v'\,v'\int_{-1}^{1}\mathrm{d}\alpha\,F\Big(\Psi_\odot - \frac{1}{2}v^2\Big)$$

$$v_0 \equiv \sqrt{-r\frac{\mathrm{d}\Psi}{\mathrm{d}r}}\Big|_{r=R_\odot}$$

$$v^2 = |\vec{v'} + \vec{v}_\oplus|^2 = v'^2 + v_\oplus^2 + 2v'v_\oplus\alpha\,,$$

$$v_\oplus = |\vec{v}_\odot + \vec{v''}_{\oplus,\mathrm{rot}}| = v_\odot + v''_{\oplus,\mathrm{rot}}\cos\gamma\cos[2\pi(t-t_0)/T]$$

$$v_{\mathrm{esc}} = \sqrt{2\Psi}\Big|_{r=R_\odot}$$

DM density profiles

$$r_s(M_{\rm vir}, c_{\rm vir}) = \frac{r_{\rm vir}(M_{\rm vir})}{c_{\rm vir}} \qquad M_{\rm vir} = 4\pi \int_0^{r_{\rm vir}} {\rm d}r \; r^2 \rho_{\rm DM}(r) = \frac{4}{3}\pi r_{\rm vir}^3 \delta_c \rho_{\rm crit}$$

Cored isothermal	$\rho_{\rm DM}(r) = \rho_s \left[1 + \left(\frac{r}{r_s}\right)^2\right]^{-1}$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{c_{\rm vir} - \tan^{-1}(c_{\rm vir})}$
Navarro–Frenk–White (NFW)	$\rho_{\rm DM}(r) = \rho_s \left(\frac{r}{r_s}\right)^{-1} \left(1 + \left(\frac{r}{r_s}\right)\right)^{-2}$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{\ln(1 + c_{\rm vir}) - c_{\rm vir}/(1 + c_{\rm vir})}$
Einasto	$\rho_{\rm DM}(r) = \rho_s \exp\left(-\frac{2}{a}\left[\left(\frac{r}{r_s}\right)^a - 1\right]\right)$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3 [2^{-\frac{3}{\alpha}} \exp(\frac{2}{\alpha}) \alpha^{\frac{3}{\alpha} - 1}]^{-1}}{\Gamma\left(\frac{3}{\alpha}\right) - \Gamma\left(\frac{3}{\alpha}, \frac{2c_{\rm vir}^\alpha}{\alpha}\right)}$
Burkert	$\rho_{\rm DM}(r) = \rho_s \left(1 + \frac{r}{r_s}\right)^{-1} \left(1 + \frac{r}{r_s}\right)^{-2}$ $\rho_s(c_{\rm vir}) = \frac{4\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{2\ln(1 + c_{\rm vir}) + \ln(1 + c_{\rm vir}^2) - 2\tan^{-1}(c_{\rm vir})}$

CDMS Si

- 2 events seen, likelihood follows a Poisson distribution
- expected background B = 4.4 (Be = 0.8, Bn = 3.6, B=Be+Bn)
- exposure of 65.8 kg days
- energy range from 5 -> 100 keV

$$\ln \mathcal{L}_{\rm CDMSSi}(2|S,B) = -S - B + 2 + 2\ln\left(\frac{S+B}{2}\right)$$

Analytical marginalization over the background:

$$\mathcal{L}^{\rm eff}_{\rm CDMSSi}(2|S) = \int_0^\infty {\rm d}B\ \mathcal{L}_{\rm CDMSSi}(2|S,B)\ p(B)$$

$$p(B) = \frac{1}{\sqrt{2\pi\sigma_B^2}}\exp\left[-\frac{(B-\bar{B})^2}{2\sigma_B^2}\right]$$

$$\bar{B} \pm \sigma_B = 4.4 \pm 0.6,$$

$$\Delta\chi^2_{\rm eff} \leq 4.2$$

$$S \leq 3.3$$

$$\ln \mathcal{L}^{\rm eff}_{\rm CDMSSi} = -S - \bar{B} + \frac{\sigma_B^2}{2} + 2 + \ln\left[\frac{\sigma_B^2 + (S + \bar{B} - \sigma_B^2)^2}{4}\right]$$

CDMS Ge

- 2 events seen, likelihood follows a Poisson distribution
- exposure of 1063.2 kg days (all runs combined)
- expected background B=1.38 +- 0.38, analytical marginalization
- energy range from 10 -> 100 keV
- used spectral information

$$\ln \mathcal{L}_{\rm CDMSGe} = -S - B + 2 + \sum_{i=1,2} \ln\left(\frac{\rm dR}{\rm dE_i} + \frac{B}{\bar{B}}\frac{{\rm d}N_B}{{\rm d}E_i}\right) + C_{\rm norm}$$

$$\begin{aligned} E_{1,2} &= 12.3, 15.5 \ \ {\rm keVnr} \\ C_{\rm norm} &= \textstyle\sum_{i=1,2} \ln[M_{\rm det} T \epsilon(qE_i)] \end{aligned}$$

$$\frac{{\rm d}N_B}{{\rm d}E} = \left[-0.00295 + 0.463\left(\frac{\rm keVnr}{E}\right)\right] / (612 \ {\rm kg \ days})$$

$$\begin{aligned} \ln \mathcal{L}^{\rm eff}_{\rm CDMSGe} = & -S - \bar{B} + \frac{\sigma_B^2}{2} + 2 + C_{\rm norm} + \\ & \ln\left[\prod_{i=1,2}\left(\frac{{\rm d}R}{{\rm d}E_i} + \frac{\bar{B} - \sigma_B^2}{\bar{B}}\frac{{\rm d}N_B}{{\rm d}E_i}\right) + \sigma_B^2 \prod_{i=1,2} \frac{1}{\bar{B}}\frac{{\rm d}N_B}{{\rm d}E_i}\right] \end{aligned}$$

$90_S\%$ $\quad \Delta\chi^2_{\rm eff} \leq 3.0$

$99_S\%$ $\quad \Delta\chi^2_{\rm eff} = 7.4$

CDMS Ge low energy

- 2-100 keV energy range
- 462 events combined into 16 bins from 2 -> 10 KeV and 9 from 10 to 100 keV
- 214 kg days

$$\ln \mathcal{L}_{\mathrm{CDMSGe(LE)}} = -\sum_{i=1}^{N_{\mathrm{bin}}} \frac{(s_i - \bar{s}_i^{\mathrm{obs}})^2}{2\sigma_i^2} + \ln \mathcal{L}_{m_{\mathrm{B}}}$$

Background due to surface events, leakage events and zero-charge events is extrolated below 5 KeV -> nuisance parameter

$$\ln \mathcal{L}_{m_{\mathrm{B}}} = -\frac{(a-\bar{a})^2}{2\sigma_a^2}$$

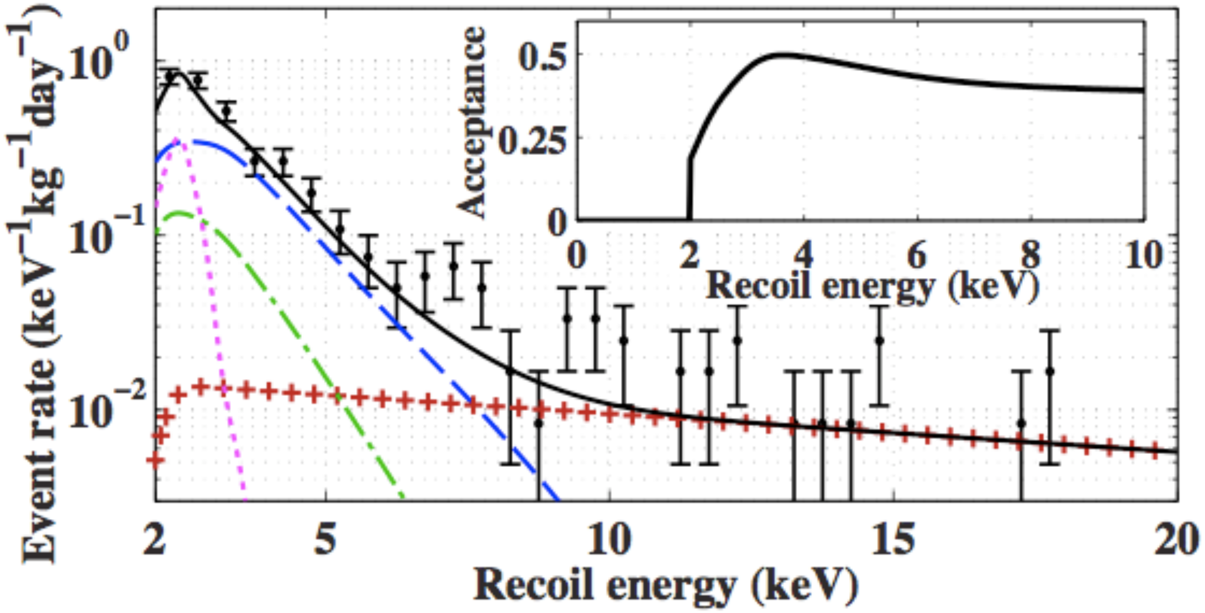
prior range flat over: $-0.60 \rightarrow -0.18$

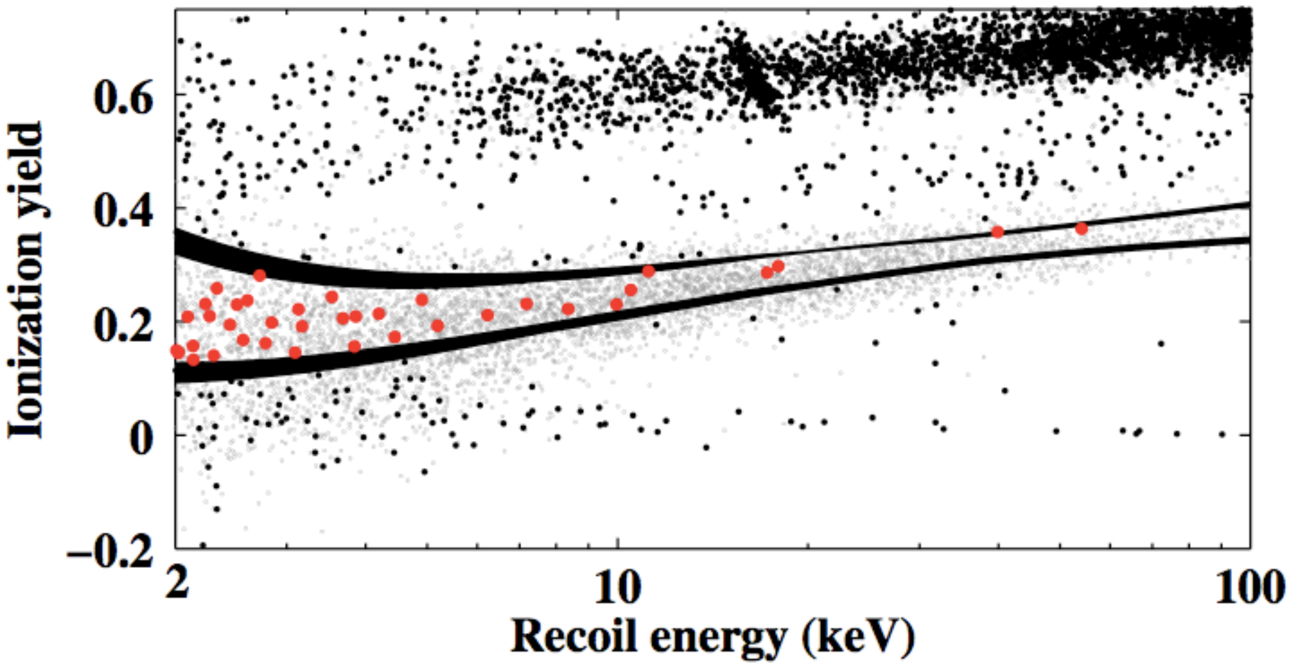
$$s_i = \frac{1}{\Delta E_i} \int_{E_i - \Delta E_i/2}^{E_i + \Delta E_i/2} \mathrm{d}E \left[\frac{\mathrm{d}R}{\mathrm{d}E} + \mathrm{m_B}(E) \right]$$

$$\mathrm{m_B}(E) = \begin{cases} \bar{\mathrm{m}}_{\mathrm{B}}(E), & E \geq 5 \ \mathrm{keVnr}, \\ 0.1 \times 10^{a[(E/\mathrm{keVnr})-5]}, & 2 < E/\mathrm{keVnr} < 5. \end{cases}$$

$$90_S\% \quad \Delta\chi^2_{\mathrm{eff}} < 4.6$$

arXiv:1011.2482





CoGeNT 2011

Germanium cryogenic detector
detector mass 0.33 kg
live time 442 days
total exposure 145.86 kg days

- Data analysis and binning follow arXiv:1106.0650 [astro-ph.CO]
- Radioactive peaks subtracted as prescribed by the collaboration
- Analysis of the total rate with a background (27 bins)
- Analysis of the modulated rate without background in 3 energy bins
- All data are corrected by the efficiency factor, ranging from 0.7 to 0.82

$$\ln \mathcal{L}_{\mathrm{TR}} = -\frac{\chi^2}{2} = -\sum_{i=1}^{27} \frac{((S_i + b_i) - C_i)^2}{2\sigma_i^2}$$

$$\ln \mathcal{L}_{MR} = -\frac{\chi^2}{2} = -\sum_{j=1}^{3} \frac{(S^i_{\mathrm{theo}} - S^i_{\mathrm{m}})^2}{2\sigma_i^2}$$

Total rate : 27 bins of width 0.1 keVee
energy range 0.5- 3.2 keVee

Modulated rate:

ΔE_i (keVee)	S_m (cpd/kg/keVee)
$0.5 - 0.9$	1.10 ± 0.39
$0.9 - 3.0$	0.60 ± 0.12
$3.0 - 4.5$	0.07 ± 0.9

3 nuisance parameters for the non modulating background

$$b_i = \frac{1}{\Delta_b} \int_{\mathcal{E}_i}^{\mathcal{E}_{i+1}} \frac{\mathrm{d}B}{\mathrm{d}\mathcal{E}} \mathrm{d}\mathcal{E}$$

$$\frac{\mathrm{d}B}{\mathrm{d}\mathcal{E}} = C + A \exp(-\mathcal{E}/\mathcal{E}_0)$$

Experiment	Parameter	Prior
CoGeNT	C	$0 \to 10$ cpd/kg/keVee
CoGeNT	$\mathcal{E}_0$	$0 \to 30$ keVee
CoGeNT	A	$0 \to 10$ cpd/kg/keVee

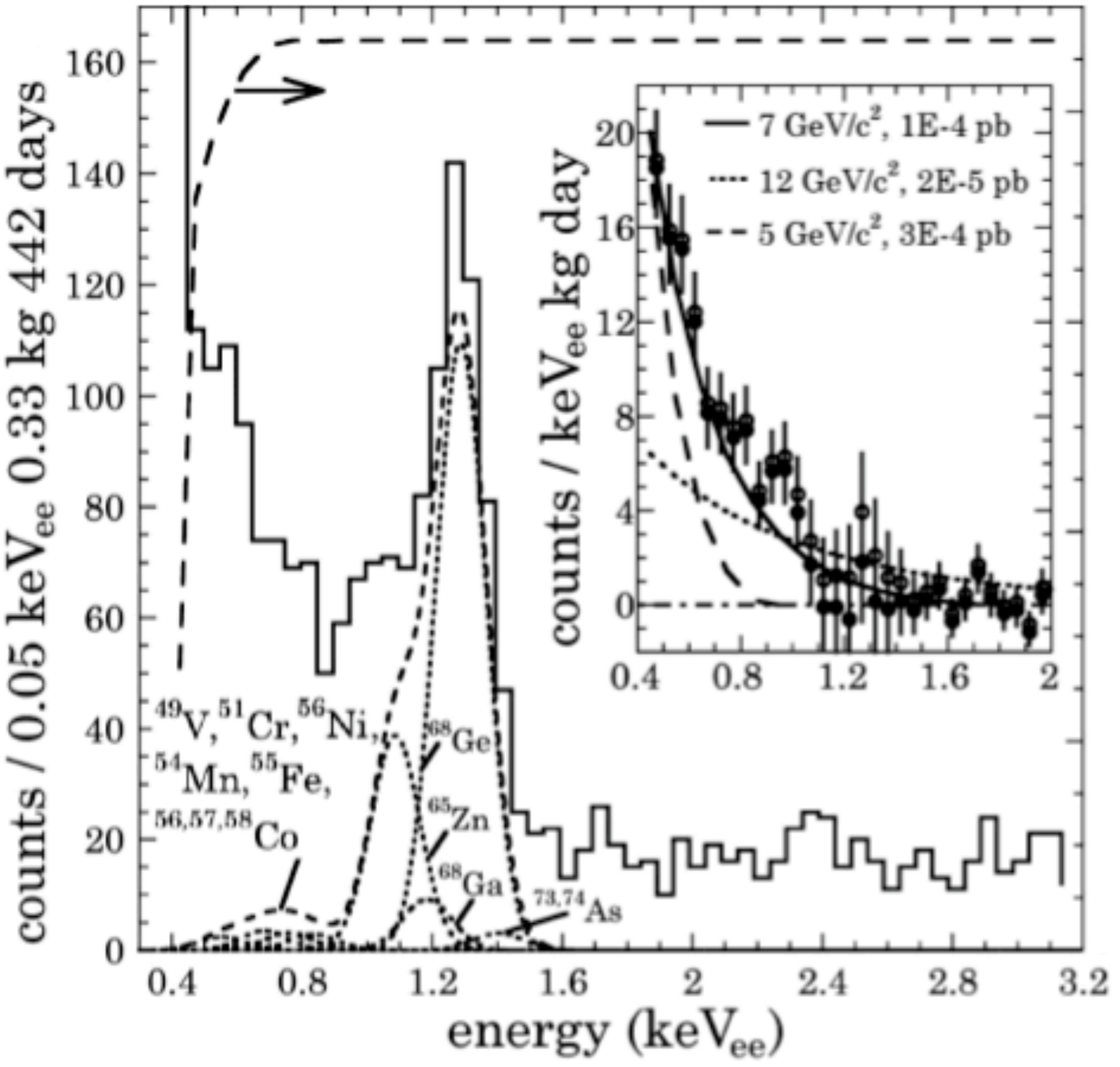
quenching factor: $\mathcal{E}(\mathrm{keVee}) = 0.19935 \times E^{1.1204}(\mathrm{keVnr})$

CoGeNT 2011

Data analysis

Radioactive peaks

arXiv:1106.0650



Element	$\mathcal{E}_p$ (keVee)	σ_p (keVee)	$\tau_{1/2}$ (days)	N_0
^{73}As	1.414	0.077	80.	12.7
^{68}Ge	1.298	0.077	271.	638.9
^{68}Ga	1.194	0.076	271.	52.8
^{65}Zn	1.096	0.075	244.	211.2
^{56}Ni	0.926	0.075	5.9	1.53
^{56,58}Co	0.846	0.074	71.	9.44
^{57}Co	0.846	0.074	271.	2.59
^{55}Fe	0.769	0.074	996.	44.9
^{55}Mn	0.695	0.073	312.	21.1
^{51}Cr	0.628	0.073	28.	2.93
^{49}V	0.564	0.073	330.	14.9

$$P^A_{\mathrm{rad}}(\mathcal{E}_{\mathrm{min}}, \mathcal{E}_{\mathrm{max}}) = \int_{\mathcal{E}_{\mathrm{min}}}^{\mathcal{E}_{\mathrm{max}}} \mathrm{Gaussian}(\mathcal{E}, \mathcal{E}_p, \sigma_p)\mathrm{d}\mathcal{E}$$

$$D^A(t_1, t_2) = \left(\exp(-\frac{\ln 2}{\tau_{1/2}} t_1) - \exp(-\frac{\ln 2}{\tau_{1/2}} t_2) \right)$$

$$N^A_{\mathrm{tot}}(\mathcal{E}_{\mathrm{min}}, \mathcal{E}_{\mathrm{max}}, t_1, t_2) = N_0 P^A_{\mathrm{rad}}(\mathcal{E}_{\mathrm{min}}, \mathcal{E}_{\mathrm{max}}) D^A(t_1, t_2)$$

Theoretical predictions for elastic spin-independent scattering off nucleus

Differential rate

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho_\odot}{m_{\mathrm{DM}}} \int_{v'>v'_{\mathrm{min}}} \mathrm{d}^3v' \, \frac{\mathrm{d}\sigma}{\mathrm{d}E} \, v' \, f(\vec{v'}(t))$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{M_{\mathcal{N}} \sigma_n^{\mathrm{SI}}}{2\mu_n^2 v'^2} \frac{\left(f_p Z + (A-Z) f_n\right)^2}{f_n^2} \mathcal{F}^2(E)$$

$$\mathcal{E} = qE$$

$$S(t) = M_{\mathrm{det}} T \int_{\mathcal{E}_1/q}^{\mathcal{E}_2/q} \mathrm{d}E \, \epsilon(qE) \, \frac{\mathrm{d}R}{\mathrm{d}E}$$

Modulated rate

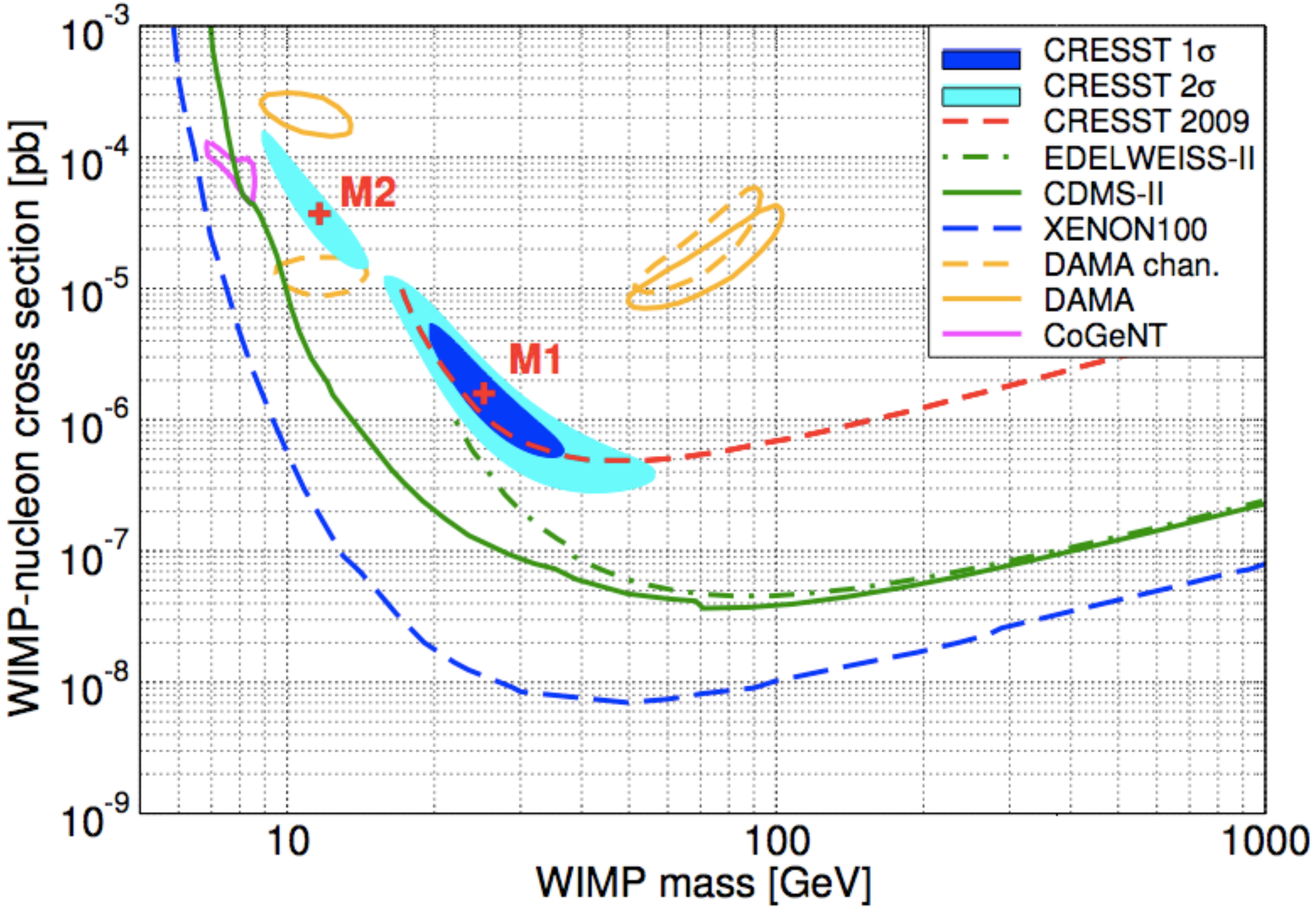
$$s = \frac{1}{\mathcal{E}_2 - \mathcal{E}_1} \sum_{X=\mathrm{Na,I}} w_X \int_{\mathcal{E}_1/q_X}^{\mathcal{E}_2/q_X} \mathrm{d}E \, \frac{1}{2} \left[\frac{\mathrm{d}R_X}{\mathrm{d}E}(\mathrm{June}\ 2) - \frac{\mathrm{d}R_X}{\mathrm{d}E}(\mathrm{Dec}\ 2) \right]$$

$$S_{\mathrm{m\%}} = \frac{R(\mathrm{June2}) - R(\mathrm{Dec2})}{R(\mathrm{June2}) + R(\mathrm{Dec2})}$$

CRESST-II

Angloher et al., arXiv:1109.0702

- 8 detector module made by CaWO4 crystals
- energy range 8/12 keV - 40 KeV
- scintillation + ionization to disentangle background (e, n, alpha, decays of Pb isotopes)
- exposure of 730 kg days with N = 67 events (background can account only for 65% of N)
- profile likelihood analysis, evidence for a signal at 4 sigma



- The exclusion limit from the CRESST commissioning run on W should be take into account as well (Brown et al. arXiv:1109.2589)

Results for various DM halos