Nonperturbative dynamics of quantum fields in de Sitter spacetime

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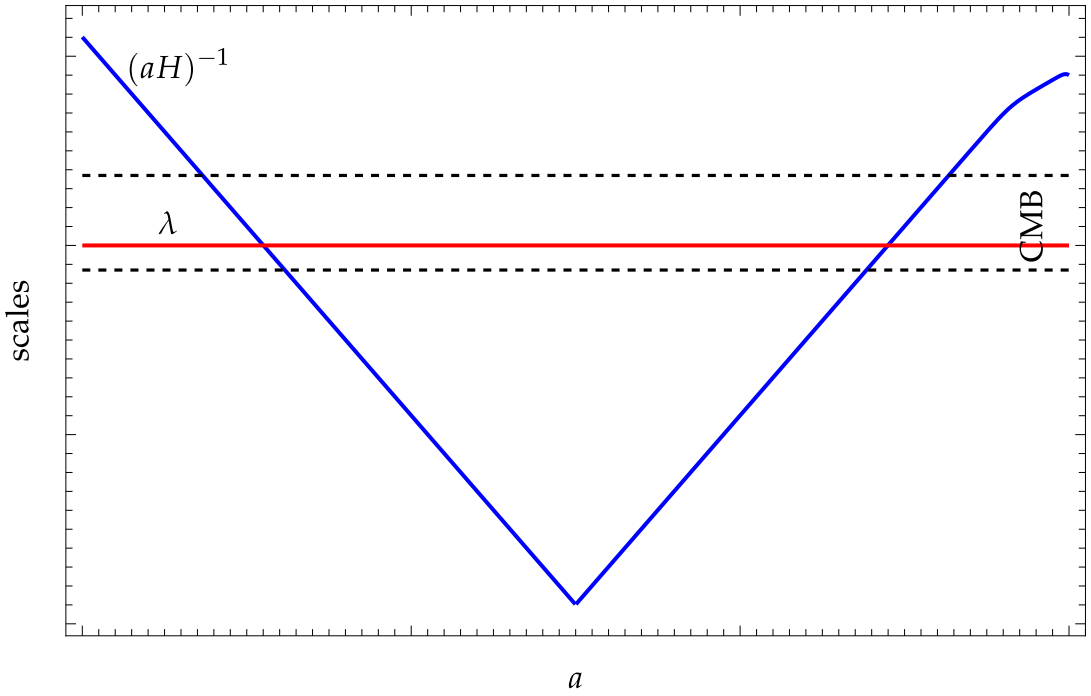
Backreaction

Unequal-time correlators

General conclusion and perspective

Quantum fields in curved spacetime and inflation

- Cosmological standard model $\rightarrow$ horizon and flatness problems
- Solved by inflation - quasi-de Sitter phase in the early universe
- It can be generated simply by a scalar field in slow roll
- The quantum fluctuations of the scalar field (and metric perturbations) are observed to the CMB



Quantum fields in curved spacetime and inflation

- ⋈ Standard approach is **perturbative**
- ⋈ What about higher order corrections ?
- ⋈ We follow a **semi-classical** approach

Nonperturbative dynamics of scalar fields in de Sitter

→ Explain the success of the standard approach

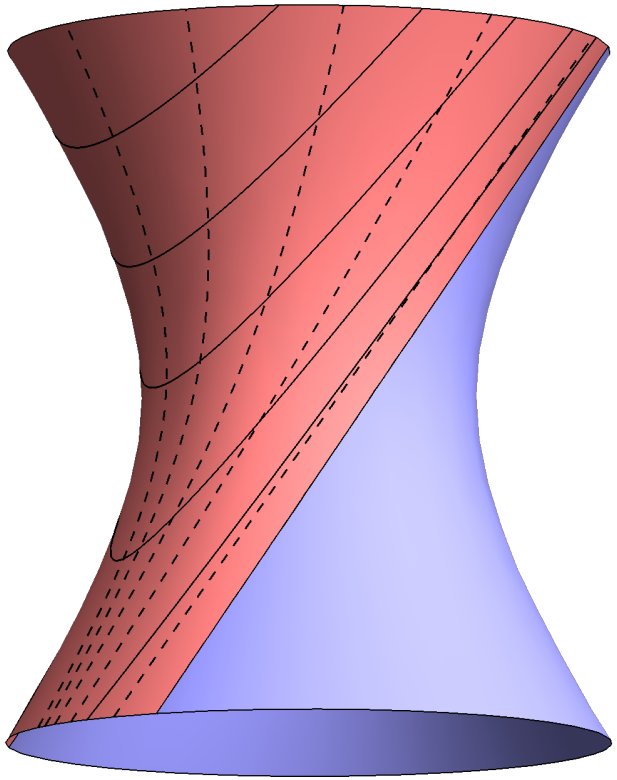
→ Use cosmological experiments to test fundamental quantum physics

→ Explore unknown effects for spectator fields

De Sitter spacetime

Maximally symmetric solution of Einstein equations with a positive cosmological constant

- We will consider the **Expanding Poincaré patch** $\rightarrow$ FLRW with constant Hubble rate H
- Scalar curvature $R \propto H^2$
- $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$
  with $a(t) = e^{Ht}$
- Conformal time, $\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}$
  $\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$
- Spacetime dimension denoted as $D = d + 1$



Free scalar field in de Sitter

Scalar **spectator** in a **semi-classical** approach

With the action

$$S = \int \mathrm{d}^D x \sqrt{-g} \left(\frac{1}{2} \phi \Box \phi - \frac{m^2}{2} \phi^2 \right)$$

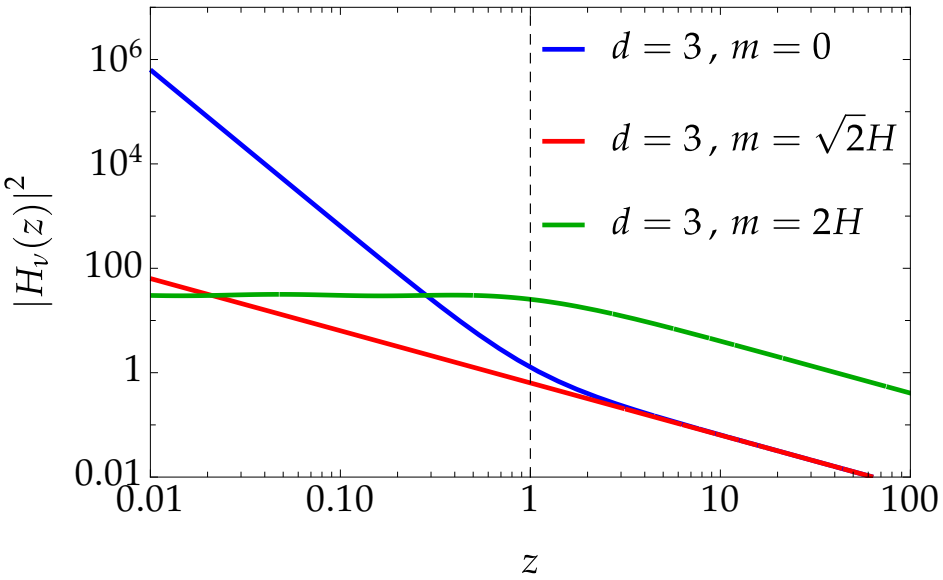
We get the Klein-Gordon equation $(-\Box + m^2)\phi = 0$ where in conformal time

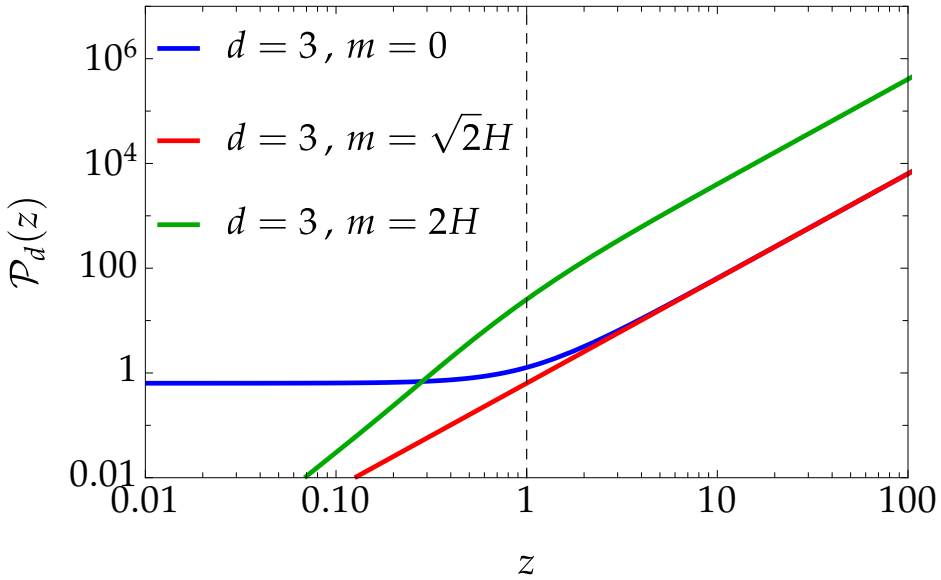
$$\Box = \frac{1}{a^2(\eta)} \left(-\partial_\eta^2 + \frac{2}{\eta} \partial_\eta + \partial_{\vec{x}}^2 \right)$$

It gives for the mode decomposition of ϕ, in the Bunch-Davies vacuum,

$$\phi(\eta, \vec{x}) \sim \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\boxed{H_\nu \left(\frac{k}{a(\eta) H} \right)} e^{i\vec{k}\cdot\vec{x}} a_k + \text{h.c.} \right), \quad \nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$$

Free scalar field in de Sitter





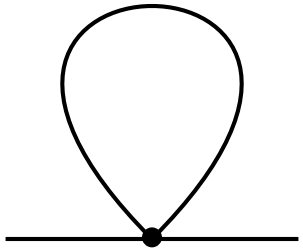
The power spectrum is a function of the physical momentum $p = k/a(\eta)$,

$$\begin{cases} \mathcal{P}_d(p) \sim H^{2\nu-1} p^{d-2\nu} & \text{for} \quad p \ll H\,, \\ \mathcal{P}_d(p) \sim p^{d-1} & \text{for} \quad p \gg H\,, \end{cases}$$

IR amplification can be interpreted as particle production, in analogy with the Schwinger effect for charged particles in a background electric field.

Infrared divergences

For a light field, $m \ll H$



$$G(x,x) \sim \frac{1}{d-2\nu} \sim \frac{H^2}{m^2}$$

$$\text{as} \quad \nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}} \approx \frac{d}{2}$$

infrared divergence as $m \to 0$

Also **secular** divergences appearing as large logarithms $\sim \log p/H$

The infrared amplification leads to infrared and secular divergences
$\to$ signals nontrivial physics missed by perturbation theory

N. C. Tsamis, R. P. Woodard '05 ; S. Weinberg '05 ; '06

Nonperturbative treatments

Variety of nonperturbative treatments :

- △ Stochastic formalism
  A. A. Starobinsky '86 ; A. A. Starobinsky, J. Yokoyama '94
- △ Dynamical RG
  C. P. Burgess et al. '10
- △ Schwinger-Dyson equations
  B. Garbrecht, G. Rigopoulos '11 ; F. Gautier, J. Serreau '13 ; '15
- △ Functional renormalization group (FRG)
  J. Serreau '13 ; A. Kaya '13 ; M. Guilleux, J. Serreau '15
- △ Euclidean de Sitter
  A. Rajaraman '10 ; M. Beneke, P. Moch '13 ; D. López Nacir et al. '19
- △ $1/N$ expansion
  F. D. Mazzitelli, J. P. Paz '89 ; A. Riotto, M. Sloth '08 ; J. Serreau '11

Two topics

Backreaction
of an interacting $O(N)$ scalar theory

Unequal-time correlators
in nonperturbative regimes

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Introduction

Backreaction

Unequal-time correlators

General conclusion and perspective

Stability of de Sitter

The nontrivial infrared physics of scalar fields in de Sitter gives a possible dynamical mechanism to solve the cosmological constant problem.

E. Mottola '85 ; I. Antoniadis et al. '86 ; N. C. Tsamis, R. P. Woodard '93 ; R. H. Brandenberger et al. '97 ; A. M. Polyakov '10 ; '12 ; D. Krotov, A. M. Polyakov '11

Problem : including metric perturbation is hard (gauge ambiguity, breaking of de Sitter isometries)

We study the backreaction in a simplified setup without metric fluctuations, using the FRG to compute how the metric is renormalized by the quantum fluctuations

$\rightarrow$ preserves de Sitter isometries

$\rightarrow$ nonperturbative approximation scheme

Functional renormalization group (FRG)

Effective action formalism :

$$e^{-i\mathcal{W}[j,g]} = \int \mathcal{D}\hat{\phi} e^{iS[\hat{\phi},g] - i\int j\hat{\phi}}, \quad \Gamma[\phi,g] = \mathcal{W}[j,g] - j\cdot\phi$$

$g_{\mu\nu}$ background metric, S action of an $O(N)$ scalar theory.

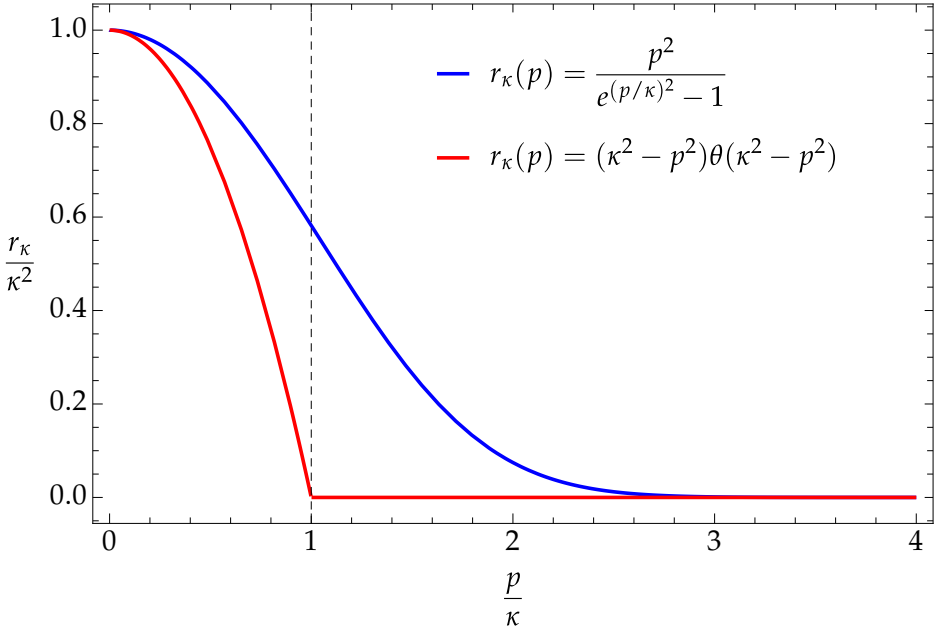
- $\Gamma[\phi,g]$ includes all quantum effects
- classical field $\phi = \langle\hat{\phi}\rangle$

Add a regulator : it defines a **continuum of coarse grained theories**

$$i\Delta S_\kappa[\hat{\phi},g] = i\int_{x,y} R_\kappa(x,y)\hat{\phi}(x)\hat{\phi}(y), \quad \Gamma_\kappa[\phi,g] = \mathcal{W}_\kappa[j,g] - j\cdot\phi - \Delta S_\kappa[\phi,g]$$

Functional renormalization group (FRG)

$$\text{For} \quad R_\kappa(x,y) = \delta(t_x - t_y) \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{i(\vec{r}_x - \vec{r}_y)\cdot\vec{p}} r_\kappa(p)$$



$$\Gamma_{\kappa\to\infty} = S$$

$$\Gamma_{\kappa\to 0} = \Gamma$$

The flow of Γ_κ obeys the **Wetterich equation**, which is IR and UV finite

$$\dot{\Gamma}_\kappa = \frac{i}{2} \operatorname{tr} \dot{R}_\kappa (\Gamma_\kappa^{(2)} + R_\kappa)^{-1}.$$

C. Wetterich '93

$\rightarrow$ Requires further approximations to be solved.

Semiclassical EOMs

The **physical values** for g and ϕ are simultaneously determined at each scale κ through the semiclassical Klein-Gordon and Einstein equations

$$\frac{\delta\Gamma_\kappa}{\delta\phi} = 0, \quad \frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0 \quad \text{or} \quad G^\kappa_{\mu\nu} = \left\langle T^\kappa_{\mu\nu} \right\rangle + \Delta T^\kappa_{\mu\nu}$$

For constant values of ϕ and a de Sitter metric : it gives the **flow of the Hubble constant** (semiclassical Friedmann equation)

Assumptions

We use the following assumptions,

- Infrared regime : $\kappa \ll H_\kappa$
- Small curvature : $m^2_{t/l,\kappa} \ll H^2_\kappa$
- Local potential approximation :

$$\Gamma_\kappa[\phi, H] = -\int \mathrm{d}^D x \sqrt{-g}\left(\frac{1}{2}(\partial\phi)^2 + N U_\kappa(\phi, H)\right)$$

- Discard higher derivative interactions
- Restrict to the flow of the effective potential U_κ
- The effective potential is captured **exactly** (nontrivial)

Flow equation

Using the regulator

$$r_\kappa(p) = H^{-D}(\kappa^2 - p^2)\theta(\kappa^2 - p^2)$$

The flow equation becomes

$$N\dot{U}_\kappa = \beta(m^2_{l,\kappa}, \kappa) + (N-1)\beta(m^2_{t,\kappa}, \kappa).$$

$$\beta(m^2, \kappa) = \frac{H^D}{\Omega_{D+1}} \frac{\kappa^2}{\kappa^2 + m^2}$$

M. Guilleux, J. Serreau '15

with the volume factor $\Omega_n = \dfrac{2\pi^{n/2}}{\Gamma(n/2)}$

Zero dimensional theory

The theory flows towards a zero dimensional theory

J. Serreau '14; M. Guilleux, J. Serreau '15

$$e^{H^{-D}\Omega_{D+1}\mathcal{W}_\kappa(j,h)} = \int \mathrm{d}^N\hat{\phi}\, e^{-H^{-D}\Omega_{D+1}\left(NU_{in}(\hat{\phi},h)+\frac{\kappa^2}{2}\hat{\phi}^2-j\cdot\hat{\phi}\right)}$$

with the initial conditions U_{in} that match the microscopic potential,

- It coincides with the equilibrium probability distribution in the stochastic formalism

  A. A. Starobinsky, J. Yokoyama '94

- It is the **effective theory for the scalar field averaged over a Hubble patch** at constant values of the field

Initial potential

Initial conditions are implemented in U_{in}, using $\chi = \frac{\phi_a^2}{2N}$

$$U_{in}(\chi, H) = a(H) + \mu(H)^2\chi + \frac{\lambda}{2}\chi^2.$$

with

$$a(H) = \alpha - \frac{\beta}{2}H^2 + \frac{\gamma}{4}H^4, \quad \mu(H)^2 = m^2 + \zeta H^2$$

- $\alpha \propto \Lambda$, cosmological constant
- $\beta H^2 \propto R$, Einstein-Hilbert term
- $\gamma H^4 \propto R^2$, does not play any role in $D = 4$

Flow of the physical quantitites

The minimization of the effective action gives in $D = 4$

$$\begin{cases} \phi_\kappa = \langle \hat{\phi} \rangle \\ \dfrac{H_\kappa^2}{4} = \dfrac{\alpha + m^2 \langle \hat{\chi} \rangle + \frac{\lambda}{2} \langle \hat{\chi}^2 \rangle + \kappa^2 (\langle \hat{\chi} \rangle - \chi_\kappa)}{\beta - 2\zeta \langle \hat{\chi} \rangle} \end{cases}$$

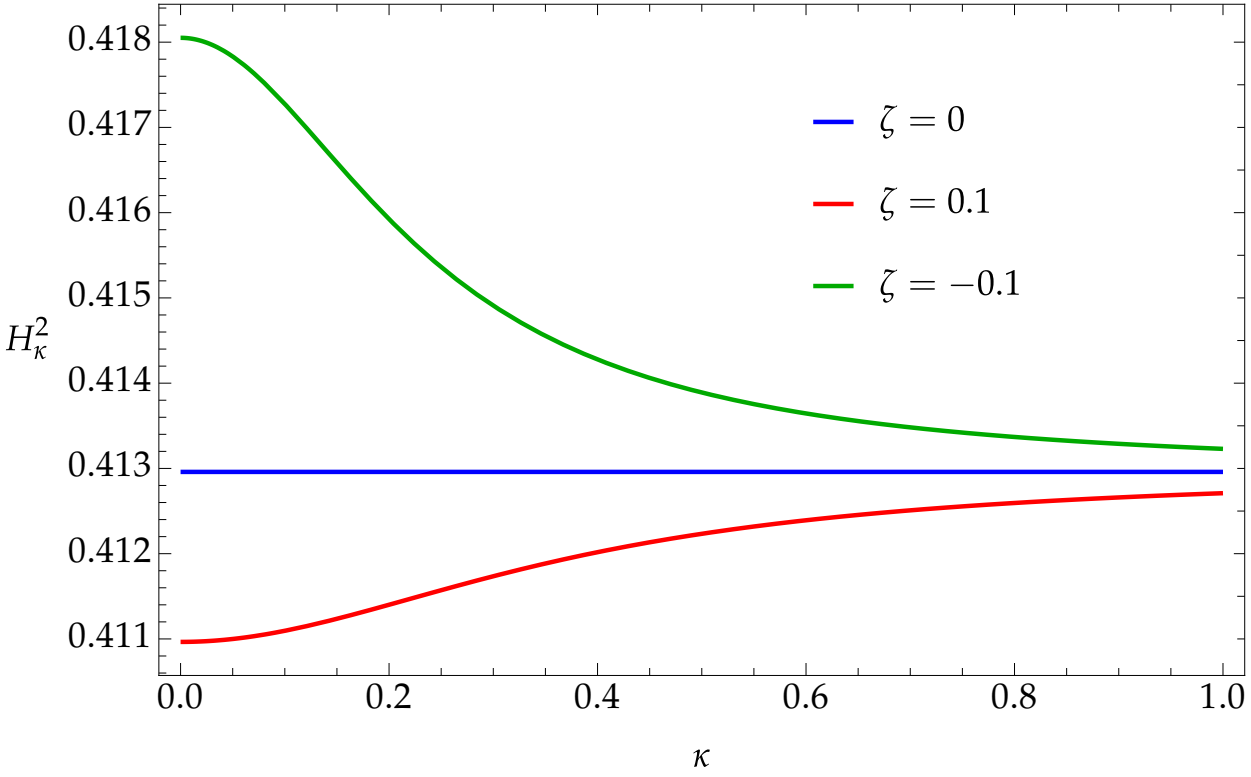
The expectation values are to be computed in the zero dimensional theory.

The classical value is $H_{\mathrm{cl}}^2 = 4\alpha/\beta$

Gaussian case

Consider the case $\lambda = 0$, we get $\chi_\kappa = 0$ and

$$H_\kappa^2 = H_{\rm cl}^2 + \frac{H_\kappa^4}{\beta\Omega}\left(1 + \frac{m^2+\kappa^2}{m^2+\kappa^2+\zeta H_\kappa^2}\right)$$



- minimally coupled case $\zeta = 0$: no flow
- non trivial effect depending on the sign of ζ

Interacting large N case

In the large N regime, we can **solve the flow analytically** while keeping track of the main effects occuring at finite N.

Focus on the massless minimally coupled case : $m^2 = \zeta = 0$.
We find $\chi_\kappa = 0$,

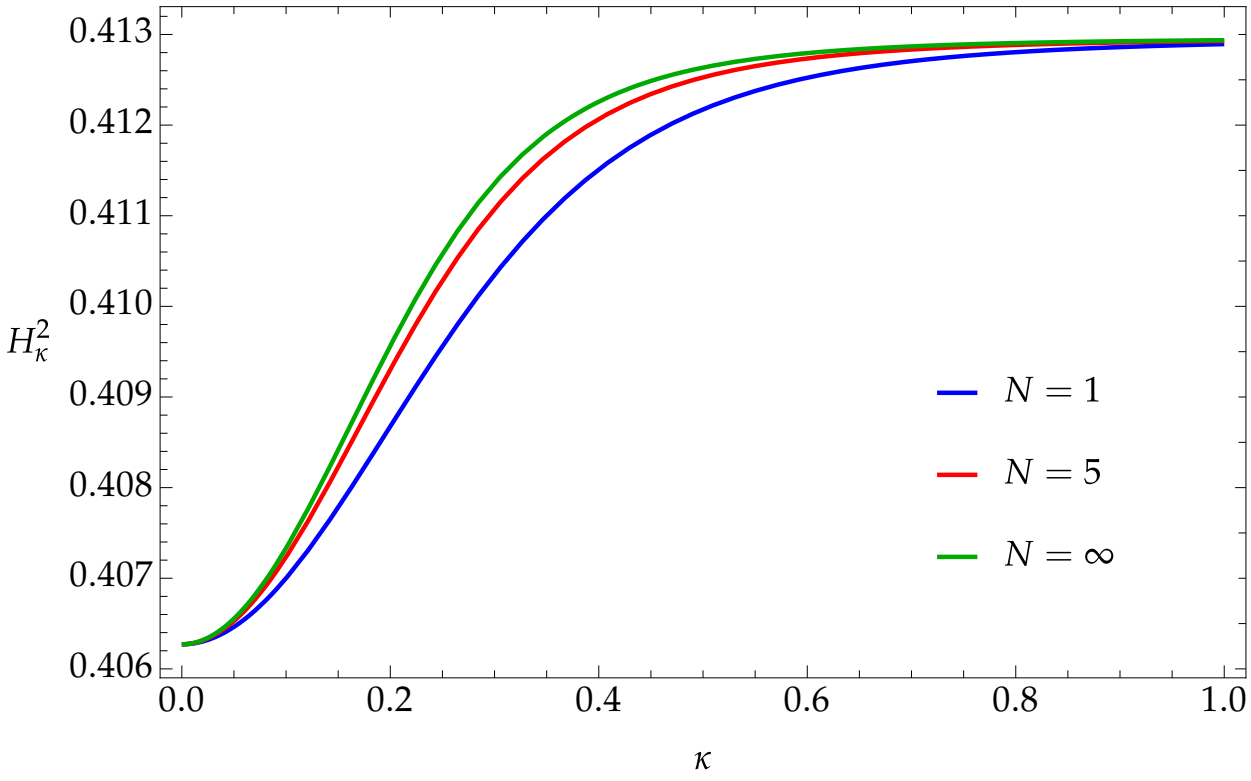
$$H_\kappa^2 = H_{\rm cl}^2 + \frac{H_\kappa^4}{\beta\Omega}\left(1 + \frac{\kappa^2}{m_{t,\kappa}^2 + \kappa^2}\right) \quad \text{with} \quad m_{t,\kappa}^2 = -\frac{\kappa^2}{2} + \sqrt{\frac{\kappa^4}{4} + \frac{\lambda H^4}{2\Omega}}$$

We have **finite asymptotic values**

$$H_\infty^2 \approx H_{\rm cl}^2 + \frac{2H_{\rm cl}^4}{\beta\Omega} \quad \text{and} \quad H_0^2 \approx H_{\rm cl}^2 + \frac{H_{\rm cl}^4}{\beta\Omega}$$

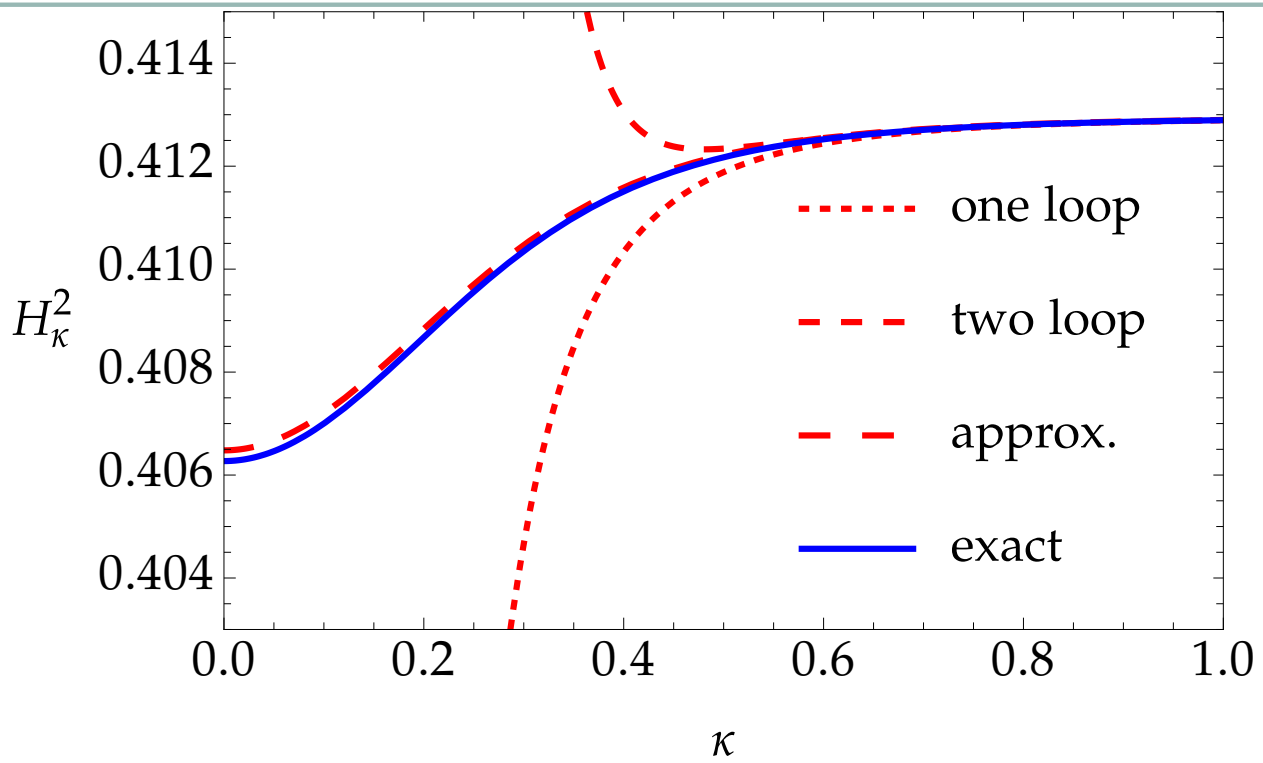
Controlled by $\dfrac{H_{\rm cl}^2}{\beta\Omega} \ll 1$

Interacting large N case



- The superhorizon modes of the massless scalar fields are greatly enhanced, drawing energy from the gravitational field
- The dynamical generation of a mass screens this effect, leading to a finite renormalization of the Hubble constant

Breakdown of perturbation theory

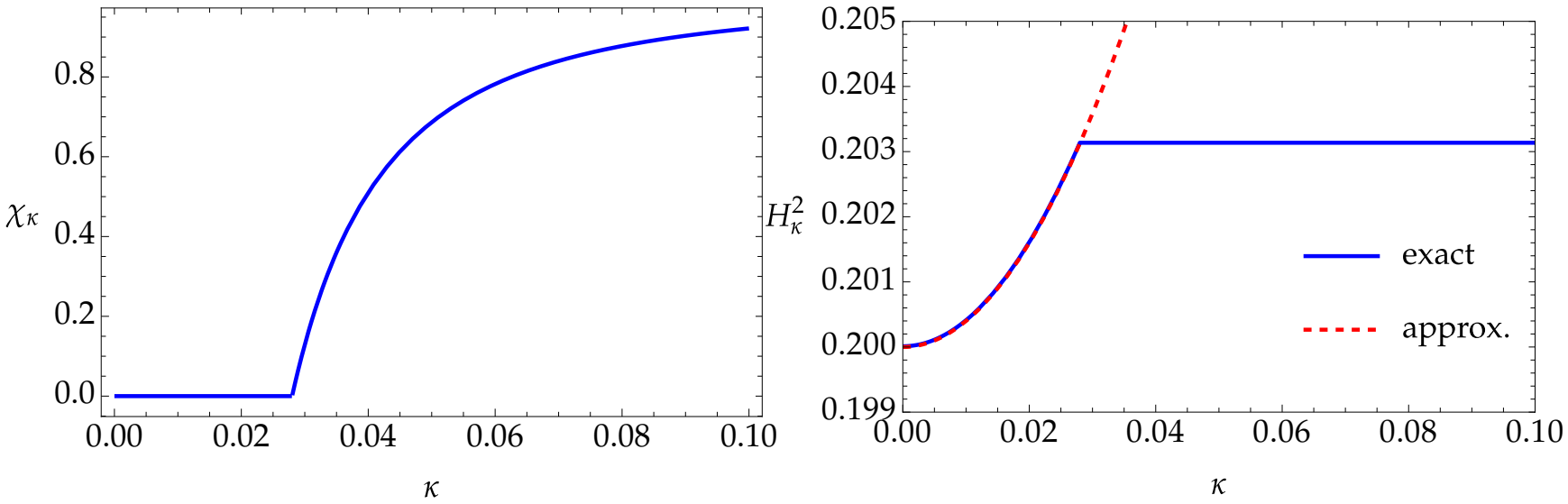


At one loop, the tadpole diagram gives :

- self energy : $M_\kappa^2 = \kappa^2\left(1 + \frac{\lambda H_\kappa^4}{2\Omega\kappa^4} + \mathcal{O}(\lambda^2)\right)$
- expansion parameter $\propto \frac{\lambda H^4}{\kappa^4}$
- $H_\kappa^2 = \frac{4\alpha}{\beta} + \frac{H_\kappa^4}{\beta\Omega}\left(1 + \frac{\kappa^2}{M_\kappa^2}\right)$
- instability

Broken phase

$$\chi_\kappa = -\frac{H_\kappa^4}{2\Omega\kappa^2} - \frac{m^2 + \zeta H_\kappa^2}{\lambda}, \quad 4\alpha - \beta H_\kappa^2 - \frac{2m^2\left(m^2 + \zeta H_\kappa^2\right)}{\lambda} + \frac{2H_\kappa^4}{\Omega} = 0$$



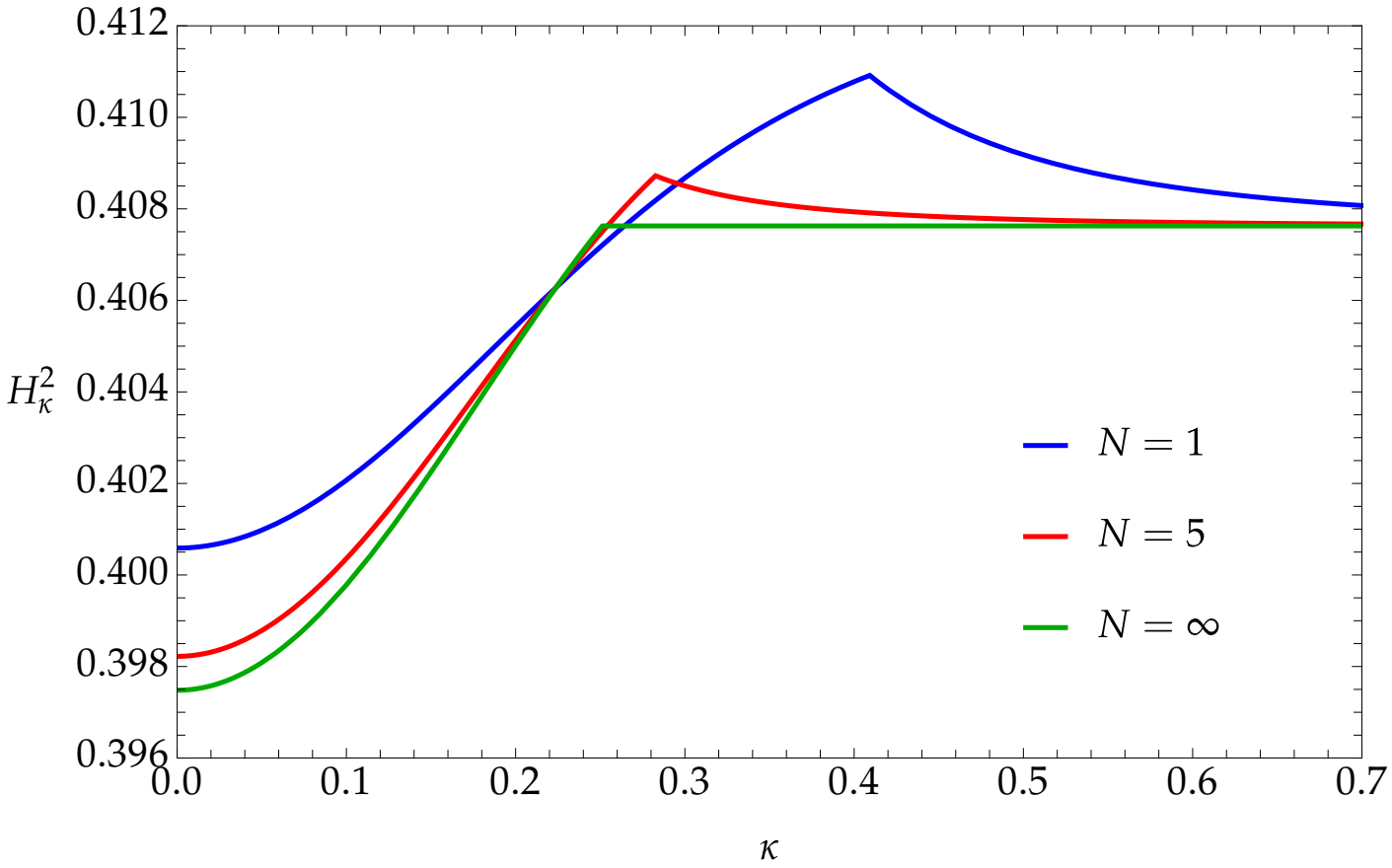
- **the symmetry is always restored**

  Serreau '11 ; '14

- **the Goldstone bosons do not renormalize H_κ!**

Broken phase

The contribution from the longitudinal mode is more and more important for decreasing N



Conclusion

The interactions of the scalar fields lead to a decrease in the value of H

The nonperturbative generation of a mass saturates the flow around a finite value $\kappa \lesssim \sqrt{\lambda H_{\rm cl}^4}$

The overall effect is controlled by a small parameter $H_{\rm cl}/M_{\rm Pl}$

For a symmetry breaking microscopic potential, the Goldstone modes do not renormalize H

- A similar result has been obtained using holographic methods (not limited by semiclassical approximation)
  J. K. Ghosh et al. '20
- This calculation is not entirely conclusive on de Sitter stability : metric fluctuations or less symmetric quantum states are not considered here

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Stochastic formalism

A. A. Starobinsky '86; A. A. Starobinsky, J. Yokoyama '94

We use rescaled variables (in $D = d + 1$ spacetime dimension)

$$\phi \to \sqrt{\frac{2H^d}{d\Omega_{D+1}}}\phi, \quad V \to \frac{2H^D}{\Omega_{D+1}}V$$

Introduce the coarse grained field φ

$$\phi_a(t,\vec{x}) = \underbrace{\varphi_a(t,\vec{x})}_{\text{long-wavelength}} + \underbrace{\int \frac{\mathrm{d}^d k}{(2\pi)^d}\theta(k - \varepsilon a(t)H)\left(\varphi_{a,k}(t)e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}} + \text{h.c.}\right)}_{\text{short-wavelength}}$$

In the Bunch-Davies vacuum, light mass limit and slow roll, φ behaves classically and verifies a stochastic equation

$$\partial_t \varphi_a + \partial_{\varphi_a} V = \xi_a \quad \text{with} \quad \langle \xi_a(t)\xi_b(t')\rangle = \delta_{ab}\delta(t - t')$$

$\xi \to$ UV modes exiting the horizon (separation in terms of physical scales)

Fokker-Planck equation

In terms of $P(\varphi_a, t)$ the probability distribution function (PDF),

$$\partial_t P = \partial_{\varphi_a}\left((\partial_{\varphi_a} V)P + \frac{1}{2}\partial_{\varphi_a} P\right)$$

Equilibrium distribution :

$$P_{\text{eq}}(\varphi_a) = \frac{e^{-2V(\varphi_a)}}{\mathcal{N}_{\text{eq}}}$$

One-point functions, $\langle \varphi^n \rangle$, can be computed analytically from the equilibrium distribution P_{eq}.

Motivations

Unequal-time correlators :
Correlation of operators at different spacetime point (here on a single Hubble patch at different times)
Example : $\langle \varphi(t)\varphi(t')\rangle$, $\langle \varphi^2(t)\varphi^2(t')\rangle$, ...

→ Numerical resolution is possible

A. A. Starobinsky, J. Yokoyama '94 ; T. Markkanen et al. '19 , '20

→ We investigate alternative ways of computing them to get some analytical result

→ They give information about

- spectral indices
- relaxation time to the stationary state
- decoherence properties

Motivations

In the following : **understanding the properties of the unequal-time correlators in the nonperturbative regime**

Different approaches, using the effective stochastic description as a starting point :

- → Using a path integral formulation of the Langevin equation
  - ☐ FRG
  - ☐ Schwinger-Dyson equations
- → Using the Fokker-Planck equation
  - ☐ $1/N$ expansion → main focus

Eigenvalue problem

$$\partial_t P = \partial_{\varphi_a}\left((\partial_{\varphi_a} V)P + \frac{1}{2}\partial_{\varphi_a} P\right) \underset{P=e^{-V}p}{\Rightarrow} \partial_t p = \left(-\frac{1}{2}\Delta_\varphi + W(\varphi_a)\right)p$$

where $W(\varphi_a) = \frac{1}{2}\left(V_{,aa} - V_{,a}^2\right)$

With eigenfunctions $\Psi_{n,\ell}$ and eigenvalues $\Lambda_{n,\ell}$,

$$\left(-\frac{1}{2}\Delta_\varphi + W(\varphi_a)\right)\Psi_{n,\ell} = \Lambda_{n,\ell}\Psi_{n,\ell}$$

The probability distribution functions reads

$$P(\varphi_a, t) = \underbrace{\frac{1}{\mathcal{N}_{\text{eq}}} e^{-2V(\varphi_a)}}_{\text{equilibrium distribution}} + e^{-V(\varphi_a)} \sum_{n\geq 1}\sum_{\ell=0}^{n} a_{n,\ell}\Psi_{n,\ell} e^{-\Lambda_{n,\ell} t}$$

In the non interacting case $\Lambda_n^{free} = nm^2$

Unequal time correlators

T. Markkanen et al. '19

For a given operator $\mathcal{A}$, the correlator $\langle \mathcal{A}(t)\mathcal{A}(t')\rangle$ can be expressed using the spectral decomposition

$$\langle \mathcal{A}(t)\mathcal{A}(t')\rangle = \sum_{n\geq 0}\sum_{\ell=0}^{n} C^{\mathcal{A}}_{n,\ell} e^{-\Lambda_{n,\ell}|t-t'|}$$

with the coefficients

$$C^{\mathcal{A}}_{n,\ell} = \left[\int \mathrm{d}^N\varphi \Psi_{0,0}(\varphi_a)\mathcal{A}\Psi_{n,\ell}(\varphi_a)\right]^2$$

Remark : the spectral index $n_{\mathcal{A}}$ of the corresponding operator at long time depends on the lowest contributing eigenvalue $\Lambda_{\mathcal{A}}$

$$n_{\mathcal{A}} - 1 \equiv \frac{\log \mathcal{P}_{\mathcal{A}}(k)}{\log k} = \frac{2}{H}\Lambda_{\mathcal{A}}$$

$1/N$ expansion of the FP equation

Using the generalized spherical harmonics, $Y_{\ell_i}(\theta_i)$,

$$\Psi_{n,\ell}(\varphi_a) = \mathcal{R}_{n,\ell}(x) Y_{\ell_i}(\theta_i) e^{-\Lambda_{n,\ell} t} \ ,$$

where $x = \sqrt{\varphi^2/N}$ and $|\ell_1| \leq \ell_2 \leq \cdots \leq \ell_{N-1} \equiv \ell$, we find the radial equation

$$-\frac{\mathcal{R}''_{n,\ell}}{2N} - \frac{N-1}{2Nx}\mathcal{R}'_{n,\ell} + \left[\frac{\ell(\ell+N-2)}{2Nx^2} + W\right]\mathcal{R}_{n,\ell} = \Lambda_{n,\ell}\mathcal{R}_{n,\ell}$$

Write explicitely the exponential factor $\mathcal{R}_{n,\ell}(x) = e^{-Nv(x)} r_{n,\ell}(x)$, where $V(\varphi_a) = Nv(x)$ to get

$$-\frac{r''_{n,\ell}}{2N} - \left(\frac{N-1}{2Nx} - v'\right) r'_{n,\ell} + \frac{\ell(\ell+N-2)}{2Nx^2} r_{n,\ell} = \Lambda_{n,\ell} r_{n,\ell}$$

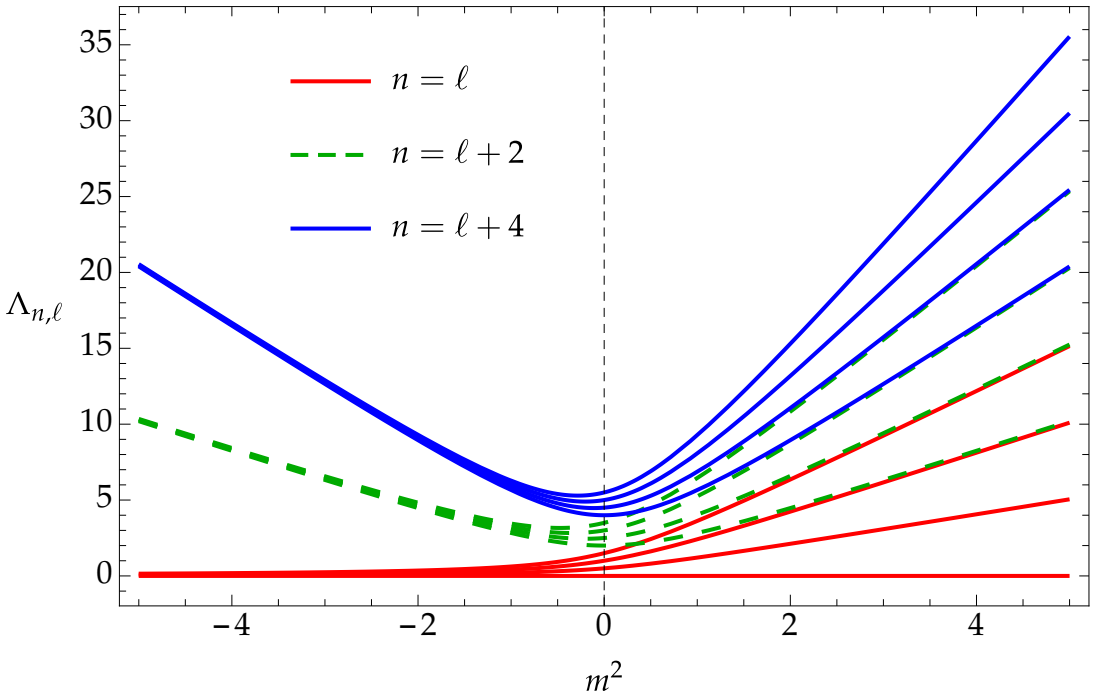
$1/N$ expansion from there gives analytical (and nontrivial) results

LO result

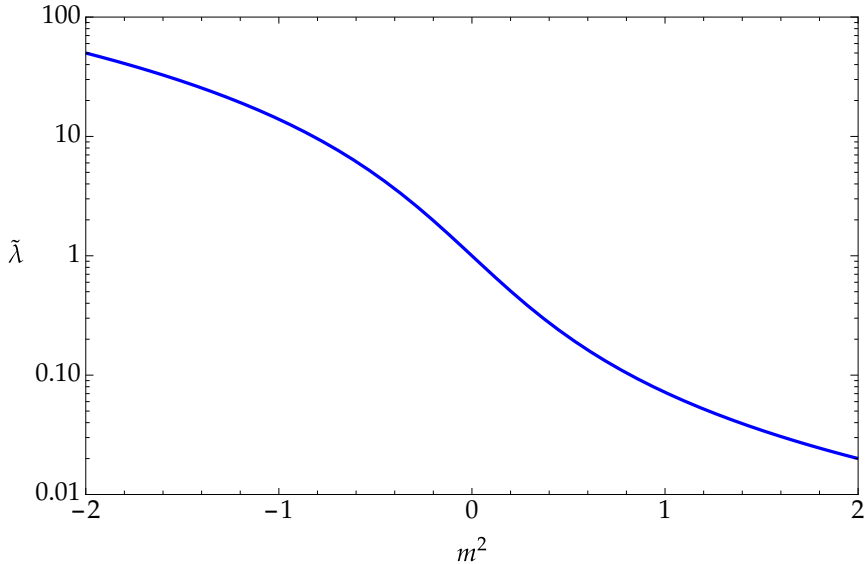
In particular, at LO, with $m_\pm^2 = \pm\frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{\lambda}{4}}$,

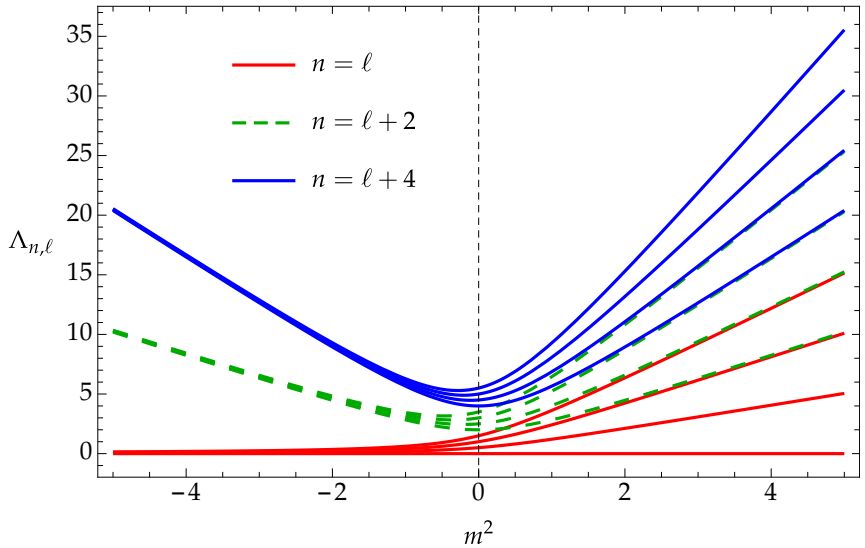
$$\Lambda_{n,\ell} = nm_+^2 + (n-\ell)m_-^2, \quad r_{n,\ell}(x) = a_0 x^\ell (1 - 2m_+^2 x^2)^{\frac{n-\ell}{2}} (1 - 2m_-^2 x^2)^{-\frac{n}{2}}$$

where $n - \ell$ is a positive even integer



Spectrum at LO



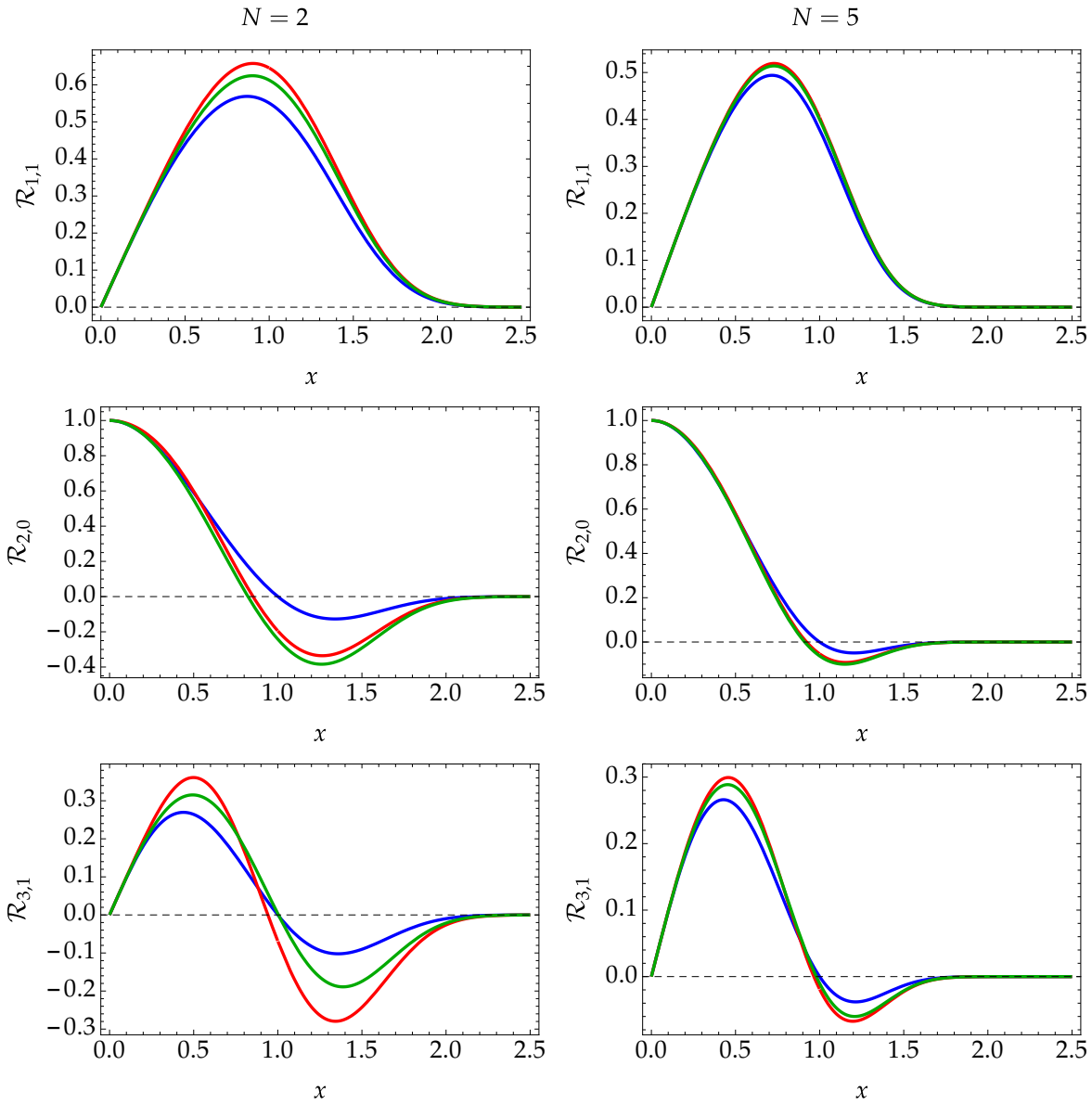


- Massless and deeply broken regime corresponds to highly nonperturbative behavior, $m_-^2 = -\tilde{\lambda} m_+^2$
- Gaussian spectrum (with different degeneracies) in the massless and deeply broken limit and fundamental frequency

$$\Lambda_{n,\ell} = (2n - \ell)\sqrt{\frac{\lambda}{4}}, \quad \Lambda_{n,\ell} = (n - \ell)\left|m^2\right|$$

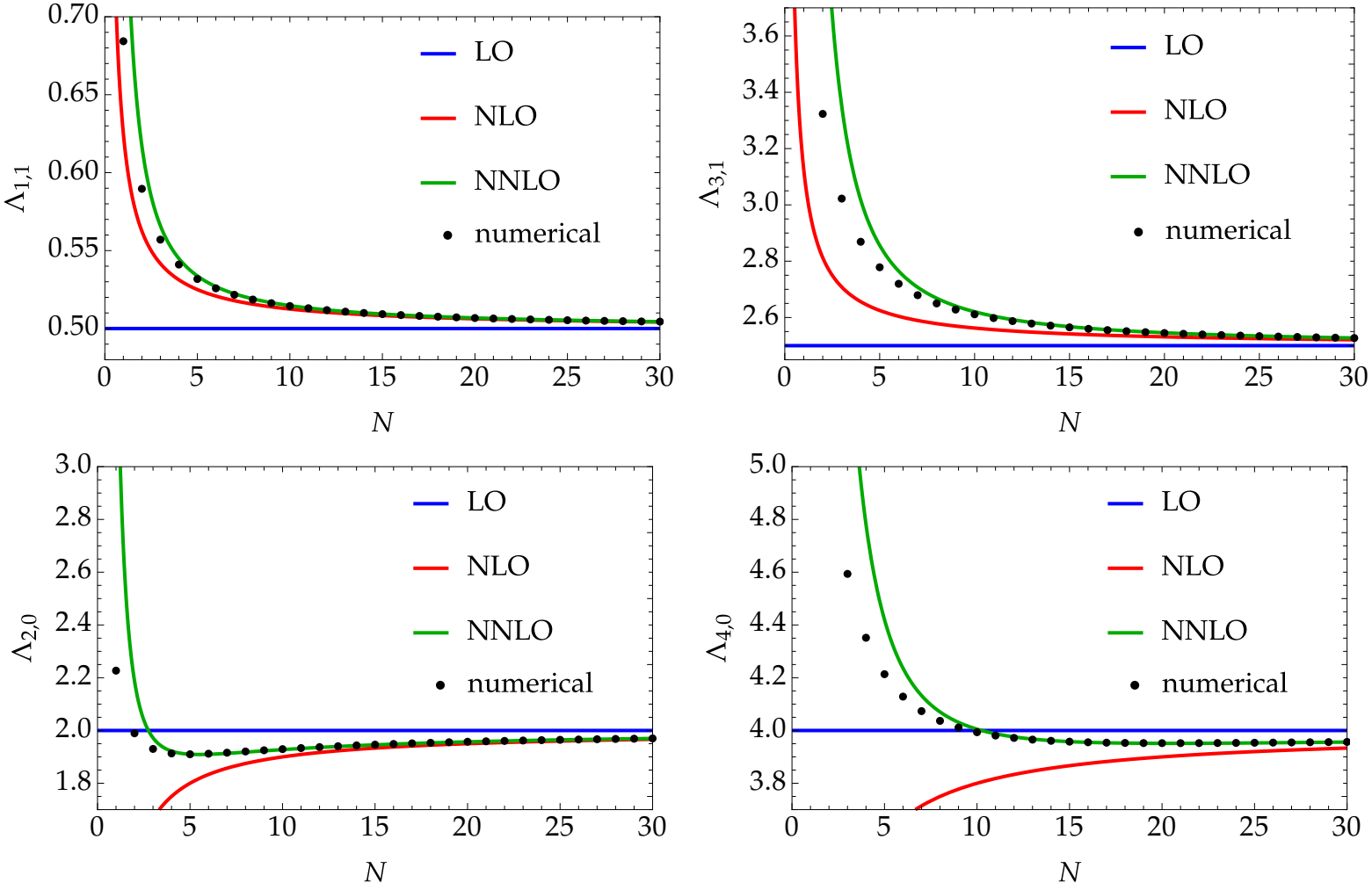
- In deeply broken limit, the scalar operators have a much smaller auto-correlation time in the deeply broken limit $\rightarrow$ **scalar sector does not couple to the Goldstone modes**

Comparison with finite N - massless case



Blue $\rightarrow$ LO
Red $\rightarrow$ NLO
Green $\rightarrow$ Numerical

Comparison with finite N - massless case



$$\frac{\Lambda_{n,\ell}}{\sqrt{\lambda}} = \frac{2n-\ell}{2}\left(1+\frac{3\ell-2}{4N}+\frac{5n^2-4\ell^2-\ell(5n-9)+2}{16N^2}+\mathcal{O}\left(\frac{1}{N^3}\right)\right)$$

Conclusion

The obtained results coincide with Lorentzian or Euclidean QFT comptations, showing the stochastic approach correctly captures such correlators

We can probe the deeply nonperturbative regime of the interacting theory using a $1/N$ expansion and going to the massless or negative square mass limit

We get analytical results for the autocorrelation time of any kind of two-point correlator

The result in the $1/N$ expansion is qualitatively good at LO and quantitatively at NLO (or NNLO) for the obtained correlators down to low values of N

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Backreaction :

- FRG is an interesting tool to study the backreaction problem along a renormalization flow
- Possible extension to less symmetric spacetime, in particular FLRW in slow-roll
- Or higher spin fields

Correlators :

- A more complete numerical analysis at finite N remains to be done
- The auto-correlation times are directly related to decoherence timescales in the early universe
- Possible applications to cosmological models with spectator fields to constraint such models and possibly find new physical effects