The Ioffe-Regel criterion and diffusion of vibrations in random lattices

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We consider diffusion of vibrations in $3d$ random lattices with translational invariance. Above frequency $\omega_{\rm IR}$ corresponding to the Ioffe-Regel crossover notion of phonons becomes meaningless. They cannot propagate through the lattice and transfer the energy. Nevertheless most of the vibrations above this frequency are delocalized. We show that they are similar to diffusons introduced by Allen, Feldman et al., Phil. Mag. B **79**, 1715 (1999) for glasses. The frequency $\omega_{\rm IR}$ correlates with position of the boson peak. Depending on the rigidity of the lattice $\omega_{\rm IR}$ can vary from zero value (when rigidity is zero and there are no phonons) to the typical frequency in the system. We show that above $\omega_{\rm IR}$ the energy in the lattice is transferred by means of diffusion of vibrational excitations. At zero rigidity a structure factor of particle displacements $S(\mathbf{q},\omega)$ coincides well with a structure factor of a random walk on a lattice. We have calculated the diffusivity $D(\omega)$ of the modes using formula of Edwards and Thouless. It is nearly a constant above $\omega_{\rm IR}$ and goes to zero at the localization threshold. The results are in a good agreement with a direct numerical solution of the Newton equations for a sample with 10^6 particles. Our findings may have important consequences for understanding mechanisms of the heat transfer in glasses.

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